CHAPTER TWO

KINEMATICS OF PARTICLES

2/4 $\int v\,dv = \int a\,ds;\quad \int_{30/3.6}^{200/3.6} v\,dv = a\int_0^{600} ds$

200 km/h $= 200/3.6$ m/s

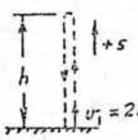
$a(600) = \frac{1}{2(3.6)^2}(\overline{30}^2 - \overline{200}^2)$

$\underline{a = -2.51\ \text{m/s}^2}$

2/5 $v^2 = v_1^2 + 2as;\quad v^2 = 4^2 + 2\,\frac{9.81}{6}(5)$

$v^2 = 32.4,\quad \underline{v = 5.69\ \text{m/s}}$

2/6



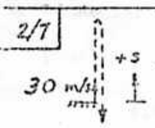
$a = -9.81\ \text{m/s}^2$

$v^2 = v_1^2 + 2as;\quad 0 = (200)^2 + 2(-9.81)h$

$\underline{h = 2040\ \text{m}}$

$v = v_1 + at;\quad -200 = 200 + (-9.81)t,\quad \underline{t = 40.8\ \text{s}}$

2/7



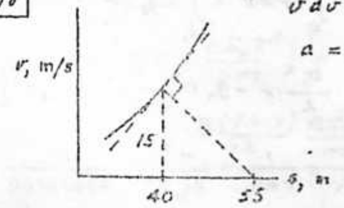
$\Delta s = v_1(t_2 - t_1) + \frac{1}{2}a(t_2^2 - t_1^2)$

$= 30(3-1) + \frac{1}{2}(-9.81)(3^2 - 1^2) = 20.8$ m

$\underline{\text{Particle rises } 20.8\ \text{m}}$

$v = v_1 + at;\quad v = 30 - 9.81(4) = \underline{-9.24\ \text{m/s}}$

2/8



$v\,dv = a\,ds$

$a = v\frac{dv}{ds} = 15\ \frac{55-40}{15}$

$= \underline{15\ \text{m/s}^2}$

2/9 $\Delta v = \int a\,dt = \int_0^4 (-5g)\,dt = -4(5)(9.81)$

$= -196.2$ m/s

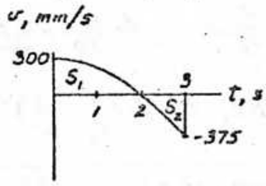
$v_{12} - 100 = -196.2,\quad \underline{v_{12} = -96.2\ \text{m/s}}$

2/10 $v\,dv = a\,ds;\quad v = a/\frac{dv}{ds} = -30/(\frac{-25}{50})$

$= \underline{60\ \text{m/s}}$

2/11 $S_1 = \int v\,dt = \int_0^2 (300 - 75t^2)\,dt = 300t - 25t^3\Big|_0^2$

$= 400$ mm



$S_2 = \int_2^3 (300 - 75t^2)\,dt$

$= 300t - 25t^3\Big|_2^3 = -175$ mm

Distance $D = 400 + 175 = \underline{575\ \text{mm}}$

Displacement $s = 400 - 175 = \underline{225\ \text{mm}}$

2/12 $v\,dv = a\,ds$ gives $a = \frac{1}{2}\frac{d(v^2)}{ds}$ constant

$a = \frac{1}{2}\,\frac{-900}{200} = -2.25\ \text{m/s}^2$

$v = v_1 + at;\quad 0 = 30 - 2.25t,\quad \underline{t = 13.33\ \text{s}}$

$v_{10.33} = 30 - 2.25(10.33) = 6.75$ m/s

$\Delta s = v_{av}\Delta t = \frac{6.75 + 0}{2}(3) = \underline{10.12\ \text{m}}$

2/13 $a = -Kv;\quad v\,dv = a\,ds;\quad v\,dv = -Kv\,ds$

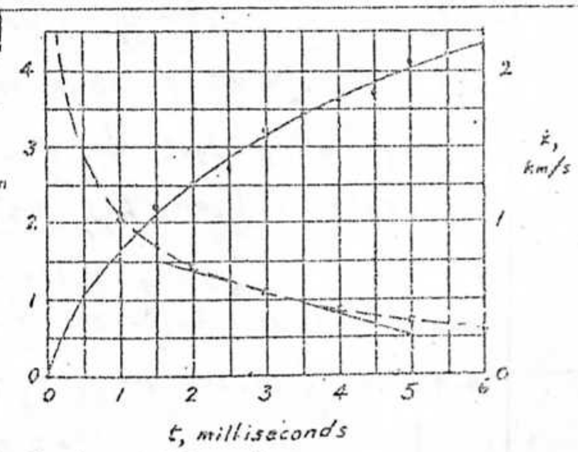
$\int_{v_0}^{0} dv = -K\int_0^S ds;\quad v_0 = KS,\quad \underline{S = v_0/K}$

$dv = a\,dt;\quad \int_{v_0}^{v_0/2}\frac{dv}{-Kv} = \int_0^t dt;\quad t = \frac{\ln 2}{K} = \underline{\frac{0.693}{K}}$

2/14 $a = \frac{dv}{dt} = -cv^n,\quad \int_{v_1}^{v}\frac{dv}{v^n} = -c\int_0^t dt$

$\frac{v^{1-n}}{1-n}\Big]_{v_1}^{v} = -ct,\quad \underline{v = \left[v_1^{\,1-n} + c(n-1)t\right]^{\frac{1}{1-n}}}$

2/15



at $t = 3$ ms, $v \doteq \underline{550\ \text{m/s}}$

$a \doteq -\frac{0.5(10^3)}{3.6(10^{-3})} = \underline{-140\ \text{km/s}^2}$

2/34 $v\,dv = a\,ds;\quad \frac{v\,dv}{-Kv^2} = ds,\ \int_{v_1}^{v_2}\frac{dv}{v} = -K\int_0^x dx$

$\ln\frac{v_2}{v_1} = -Kx,\quad K = \frac{1}{x}\ln\frac{v_1}{v_2} = \frac{1}{400}\ln\frac{160}{30} = \underline{4.18(10^{-3})\ \text{m}^{-1}}$

$\frac{dv}{dt} = -Kv^2,\ \int_{v_1}^{v_2}\frac{dv}{v^2} = -Kt,\ t = \frac{1}{K}\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$

$t = \frac{10^3}{4.18}\left(\frac{1}{30} - \frac{1}{160}\right)3.6 = \underline{23.3\ \text{s}}$

2/35 $v\,dv = a\,ds;\quad a = \frac{T - 4.50v^2}{m}$

$\int_0^v \frac{m\,v\,dv}{T - 4.50v^2} = \int_0^s ds;\quad s = \frac{m}{9.00}\ln\frac{T}{T - 4.50v^2}$

$s = \frac{16\,000}{9.00}\ln\frac{250}{250 - 4.50\left(14\,\frac{1.852}{3.6}\right)^2}$

$= 1\,778\ \ln 15.08 = 1\,778(2.714) = 4820\ \text{m or } 4.82\ \text{km}$

$s = \frac{4.82}{1.852} = \underline{2.60\ \text{mi (nautical)}}$

2/36 $a = \frac{d^2x}{dt^2} = Kt - k^2x,\quad \frac{d^2x}{dt^2} + k^2x = Kt$

$x = x_c + x_p = A\sin kt + B\cos kt + \frac{K}{k^2}t$

$\dot{x} = Ak\cos kt - Bk\sin kt + \frac{K}{k^2} = 0$ for $t = 0$

$0 = Ak - 0 + \frac{K}{k^2},\quad A = -K/k^3$

$x = 0$ when $t = 0$, so $0 = 0 + B + 0,\ B = 0$

Thus $\quad \underline{x = \frac{K}{k^3}\left(kt - \sin kt\right)}$

2/37 0–4 s; $\quad a = 5gt = \frac{dv}{dt};\ \int_0^v dv = \int_0^t 5gt\,dt$

$v = \frac{5}{2}gt^2;\quad v_4 = \frac{5}{2}g(4)^2 = 40g = 392\ \text{m/s}$

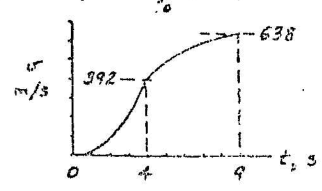
4–9 s; $\ a = 10g - 2gt'$ where t' counted from $t = 4$ s

$\frac{dv}{dt'} = 2g(5 - t'),\ \int_{40g}^{v} dv = 2g\int_0^{t'}(5 - t')dt'$

$v = 40g + 2g\left(5t' - \frac{t'^2}{2}\right)$

0–4 s; $\ s = \int v\,dt = \frac{5}{2}g\int_0^4 t^2dt = \frac{5}{2}(9.81)\frac{4^3}{3} = 523\ \text{m}$

4–9 s; $\ \Delta s = \int_0^5 g(40 + 10t' - t'^2)dt' = g\left(40t' + 5t'^2 - \frac{t'^3}{3}\right)_0^5$



$= 2780\ \text{m}$

Total $\ \underline{s = 3.30\ \text{km}}$

2/38 $\frac{dv}{dt} = ke^{-bt} - cv - g;\quad \frac{dv}{dt} + cv = ke^{-bt} - g$

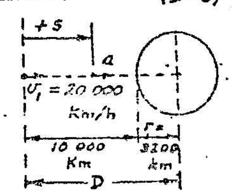
In the standard form for the solution of first order linear equation, $e^{-\int c\,dt} = e^{-ct}$

& $\ v = Ae^{-ct} + e^{-ct}\int\left(ke^{-bt} - g\right)e^{ct}dt$

$= Ae^{-ct} + \frac{k}{c-b}e^{-bt} - \frac{g}{c}$; $\ v = 0$ when $t = 0$ so $A = \frac{g}{c} - \frac{k}{c-b}$

Thus $\quad \underline{v = \frac{g}{c}\left(e^{-ct} - 1\right) + \frac{k}{c-b}\left(e^{-bt} - e^{-ct}\right)}$

2/39 $a = g_0\frac{r^2}{(D-s)^2}$ where $g_0 = 3.73\ \text{m/s}^2$ at surface



$\int_{v_1}^{v} v\,dv = \int_0^s a\,ds$

$\frac{1}{2}(v^2 - v_1^2) = \int_0^s g_0\frac{r^2}{(D-s)^2}ds$

$= g_0r^2\left(\frac{1}{D-s} - \frac{1}{D}\right)$

$v^2 = v_1^2 + 2g_0r^2\frac{s}{(D-s)D}$

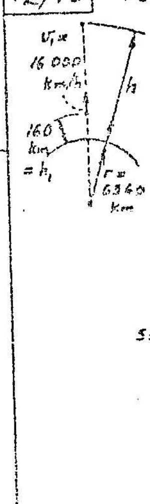
So for $s = 10\,000$ km,

$v^2 = (20\,000)^2 + 2(3.73)(3200)^2\frac{10\,000}{(3200)(13\,200)}\frac{(3600)}{1000}$

$= 400(10^6) + 234(10^6) = 634(10^6)$

$\underline{v = 25\,200\ \text{km/h}}$ at impact

2/40 $+s$ = away from earth; $\ a = -g = -g_0\frac{r^2}{(s+r)^2}$



$\int_{v_1}^{0} v\,dv = \int_{h_1}^{h} -g_0\frac{r^2}{(s+r)^2}ds$

$\frac{v_1^2}{2} = -g_0r^2\frac{1}{s+r}\Big|_{h_1}^{h} = g_0r^2\left[\frac{1}{h_1+r} - \right.$

$\frac{1}{r+h} = \frac{1}{r+h_1} - \frac{v_1^2}{2g_0r^2} = \frac{1}{6360+160} - \frac{(16\,00}{2(9.833}$

$= 1.534(10^{-4}) - 0.2483(10^{-4}) = 1.285$

So $\ 6360 + h = \frac{10^4}{1.285} = 7780\ \text{km},\ \underline{h = 1420}$

2/41 $\displaystyle\int_{v_1}^{v} dv = \int_{t_1}^{t} a\,dt\ ;\ v = v_1 + \frac{eE_0}{m}\int_{\pi/4\omega}^{t} \sin\omega t\,dt$

$$= v_1 + \frac{eE_0}{m\omega}\left(\frac{1}{\sqrt{2}} - \cos\omega t\right)$$

For $t = t_2 = \frac{3\pi}{4\omega}$, $v = v_2 = v_1 + \frac{eE_0}{m\omega}\sqrt{2}$

$\ell = v_2(t_3 - t_2) = \underline{\left(v_1 + \frac{eE_0\sqrt{2}}{m\omega}\right)\left(\frac{3\pi}{2\omega}\right)}$

$$s = \int_{t_1}^{t_2} v\,dt = \left(v_1 + \frac{eE_0}{m\omega\sqrt{2}}\right)\left(\frac{2\pi}{4\omega} - \frac{\pi}{4\omega}\right) - \frac{eE_0}{m\omega}\int_{\pi/4\omega}^{3\pi/4\omega}\cos\omega t\,dt$$

$$s = \underline{\frac{\pi}{2\omega}\left(v_1 + \frac{eE_0}{m\omega\sqrt{2}}\right)} + 0$$

2/47 $\alpha = \frac{d\omega}{dt}$, $\displaystyle\int_{\omega_0}^{0}\frac{d\omega}{-(b+c\omega^2)} = \int_0^t dt$

$$\frac{-1}{\sqrt{bc}}\tan^{-1}\frac{\omega\sqrt{bc}}{b}\Bigg]_{\omega_0}^{0} = t,\quad \underline{t = \frac{1}{\sqrt{bc}}\tan^{-1}\left(\omega_0\sqrt{\frac{c}{b}}\right)}$$

2/48 $\omega = 20(1 + 2t/3 - t^2/3)$

$\alpha = \dot\omega = 40/3 - 40t/3$

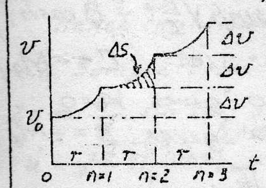
For $t = 3$ s, $\alpha = 40(1-3)/3 = \underline{-26.7 \text{ rad/s}^2}$

$$N = \frac{\theta}{2\pi} = \frac{1}{2\pi}\int\omega\,dt = \frac{1}{2\pi}\int_2^3 20\left(1 + \frac{2t}{3} - \frac{t^2}{3}\right)dt$$

$$= \frac{10}{\pi}\left[t + \frac{t^2}{3} - \frac{t^3}{9}\right]_2^3 = \frac{10}{\pi}\,\frac{5}{9} = \underline{1.768 \text{ rev}}$$

2/42 $\dot v = \frac{eE}{m} = \frac{eE_0}{m\tau}t$, $\Delta v = \displaystyle\int_0^{\tau}\frac{eE_0}{m\tau}t\,dt = \frac{eE_0\tau}{2m}$

(for each interval)



so $\underline{v_n = v_0 + \frac{neE_0\tau}{2m}}$

Due to Δv, $\Delta s = \displaystyle\int v\,dt = \int_0^{\tau}\frac{eE_0t^2}{2m\tau}dt = \frac{eE_0\tau^2}{6m}$

For arithmetic sum $S = a + (a+d) + (a+2d) + \cdots$

$S = n\frac{a+\ell}{2}$, ℓ = last term $= a + (n-1)d$

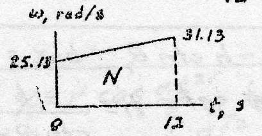
$d = \Delta v(\tau) = \frac{eE_0\tau^2}{2m}$; $S = na + \frac{n(n-1)}{2}d$

$$S = D = n\left(v_0\tau + \frac{eE_0\tau^2}{6m}\right) + \frac{n(n-1)}{2}\frac{eE_0\tau^2}{2m}$$

$$\underline{D = nv_0\tau + \frac{n(3n-1)}{12}\frac{eE_0\tau^2}{m}}$$

2/49 $\alpha = \frac{30}{60} = \frac{1}{2}$ rad/s², constant

$\omega = \omega_0 + \alpha t$, $\omega_{12} = 4(2\pi) + \frac{1}{2}(12) = 31.13$ rad/s



$N = \int\omega\,dt = \frac{25.13 + 31.13}{2}12$

$= 337.6$ rad

$N = \frac{337.6}{2\pi} = \underline{53.7 \text{ rev}}$

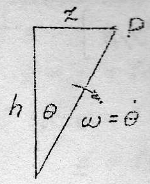
2/50 From $\omega\,d\omega = \alpha\,d\theta$, $\alpha = \frac{1}{2}\frac{d(\omega^2)}{d\theta}$

so $\alpha = \frac{1}{2}\,\frac{1600 - 900}{100(2\pi)} = 0.557$ rad/s²

$\Delta\omega = \int\alpha\,dt$, $\sqrt{1600} - \sqrt{900} = 0.557t$,

$t = \frac{40 - 30}{0.557} = \underline{17.95 \text{ s}}$

2/45 $x = h\tan\theta$, $\dot x = h\sec^2\theta\,\dot\theta$



$a = \ddot x = 2h\sec^2\theta\tan\theta\,\dot\theta^2 + 0$

$\underline{a = 2h\omega^2\sec^2\theta\tan\theta}$

2/51 $\omega = \omega_0 + k\theta = d\theta/dt$

$\displaystyle\int_0^{\theta}\frac{d\theta}{\omega_0 + k\theta} = \int_0^t dt$; $t = \frac{1}{k}\ln\frac{\omega_0 + k\theta}{\omega_0}$

$\frac{\omega_0 + k\theta}{\omega_0} = e^{kt}$, $\underline{\theta = \frac{\omega_0}{k}(e^{kt} - 1)}$

$\omega = \dot\theta = \underline{\omega_0 e^{kt}}$; $\alpha = \dot\omega = \underline{\omega_0 k e^{kt}}$

2/46 $N = \int\omega\,dt$ = area under curve

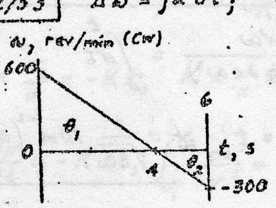
$= 5.4$ rev

2/52 $\alpha = \frac{d\omega}{dt} = \frac{1}{I}(M - k\omega^2)$

$$\int_0^{\omega}\frac{d\omega}{M - k\omega^2} = \frac{1}{I}\int_0^t dt,\quad \frac{I}{\sqrt{MK}}\tanh^{-1}\frac{\sqrt{MK}\,\omega}{M}\Bigg]_0^{\omega} = t$$

$$\frac{\sqrt{MK}}{M}\omega = \tanh\frac{\sqrt{MK}\,t}{I},\quad \underline{\omega = \sqrt{\frac{M}{k}}\tanh\frac{\sqrt{Mk}\,t}{I}}$$

2/53 $\Delta\omega = \int \alpha\,dt;\ -300 - \omega_1 = -150(6)$



$\omega_1 = 600$ rev/min CW

$\Delta\theta = \int \omega\,dt$

$\theta_1 = \frac{1}{2}\left(\frac{-600}{-150}\right)\frac{600}{60} = 20$ rev CW

$\theta_2 = \frac{1}{2}(2)\frac{300}{60} = 5$ rev CCW

$N = \theta_1 + \theta_2 = \underline{25}$ rev (CW + CCW)

2/54 During deceleration $\omega = 120 - 6\theta$ rev/min

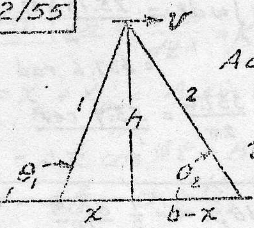
$\omega\,d\omega = \alpha\,d\theta,\ \omega\frac{d\omega}{d\theta} = \alpha$ but $d\omega = -6\,d\theta$

so $\omega(-6) = \alpha = \frac{d\omega}{dt}$. Thus $\int_{120}^{60}\frac{d\omega}{\omega} = -6\int_0^t dt$

$\ln\frac{120}{60} = 6t,\ t = \frac{1}{6}\ln 2 = \frac{0.6931}{6} = 0.1155$ min

or $t = 0.1155(60) = \underline{6.93\text{ s}}$

2/55 $x = -h\,\text{ctn}\,\theta_1,\ \ b - x = h\,\text{ctn}\,\theta_2$

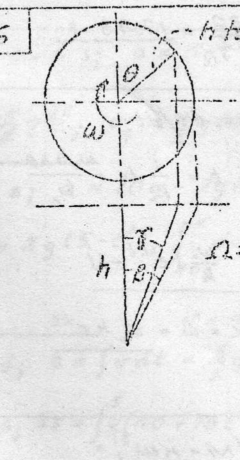


Add & get $h = \dfrac{b}{\text{ctn}\,\theta_2 - \text{ctn}\,\theta_1}$

$v = \dot{x} = h\csc^2\theta_1\,\dot{\theta}_1,\ \ \omega_1 = \dot{\theta}_1$

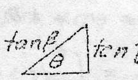
$v = \underline{\dfrac{b\omega_1\csc^2\theta_1}{\text{ctn}\,\theta_2 - \text{ctn}\,\theta_1}}$

2/56



$-h\tan\beta;\ h\tan\gamma = h\tan\beta\sin\theta$

$\sec^2\gamma\,\dot{\gamma} = \tan\beta\cos\theta\,\dot{\theta}$



$\cos\theta = \dfrac{\sqrt{\tan^2\beta - \tan^2\gamma}}{\tan\beta}$

& $\omega = \dot{\theta}$

$\Omega = \dot{\gamma} = \underline{\omega\cos^2\gamma\sqrt{\tan^2\beta - \tan^2\gamma}}$

2/57 $\alpha = \ddot{\theta} = -K\theta$, General solution [of]

diff. eq. $\ddot{\theta} + K\theta = 0$ is

$\theta = A\sin bt + B\cos bt$ where A, [B]

are constants. Substitute & get

$-Ab^2\sin bt - Bb^2\cos bt + KA\sin bt + KB\cos$ [bt]

Which requires $A(K - b^2) = 0,\ b = \sqrt{K}$.

$\theta = \theta_0,\ \dot{\theta} = 0,\ t = 0$ so $\theta = A\sqrt{K},\ A = 0$

$\theta_{max} = \theta_0$ when $t = 0$, so $\theta_0 = B$ & $\theta = \theta_0\cos\sqrt{K}t$

$\omega = \dot{\theta} = -\theta_0\sqrt{K}\sin\sqrt{K}t,\ \ \underline{|\omega|_{max} = \theta_0\sqrt{K}}$

2/58 $\alpha = \ddot{\theta} = \frac{M_0}{I} + \frac{K}{I}\theta,\ \ \frac{d^2\theta}{dt^2} - \frac{K}{I}\theta = \frac{M_0}{I}$

Solution is $\theta = A\sinh\sqrt{\frac{K}{I}}t + B\cosh\sqrt{\frac{K}{I}}t$ [+ C]

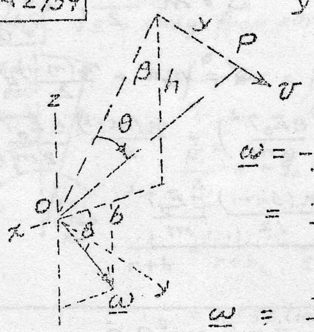
Substitute into D.E. - gives $C = -M_0/K$

Also $\dot{\theta} = 0$ when $t = 0$ gives $A = 0$

& $\theta = 0$ when $t = 0$ gives $B = -M_0/K$

Thus $\theta = \underline{\dfrac{M_0}{K}\left(\cosh\sqrt{\frac{K}{I}}\,t - 1\right)}$

***2/59**



$y = \sqrt{b^2 + h^2}\tan\theta$

$v = \dot{y} = \sqrt{b^2 + h^2}\sec^2\theta\,\dot{\theta}$

where $\dot{\theta} = \omega$

$\boldsymbol{\omega} = -\mathbf{i}\,\omega\cos\beta - \mathbf{k}\,\omega\sin\beta$

$= \dfrac{-v\cos^2\theta}{\sqrt{b^2 + h^2}}(\mathbf{i}h + \mathbf{k}b)\dfrac{1}{\sqrt{b^2 + h^2}}$

$\boldsymbol{\omega} = \underline{\dfrac{-v\cos^2\theta}{b^2 + h^2}(\mathbf{i}h + \mathbf{k}b)}$

[Note: $\omega_z \neq -v/b$ & $\omega_x \neq -v/h$]

2/63 $x = 2t^2 + 3t$, $\dot{x} = 4t + 3$, $\ddot{x} = 4$ m/s^2

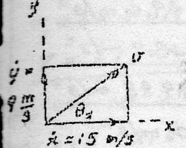
$y = t^3/3 - 8$, $\dot{y} = t^2$, $\ddot{y} = 2t$ m/s^2

When $t = 3$ s, $\dot{x} = 4(3) + 3 = 15$ m/s

$\dot{y} = (3)^2 = 9$ m/s

$\ddot{x} = 4$ m/s^2

$\ddot{y} = 2(3) = 6$ m/s^2

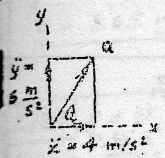


$v = \sqrt{(15)^2 + 9^2} = \underline{17.49 \text{ m/s}}$

with $\theta_x = \tan^{-1} \frac{9}{15} = \underline{31.0°}$

$a = \sqrt{4^2 + 6^2} = \underline{7.21 \text{ m/s}^2}$

with $\theta_x = \tan^{-1} \frac{6}{4} = \underline{56.3°}$



2/64 Angle θ for maximum range and hence minimum u is $45°$.

From Sample Prob. 2/60 range is

$2s = \frac{u^2}{g} \sin 2\theta$, so $10(10^3) = \frac{u^2}{9.81}$ (1)

$u = \sqrt{9.81(10^4)} = \underline{313 \text{ m/s}}$

2/65 Real coordinates $X = 0.25x$, $Y = 1.25y$, $T = 0.04t$

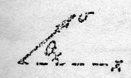
so $x = 4X$, $y = 0.8Y$, $t = 25T$

& $4X = 50 + 0.1(25T)^2$, $X = 12.5 + 15.62\,T^2$

$0.8Y = 50 + 0.0015(25T)^3/3 - 0.04(25T)^2/2$,

$Y = 62.5 + 29.3\,T^3/3 - 31.25\,T^2/2$

For actual motion $v_X = dX/dT = 31.25\,T$



$v_Y = dY/dT = 29.3\,T^2 - 31.25\,T$

$a_X = 31.25$ m/s^2

$\theta_x = \tan^{-1} \frac{v_Y}{v_X} = \underline{41.2°}$ $\quad a_Y = 58.6\,T - 31.25$

$T = 2$ s, $v_X = 62.5$ m/s, $v_Y = 54.7$ m/s

$a = \sqrt{31.25^2 + 85.9^2} = \underline{91.4 \text{ m/s}^2}$ $\quad a_X = 31.25$ m/s^2, $a_Y = 85.9$ m/s^2

2/66 $\underline{v} = \dot{\underline{r}} = (2t^2 - 3t)\underline{i} + \frac{t^3}{3}\underline{j}$

$\underline{a} = \dot{\underline{v}} = (4t - 3)\underline{i} + t^2\underline{j}$

$t = 3$ s, $\underline{v} = (18 - 9)\underline{i} + 9\underline{j} = 9\underline{i} + 9\underline{j}$ m/s

$\underline{a} = (12 - 3)\underline{i} + 9\underline{j} = 9\underline{i} + 9\underline{j}$ m/s^2

$\left.\begin{array}{l} \underline{v = 9\sqrt{2} \text{ m/s}} \\ \underline{a = 9\sqrt{2} \text{ m/s}^2} \end{array}\right\}$ $\underline{v}$ & $\underline{a}$ have same direction, so at $t = 3$ s particle is at inflection point on curve.

2/67 $y = 1 + x^2/10$, $\dot{y} = \frac{dy}{dx}\dot{x} = \frac{x}{5}\dot{x}$

But $x = 2t^3/3$, so $\dot{y} = \frac{2t^3}{15} 2t^2 = \frac{4}{15} t^5$

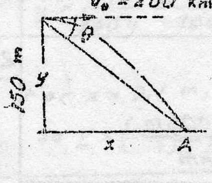
$\ddot{y} = \frac{4}{3} t^4 = \frac{4}{3} 2^4 = \frac{64}{3}$ m/s^2

$\dot{x} = 2t^2$, $\ddot{x} = 4t = 4(2) = 8$ m/s^2 for $t = 2$ s

$a = \sqrt{(64/3)^2 + 8^2} = \underline{22.8 \text{ m/s}^2}$

2/68 For y-motion; $a_y = g = 9.81$ m/s^2

$y = \frac{1}{2} g t^2$



For x-motion; $a_x = 0$, $x = v_0 t$

so $y = \frac{g}{2}\frac{x^2}{v_0^2}$ & $x = v_0\sqrt{\frac{2y}{g}}$

$x = \frac{200}{3.6}\sqrt{\frac{2(150)}{9.81}} = 307$ m

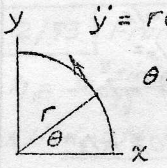
$\theta = \tan^{-1}\frac{150}{307} = \underline{26.0°}$

2/69 $x = r\cos\frac{\alpha t^2}{2}$, $\dot{x} = -r\alpha t \sin\frac{\alpha t^2}{2}$

$\ddot{x} = -r\alpha \sin\frac{\alpha t^2}{2} - r\alpha^2 t^2 \cos\frac{\alpha t^2}{2}$

$y = r\sin\frac{\alpha t^2}{2}$, $\dot{y} = r\alpha t \cos\frac{\alpha t^2}{2}$

$\ddot{y} = r\alpha \cos\frac{\alpha t^2}{2} - r\alpha^2 t^2 \sin\frac{\alpha t^2}{2}$



$\theta = \frac{1}{2}\alpha t^2$, α = angular accel. of radial line

(a) $t = 0$, $\underline{a_x = \ddot{x} = 0}$; $\underline{a_y = \ddot{y} = r\alpha}$

(b) $\alpha t^2/2 = \pi$, $a_x = \ddot{x} = \underline{2\pi r\alpha}$, $a_y = \ddot{y} = \underline{-r\alpha}$

2/70 $\ddot{y} = 8t$, $\dot{y} = 4t^2 + C_1$; $\dot{y} = 2$ m when $t = 0$,

so $C_1 = 2$ m &

$\dot{y} = 4t^2 + 2$

$\ddot{x} = 4t$, $\dot{x} = 2t^2 + C_2$; $\dot{x} = 0$ when $t = 0$, so $C_2 = 0$

$x = 2t^3/3 + C_3$; $x = 0$ when $t = 0$, so $C_3 = 0$

Eliminate t & get $y = 4\sqrt[3]{\frac{9x^2}{4}} + 2$

or $\underline{(y - 2)^3 = 144\,x^2}$

When $x = 18$ m, $\frac{2t^3}{3} = 18$, $t^3 = 27$, $t = 3$ s

$\dot{x} = 2(9) = 18$ m/s

$\dot{y} = 8(3) = 24$ m/s

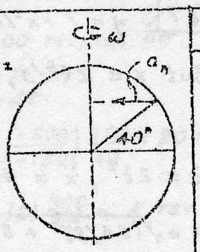
$v = \sqrt{18^2 + 24^2} = \underline{30 \text{ m/s}}$

2/71

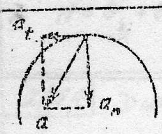
$a = a_n = r\omega^2 = R\cos\gamma\,\omega^2$

$= \frac{12.742}{2}(10^6)\cos 40^\circ (0.729)^2(10^{-4})^2$

$= \underline{0.0259\ m/s^2}$



2/72



$a_n = \sqrt{a^2 - a_t^2} = \sqrt{100^2 - 60^2}$

$= 80\ m/s^2$

$a_n = v^2/r,\ r = \frac{v^2}{a_n} = \frac{10^2}{80} = \underline{1.25\ m}$

2/73

$a_n = g = 9.81\ m/s^2$

$a_n = v^2/\rho,\ \rho = \frac{v^2}{a_n} = \frac{\overline{620}^2}{9.81} = \underline{39\ 200\ m}$ or $\underline{39.2\ km}$

2/74

$a_n = v\dot\theta = g,\ \dot\theta = \frac{9.81}{720\times10^3/3600} = 0.0491\ \frac{rad}{s}$

or $\dot\theta = 0.0491\ \frac{180}{\pi} = \underline{2.81\ deg/s}$

2/75

For A; $\omega^2 = \omega_1^2 + 2\alpha\theta,\ \alpha = \frac{\overline{120}^2 - \overline{20}^2}{2(5)}\ \frac{2\pi}{(60)^2}$

$= 2.44\ rad/s^2$

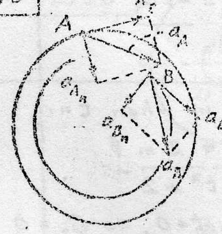
For B; $v_B = v_A = r_A\omega_A = 0.1\ \frac{30(2\pi)}{60} = 0.314\ m/s$

$a_n = v^2/r = \overline{0.314}^2/0.15 = 0.658\ m/s^2$

$a_{t_B} = a_{t_A} = r_A\alpha_A = 0.1(2.44) = 0.244\ m/s^2$

$a_P = \sqrt{a_n^2 + a_t^2} = \sqrt{\overline{0.658}^2 + \overline{0.244}^2} = \underline{0.702\ m/s^2}$

2/76



Given: $a_{A_t} = 1\ m/s^2$

$a_{B_n} = 0.6\ m/s^2$

$a_t = r\alpha;\ \alpha = 1/0.2 = 5\ rad/s^2$

$a_{B_t} = 0.15(5) = 0.75\ m/s^2$

$a_n = r\omega^2;\ \omega = \sqrt{0.6/0.15} = 2\ \frac{rad}{s}$

$v = r\omega,\ v_A = 0.2(2) = \underline{0.4\ m/s}$

$r_A = 0.2\ m,\ r_B = 0.15\ m$ $\quad a_B = \sqrt{\overline{0.6}^2 + \overline{0.75}^2} = \underline{0.960\ m/s^2}$

2/77

$\underline{r} = r\underline{r}_1\ ;\ \underline{v} = \dot{\underline{r}} = r\dot{\underline{r}}_1\ ;$ but $\dot{\underline{r}}_1 = \dot\theta\underline{\theta}_1$

so $\underline{v} = r\dot\theta\underline{\theta}_1\ ;\ \underline{a} = \ddot{\underline{r}} = \dot{\underline{v}} = r\dot\theta\dot{\underline{\theta}}_1 + r\dot\theta(-\underline{r}_1\dot\theta)$

But $\ddot\theta = 0$, so $\underline{a} = -r\dot\theta^2\underline{r}_1$

2/78

$|\Delta \underline{v}| = 2v\sin\frac{\Delta\theta}{2} = 20\sin\frac{\Delta\theta}{2}$

$\Delta t = \frac{\Delta s}{v} = \frac{r\Delta\theta}{v} = \frac{2\Delta\theta}{10} = \frac{\Delta\theta}{5}$

$(a_n)_{AV} = \frac{|\Delta v|}{\Delta t} = 100\ \frac{\sin\Delta\theta/2}{\Delta\theta}$

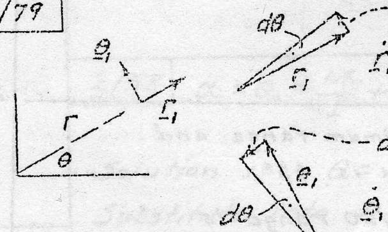
$30^\circ,\ \Delta\theta = 0.5236\ rad,\ a_{n_{AV}} = 100\ \frac{0.2588}{0.5236} = \underline{49.4\ m/s^2}$

$15^\circ,\ \Delta\theta = 0.2618\ rad,\ a_{n_{AV}} = 100\ \frac{0.1305}{0.2618} = \underline{49.9\ m/s^2}$

$5^\circ,\ \Delta\theta = 0.0873\ rad,\ a_{n_{AV}} = 100\ \frac{0.0436}{0.0873} = \underline{49.98\ m/s^2}$

Instantaneous $a_n = v^2/r = 100/2 = 50\ m/s^2$

2/79



$d\underline{r}_1 = |\underline{r}_1|\,d\theta\,\underline{\theta}_1$

$\dot{\underline{r}}_1 = \frac{d\underline{r}_1}{dt} = \frac{d\theta}{dt}\underline{\theta}_1 = \underline{\underline{\theta}_1\dot\theta}$

$d\underline{\theta}_1 = |\underline{\theta}_1|\,d\theta(-\underline{r}_1)$

$\dot{\underline{\theta}}_1 = \frac{d\underline{\theta}_1}{dt} = \frac{d\theta}{dt}(-\underline{r}_1) = \underline{-\underline{r}_1\dot\theta}$

2/80

$a_r = \ddot r - r\dot\theta^2,\ a_\theta = r\ddot\theta + 2\dot r\dot\theta$

$\dot\theta = 80\ \frac{2\pi}{60} = 8.38\ rad/s$

$\ddot\theta = -280\ \frac{2\pi}{60} = -29.3\ rad/s^2$

$r = 0.25\ m$, $\dot r = -0.3\ m/s$, $\ddot r = 0$ $\quad \underline{a} = a_r\underline{r}_1 + a_\theta\underline{\theta}_1$

so $\underline{a} = (0 - 0.25(8.38)^2)\,\underline{r}_1 + (0.25[-29.3] + 2[-0.3][8.38])\,\underline{\theta}_1$

$= \underline{-17.55\,\underline{r}_1 - 12.36\,\underline{\theta}_1\ m/s^2}$

2/81

$a_r = \ddot r - r\dot\theta^2;\ 0 = \ddot r - 0.25(4)^2,\ \underline{\ddot r = 4\ m/s^2}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta;\ 0 = 0.25(16) + 2\dot r(4),\ \underline{\dot r = -0.5\ m/s}$

2/82

For $\theta = 0$, $a_r = \ddot r - r\dot\theta^2$ where $\dot\theta = v/r$

$a_n = v^2/\rho$

But $a_r = -a_n$, so $\ddot r - v^2/r = -v^2/\rho$

$\ddot r = v^2\left(\frac{1}{r} - \frac{1}{\rho}\right) = \underline{-v^2\left(\frac{1}{\rho} - \frac{1}{r}\right)}$

2/83 $r = 2t^2$, $\dot r = 4t$, $\ddot r = 4\ \text{m/s}^2$

$\theta = 0.4\sin\frac{\pi t}{3}$, $\dot\theta = \frac{4\pi}{30}\cos\frac{\pi t}{3}$, $\ddot\theta = -\frac{4\pi^2}{90}\sin\frac{\pi t}{3}$

For $t = 2$ s, $r = 8$ m, $\dot r = 8$ m/s, $\ddot r = 4\ \text{m/s}^2$

$\dot\theta = \frac{4\pi}{30}\left(-\frac{1}{2}\right) = -0.209$ rad/s, $\ddot\theta = -\frac{4\pi^2\sqrt3}{90\cdot 2} = -0.380\ \text{rad/s}^2$

$v_r = \dot r = 8$ m/s; $v_\theta = r\dot\theta = 8(-0.209) = -1.676$ m/s

$\underline{v} = 8\underline{e}_r - 1.676\,\underline{e}_\theta$ m/s

$a_r = \ddot r - r\dot\theta^2 = 4 - 8(-0.209)^2 = 3.65\ \text{m/s}^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 8(-0.380) + 2(8)(-0.209) = -6.39\ \text{m/s}^2$

$\underline{a} = 3.65\,\underline{e}_r - 6.39\,\underline{e}_\theta\ \text{m/s}^2$

2/84 $\dot r \neq v$, $\dot{\underline r} \neq \underline v$, $\ddot r \neq a$, $\ddot{\underline r} \neq \underline a$ since scalar $\neq$ vector

$\dot r \neq v$ since $v \neq (v_r = \dot r)$

$\ddot r \neq a$ since $\ddot r$ is only one part of magnitude of a_r

$\dot{\underline r} \neq \dot r\underline e_r$ since $\underline v = \dot{\underline r}$ also contains a θ-component

$\ddot{\underline r} \neq \ddot r\underline e_r$ since $\underline a = \ddot{\underline r}$ contains another r-component and also the θ-component

$\dot{\underline r} \neq r\dot\theta\underline e_\theta$, since $\dot{\underline r} = \underline v$ also contains an r-component

2/85 $a_\theta = r\ddot\theta + 2\dot r\dot\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot\theta)$

From graph, at $t = T$ s, $\frac{d}{dt}(r^2\dot\theta) \approx \frac{24-12}{10-4} = 2\ \text{m}^2/\text{s}^2$

For $r = 0.5$ m, $a_\theta = \frac{1}{0.5}(2) = \underline{4\ \text{m/s}^2}$

2/86 $r = b - c\cos\theta$, $\dot r = c\omega\sin\theta$, $\ddot r = c\omega^2\cos\theta$

$a_r = \ddot r - r\dot\theta^2 = c\omega^2\cos\theta - (b - c\cos\theta)\omega^2 = (2c\cos\theta - b)\omega^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2c\omega^2\sin\theta$

$a = \sqrt{a_r^2 + a_\theta^2} = \underline{\omega^2\sqrt{4c^2 - 4bc\cos\theta + b^2}}$

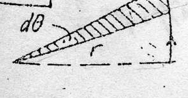
2/87 $a_n = g = v^2/\rho$, $\rho = \frac{v^2}{g} = \frac{(720)^2}{9.81} = 52\,800$ m $= \underline{52.8\ \text{km}}$

2/88 Horizontal velocity constant at 72 km/h = 20 m/s

At top of trajectory $a_n = \frac{v^2}{\rho} = g$,

$\rho = \frac{v^2}{g} = \frac{(20)^2}{9.81} = \underline{40.8\ \text{m}}$

2/89



$dA = \frac{1}{2}r\,d\theta\,(r) = \frac{1}{2}r^2 d\theta$

$\dot A = \frac{1}{2}r^2\dot\theta = $ constant since

$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot\theta) = 0$

2/90

$a_n = g\sin 30° = 7.32(0.5) = 3.66\ \text{m/s}^2$

or $a_n = \frac{3.66}{1000}(3600)^2 = 47\,400\ \text{km/h}^2$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{(40\,000)^2}{47\,400} = \underline{33\,700\ \text{km}}$

$\dot v = a_t = 7.62 - 7.32\frac{\sqrt3}{2} = \underline{1.28\ \text{m/s}^2}$

2/91 $x = 10y^2$, $\dot x = 20y\dot y$; but $\dot x = 0.1$ m/s const,

so $y\dot y = 1/200$, $\dot y^2 + y\ddot y = 0$, $\ddot y = -\frac{\dot y^2}{y}$ & $\ddot x = 0$

$\dot y = \frac{1}{200y} = \frac{1}{200}\sqrt{\frac{10}{x}}$; For $x = 0.1$ m, $\underline a = \ddot y\underline j = -\frac{10\sqrt{10}}{(200)^2 x\sqrt{x}}\underline j$

$= \underline{-0.0250\,\underline j\ \text{m/s}^2}$

For $x = 0.1$ m, $\dot y = \frac{1}{200}\sqrt{\frac{10}{0.1}} = 0.05$ m/s

so $\underline v = \left(20\sqrt{\frac{0.1}{10}}\,\underline i + \underline j\right)0.05 = \underline{0.05(2\underline i + \underline j)\ \text{m/s}}$

2/92 $\overline{OA} = r = 2b\cos\theta$, $\dot r = -2bK\sin\theta$, $\ddot r = -2bK^2\cos\theta$

where $K = \dot\theta = $ const.

$a_r = \ddot r - r\dot\theta^2 = -2bK^2\cos\theta - 2bK^2\cos\theta = -4bK^2\cos\theta$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 - 4bK^2\sin\theta$

$a = \sqrt{a_r^2 + a_\theta^2} = \underline{4K^2 b}$ independent of θ

2/93 From Sample Prob. 2/60,

$h = \frac{u^2\sin^2\theta}{2g}$, $R = 2s = \frac{u^2\sin 2\theta}{g}$

Eliminate θ; $\sin\theta = \frac{\sqrt{2gh}}{u}$ so $\cos\theta = \sqrt{1 - \frac{2gh}{u^2}}$

Thus $R = \frac{2u^2}{g}\sin\theta\cos\theta = \frac{2u^2}{g}\frac{\sqrt{2gh}}{u}\sqrt{1 - \frac{2gh}{u^2}}$

Simplify & get $\underline{R = 4h\sqrt{\frac{u^2}{2gh} - 1}}$

2/94 $r = r_0 e^{n\theta}$, $\dot r = r_0 n\dot\theta e^{n\theta} = r_0 nKe^{n\theta}$

$\ddot r = r_0 n^2K^2 e^{n\theta} = rn^2K^2$

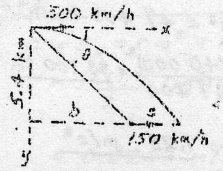
$a_r = \ddot r - r\dot\theta^2 = rn^2K^2 - rK^2 = (n^2-1)rK^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2rnK^2$

$a = \sqrt{a_r^2 + a_\theta^2} = rK^2\sqrt{(n^2-1)^2 + 4n^2} = \underline{RK^2(n^2+1)}$, $r = R$

2/95

Time of drop, $h=\frac{1}{2}gt^2$



$t=\sqrt{2(5400)/9.81}=33.2\ \text{s}$

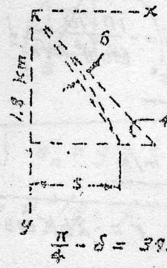
$s=vt=\frac{150}{3.6}33.2=1382\ \text{m}$

$b+s=ut=\frac{500}{3.6}33.2=4610\ \text{m}$

$\theta=\tan^{-1}\frac{5400}{4610-1382}=\underline{59.2°}$

2/96

$y=ut\cos45°+\frac{1}{2}gt^2;\ u=\frac{1000}{3.6}=278\ \text{m/s}$



$1800=278(0.707)t+\frac{1}{2}9.81t^2$

solve & get $t=7.69$ s (or $t=-47.7$ s)

$s=ut\sin45°=278(7.69)(0.707)=1510\ \text{m}$

$\tan(\frac{\pi}{4}-\delta)=\frac{1510}{1800}=0.8389$

$\frac{\pi}{4}-\delta=39.99°,\ \delta=45°-39.99°=\underline{5.01°}$

2/97

$a_n=g=\frac{v^2}{\rho}$ where $g=g_0\left(\frac{R}{R+h}\right)^2$

$\rho=\frac{1}{g_0}\left(\frac{R+h}{R}\right)^2v^2=\frac{1}{9.821(10^{-3})}\left(\frac{\frac{12742}{2}+400}{12742/2}\ \frac{32000}{3600}\right)^2=\underline{9090\ \text{km}}$

2/98

$a_n=\frac{v^2}{\rho}=\frac{v^2}{R+h}$ where $R=1738$ km

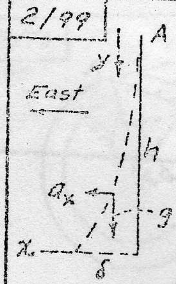
$g_0=1.62\ \text{m/s}^2$

$a_n=g=g_0\left(\frac{R}{R+h}\right)^2=1.62\left(\frac{1738}{1738+160}\right)^2=1.359\ \text{m/s}^2$

$v^2=(1738+160)1.359(3600)^2(10^{-3})=33.42(10^6)\ (\text{km/h})^2$

$\underline{v=5781\ \text{km/h}}$

2/99



$a_y=g,\ v_y=gt$

$a_x=2v_y\omega\cos\gamma=2gt\omega\cos\gamma$

$\dot{x}=\int_0^t a_x\,dt=gt^2\omega\cos\gamma$

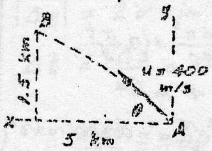
$x=\int_0^t\dot{x}\,dt=\frac{1}{3}gt^3\omega\cos\gamma=\delta$

But time of fall is given by $h=\frac{1}{2}gt^2$

so $\delta=\left(\frac{2h}{g}\right)^{3/2}\frac{g\omega}{3}\cos\gamma,\quad \underline{\delta=\frac{2\sqrt{2}}{3}\omega h\sqrt{\frac{h}{g}}\cos\gamma}$

2/100

Eq. of trajectory (Sample Prob. 2/6)



$y=x\tan\theta-\frac{gx^2}{2u^2}\sec^2\theta$

$=x\tan\theta-\frac{gx^2}{2u^2}(1+\tan^2\theta)$

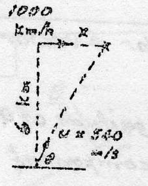
$1500=5000\tan\theta-\frac{9.81(5000)^2}{2(400)^2}(1+\tan^2\theta)$

or $\tan^2\theta-6.524\tan\theta+2.957=0$

Solution gives $\underline{\theta=26.1°}$ or $\underline{\theta=80.6°}$

2/101

Aircraft: $x=\frac{1000}{3.6}t$



Shell: $x=540t\cos\theta$

$6000=540t\sin\theta-\frac{9.81}{2}t^2$

From 1st two equations,

$\cos\theta=\frac{1000}{3.6(540)}=0.5144,\ \underline{\theta=59.0°}$

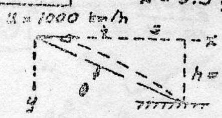
$\sin\theta=0.8575$

$6000=540(0.8575)t-\frac{9.81}{2}t^2$

or $t^2-94.41t+1223=0,\ \underline{t=15.50\ \text{s}}$ (& $t=78.9$ s)

2/102

$\ddot{x}=0.5g,\ \dot{x}=u+0.5gt,\ x=ut+0.25gt^2$



$\ddot{y}=g,\ \dot{y}=gt,\ y=\frac{1}{2}gt^2$

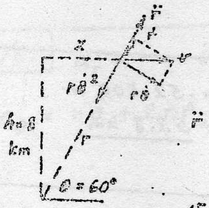
so $t=\sqrt{2h/g},\ s=u\sqrt{\frac{2h}{g}}+\frac{h}{2}$

$\theta=\tan^{-1}\frac{h}{u\sqrt{\frac{2h}{g}}+\frac{h}{2}}=\tan^{-1}\frac{1500}{\frac{1000}{3.6}\sqrt{\frac{3000}{9.81}}+\frac{1500}{2}}=\tan^{-1}0.2675$

$=\underline{14.98°}$

2/103

Acceleration in all directions is zero, so



$a_r=\ddot{r}-r\dot{\theta}^2=0,\ \ddot{r}=r\dot{\theta}^2$

$r=h/\sin\theta,\ r=8\frac{2}{\sqrt{3}}=9.24\ \text{km}$

$\ddot{r}=9.24(-0.025)^2=0.00577\ \text{km/s}^2$

$=\underline{5.77\ \text{m/s}^2}$

$\dot{\theta}=-0.025$

$v=|r\dot{\theta}|/\sin\theta=\frac{|h\dot{\theta}|}{\sin^2\theta}$ — $h\csc\theta$

$v=\frac{8000(0.025)}{(\sqrt{3}/2)^2}=267\ \text{m/s}=\underline{960\ \text{km/h}}$

2/104

$a_t=-ks;\ \int_{v_0}^{v_A}v\,dv=\int_0^s-ks\,ds,\ v_0^2=v_A^2+ks^2$

$144=36+k(324),\ k=\frac{1}{3}\ \text{s}^{-2};\ a_t=-\frac{1}{3}18=-6\ \text{m/s}^2$

$a_n=\sqrt{a^2-a_t^2}=\sqrt{10^2-6^2}=8\ \text{m/s}^2=v_A^2/\rho,\ \rho=\frac{36}{8}=\underline{4.5\ \text{m}}$

2/105 $v = 2 + 0.3t^2$, $a_t = \dot{v} = 0.6t = 0.6(2) = 1.2 \text{ m/s}^2$ for $t = 2$ s

$a^2 = a_t^2 + a_n^2$, $a_n^2 = \overline{2.4}^2 - \overline{1.2}^2 = 4.32$, $a_n = 2.078 \text{ m/s}^2$

$v_{t=2} = 2 + 0.3(2)^2 = 3.2$ m/s

$a_n = v^2/\rho$, $\rho = v^2/a_n = \overline{3.2}^2/2.078 = \underline{4.93 \text{ m}}$

2/106 $a_n = v^2/\rho = 6^2/2 = 18 \text{ m/s}^2$; $a_t = \sqrt{a^2 - a_n^2} = \sqrt{30^2 - 18^2} = 24 \text{ m/s}^2$

$a_t = \rho\ddot{\theta} + \dot{\rho}\dot{\theta}$, $\dot{\theta} = v/\rho = 6/2 = 3$ rad/s

$24 = 2(3) + \dot{\rho}(3)$, $\underline{\dot{\rho} = 6 \text{ m/s}}$

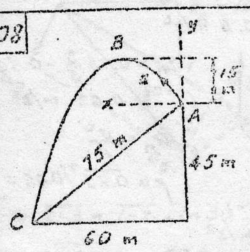
2/107 $r = K\theta$, $\dot{r} = K\dot{\theta}$, $\ddot{r} = K\ddot{\theta} = K\alpha$, $\dot{\theta}^2 = 2\int_{\pi/4}^{3\pi/4} \alpha\, d\theta = \alpha\pi$

$a_r = \ddot{r} - r\dot{\theta}^2 = K\alpha - \frac{3}{4}K\pi^2\alpha = K\alpha(1 - 3\pi^2/4)$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{3K\pi\alpha}{4} + 2K\pi\alpha = K\alpha(11\pi/4)$

$a = \sqrt{a_r^2 + a_\theta^2} = K\alpha\sqrt{1 + \frac{97}{16}\pi^2 + \frac{9}{16}\pi^4} = \underline{10.76\, K\alpha}$

2/108



y-dir., $v^2 = v_1^2 + 2as$

$s = v_1 t + \frac{1}{2}at^2$

$0 = u_y^2 - 2g(15)$ A to B

$u_y = \sqrt{30g} = 17.16$ m/s

$-45 = 17.16t - \frac{1}{2}9.81t^2$

$t = 5.25$ s (or $t = -1.749$ s)

x-dir., $60 = u_x(5.25)$, $u_x = 11.44$ m/s

$u = \sqrt{11.44^2 + 17.16^2} = \underline{20.6 \text{ m/s}}$

2/109 $r = r_0 + b_0 \sin 2\pi nt$, $\dot{r} = 2\pi b_0 n \cos 2\pi nt$

$\ddot{r} = -4\pi^2 b_0 n^2 \sin 2\pi nt$

$\dot{\theta} = \omega$, $\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2$, $a_r = -4\pi^2 b_0 n^2 \sin 2\pi nt - (r_0 + b_0 \sin 2\pi nt)\omega^2$

$= -(4\pi^2 n^2 + \omega^2) b_0 \sin 2\pi nt - r_0\omega^2$

$|a_r|_{max} = \underline{(4\pi^2 n^2 + \omega^2) b_0 + r_0\omega^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, $a_\theta = 0 + 4\pi b_0 n\omega \cos 2\pi nt$

$|a_\theta|_{max} = \underline{4\pi b_0 n\omega}$

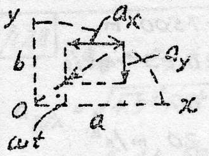
2/110 $x = a\cos\omega t$, $\dot{x} = -a\omega \sin\omega t$, $\ddot{x} = -a\omega^2 \cos\omega t$

$y = b\sin\omega t$, $\dot{y} = b\omega\cos\omega t$, $\ddot{y} = -b\omega^2 \sin\omega t$

$\cos^2\omega t + \sin^2\omega t = 1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$, an ellipse

$|a_x|_{max} = |\ddot{x}|_{max} = a\omega^2$ when $t = \frac{n\pi}{\omega}$, $n = 0, 1, 2, ----$

$|a_y|_{max} = |\ddot{y}|_{max} = b\omega^2$ when $t = \frac{n\pi}{2\omega}$, $n = 1, 3, 5, ---$

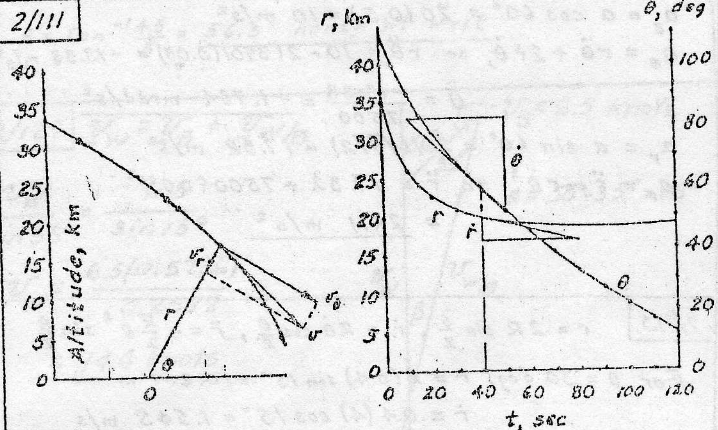


$\frac{a_y}{a_x} = \frac{-b\omega^2 \sin\omega t}{-a\omega^2\cos\omega t} = \frac{b}{a}\tan\omega t$

$\frac{y}{x} = \frac{b\sin\omega t}{a\cos\omega t} = \frac{b}{a}\tan\omega t$

Therefore accel. is directed toward O

2/111



$t = 40$ s, $r = 20\ 100$ m

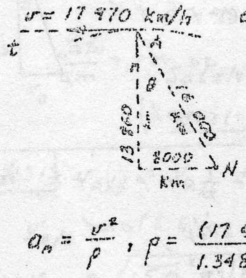
$\dot{r} = -77.5$ m/s

$\dot{\theta} = -0.014$ rad/s

$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2} = \sqrt{(77.5)^2 + (20\ 100)^2(0.014)^2} = 284$ m/s

or $\underline{v = 1020 \text{ km/h}}$

2/112 Accel. directed from A to N is



$a = g = g_0\left(\frac{R}{R+h}\right)^2$

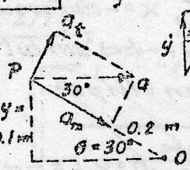
$= 9.821\left(\frac{6370}{16\ 000}\right)^2 = 1.557 \text{ m/s}^2$

$a_n = 1.557 \cos\theta = 1.557(0.866)$

$= 1.348 \text{ m/s}^2$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{(17\ 970)^2}{1.348(10^{-3})(3600)^2} = \underline{18\ 480 \text{ km}}$

2/113 $\dot y = 2$ m/s constant so $\ddot y = 0$



& accel. can have no vertical component

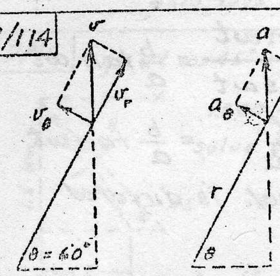
$v = \dot y/\cos 30° = 2/0.866 = 2.31$ m/s

$a_n = \dfrac{v^2}{r} = \dfrac{2.31^2}{0.2} = 26.7$ m/s²

$a_t = 26.7 \tan 30° = 15.40$ m/s²

$a_t = r\ddot\theta$, $\ddot\theta = 15.40/0.2 = \underline{77.0 \text{ rad/s}^2}$

2/114



Given $r = 7500$ m

$\dot\theta = 0.03$ rad/s

$\theta = 60°$

$a = 20$ m/s²

$v_\theta = r\dot\theta = 7500\,(0.03) = 225$ m/s

$v = 225/\cos 60° = 450$ m/s

$v_r = \dot r = 225 \tan 60° = 390$ m/s

$a_\theta = a\cos 60° = 20(0.5) = 10$ m/s²

$a_\theta = r\ddot\theta + 2\dot r\dot\theta$, so $r\ddot\theta = 10 - 2(390)(0.03) = -13.38$ m/s²

$\ddot\theta = \dfrac{-13.38}{7500} = \underline{-1.784 \text{ mrad/s}^2}$

$a_r = a\sin 60° = 20(\sqrt{3}/2) = 17.32$ m/s²

$a_r = \ddot r - r\dot\theta^2$, so $\ddot r = 17.32 + 7500\,(0.03)^2$

$= \underline{24.1 \text{ m/s}^2}$

2/115 $r = 2R\sin\frac{\theta}{2}$, $\dot r = R\dot\theta\cos\frac{\theta}{2}$, $\ddot r = -\frac{R}{2}\dot\theta^2\sin\frac{\theta}{2}$

For $\theta = 30$ deg, $r = 2(0.4)\sin 15° = 0.207$ m

$\dot r = 0.4(4)\cos 15° = 1.545$ m/s

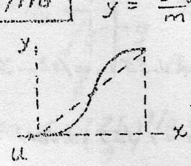
$\ddot r = -\frac{0.4}{2}(4)^2\sin 15° = -0.828$ m/s²

$a_r = \ddot r - r\dot\theta^2 = -0.828 - 0.207(4)^2 = -4.14$ m/s²

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(1.545)(4) = 12.36$ m/s²

$a = \sqrt{4.14^2 + 12.36^2} = \underline{13.04 \text{ m/s}^2}$

2/116 $\ddot y = \frac{eE_0}{m}\sin pt$, $\dot y = -\frac{eE_0}{mp}\cos pt + K$



$\dot y = 0$ when $t = 0$, so $K = +\frac{eE_0}{mp}$

& $\dot y = \frac{eE_0}{mp}(1 - \cos pt)$

$x = ut$

$y = \frac{eE_0}{mp}\left(t - \frac{1}{p}\sin pt\right) + k$, $k = 0$

For one complete cycle, $t = 2\pi/p$, and

$\underline{x = 2\pi u/p}$ & $\underline{y = \frac{eE_0}{mp^2}2\pi}$

2/117 $\ddot y = -g$, $\dot y = u\sin 45° - gt$, $y = ut\sin 45° - \frac{1}{2}gt^2$

$\ddot x = 0$, $\dot x = u\cos 45°$, $x = ut\cos 45°$

$\dfrac{dy}{dx} = \dfrac{\dot y}{\dot x} = 1 - \dfrac{gt}{u}\sqrt{2} = 1 - \dfrac{2gx}{u^2}$, $x = \dfrac{u^2}{2g}\left(1 - \dfrac{dy}{dx}\right)$

$\dfrac{dy}{dx} = \tan 20° = 0.3640$, $x = \dfrac{(15)^2(10^6)}{(3600)^2(2)(9)(10^{-3})}(1 - 0.364) = 613$ km

$t = \dfrac{x}{u\cos 45°} = \dfrac{613}{15\,000}\sqrt{2} = 0.0578$ h or $\underline{3 \text{ min } 28 \text{ sec}}$

$h = y = \dfrac{15(10^3)(0.0578)}{\sqrt{2}} - \dfrac{9(10^{-3})}{2}(0.0578)^2(3600)^2 = \underline{418 \text{ km}}$

2/118 Eq. of path: $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

Let $m = \tan\theta$; $\dfrac{1}{\cos^2\theta} = 1 + m^2$

Thus, $y = mx - \dfrac{gx^2}{2u^2}(1 + m^2)$

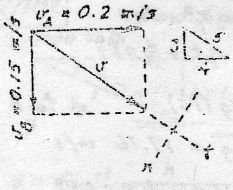
or $m^2 - \left(\dfrac{2u^2}{gx}\right)m + \left(1 + \dfrac{2u^2 y}{gx^2}\right) = 0$

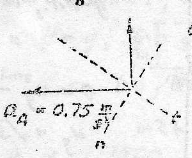
Roots are equal if discriminant $= 0$,

Thus $\dfrac{4u^4}{g^2x^2} - 4\left(1 + \dfrac{2u^2y}{gx^2}\right) = 0$, $\underline{y = \dfrac{u^2}{2g} - \dfrac{gx^2}{2u^2}}$

a vertical parabola

***2/119** $v = \sqrt{0.15^2 + 0.20^2} = 0.25$ m.. (direction of path)





$a_n = 0.75\,\frac{3}{5} - 0.50\,\frac{4}{5}$

$= 0.05$ m/s²

$a_n = \dfrac{v^2}{\rho}$

$\rho = 0.25^2/0.05 = \underline{1.25 \text{ m}}$

$a_t = -0.75\,\frac{4}{5} - 0.50\,\frac{3}{5} = -0.90$ m/s²

$a_t = \frac{d}{dt}(\rho\omega) = \dot\rho\,\frac{v}{\rho} + \rho\dot\omega$, $\dot\rho$ cannot be found until $\dot\omega$ is known.

***2/120** $a_r = \ddot r - r\dot\theta^2 = 0$, $\dfrac{d^2r}{dt^2} - K^2 r = 0$

Try $r = A\cosh Kt + B\sinh Kt$

Substitute & get

$AK^2\cosh Kt + BK^2\sinh Kt - AK^2\cosh Kt - BK^2\sinh Kt = 0$

Thus r is the solution.

When $r = r_0$, $t = 0$; Thus $r_0 = A + 0$, $A = r_0$

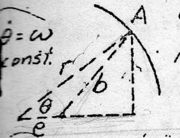
" $t = 0$, $\dot r = 0$; $r_0 K\sinh Kt + BK\cosh Kt = 0$

$0 + BK = 0$, $B = 0$

Thus equation of path is

$\underline{r = r_0\cosh Kt}$

2/121 $(r\sin\theta)^2 + (r\cos\theta - e)^2 = b^2$,



$r^2 - 2re\cos\theta + e^2 = b^2$

$\cancel{2}r\dot r - \cancel{2}\dot r e\cos\theta + \cancel{2}re\omega\sin\theta = 0$

For $\theta = \pi/2$, $\dot r = -e\omega$

$\dot r^2 + r\ddot r - \ddot r e\cos\theta + re\dot\omega\sin\theta + \dot r e\omega\sin\theta + re\omega^2\cos\theta = 0$

For $\theta = \pi/2$, $\dot r^2 + r\ddot r + 2\dot r e\omega = 0$, $\ddot r = \dfrac{e^2\omega^2}{\sqrt{b^2-e^2}}$

$v_r = \dot r = -e\omega$, $v_\theta = r\dot\theta = \sqrt{b^2-e^2}\,\omega$.

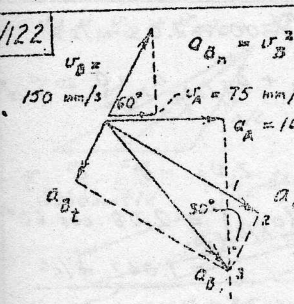
$v = \sqrt{v_r^2 + v_\theta^2} = \omega\sqrt{b^2-e^2+e^2} = \underline{b\omega}$

$a_r = \ddot r - r\dot\theta^2 = \dfrac{e^2\omega^2}{\sqrt{b^2-e^2}} - \sqrt{b^2-e^2}\,\omega^2 = \dfrac{2e^2-b^2}{\sqrt{b^2-e^2}}\omega^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(-e\omega)\omega = -2e\omega^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \underline{\dfrac{b^2\omega^2}{\sqrt{b^2-e^2}}}$

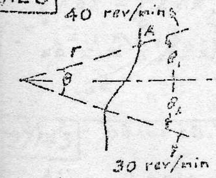
2/122 $a_{B_n} = v_B^2/r = (150)^2/150 = 150$ mm/s²



$a_{B_t} = \overline{2\text{-}3} = \overline{1\text{-}2}\ \text{ctn}\,30° = (150 - 100/0.866)\sqrt{3} = 59.8$ mm/s²

$\alpha = a_{B_t}/150 = \dfrac{59.8}{150} = 0.399$ rad/s, $\underline{\ddot\theta = -0.399 \text{ rad/s}^2}$

2/123 $r = 100 - 75\cos\theta$, $\dot r = 75\dot\theta\sin\theta$, $\ddot r = 75\dot\theta^2\cos\theta$, $\ddot\theta = 0$



$\dot\theta_1 = \dfrac{40(2\pi)}{60} = \dfrac{4}{3}\pi$ rad/s

$\dot\theta_2 = 30\dfrac{2\pi}{60} = \pi$ rad/s

$\theta = \theta_1 + \theta_2$, $\dot\theta = \dot\theta_1 + \dot\theta_2 = \dfrac{7\pi}{3}$ rad/s

$a_r = \ddot r - r\dot\theta_1^2$, $a_\theta = r\ddot\theta_1 + 2\dot r\dot\theta_1$

For $\theta = 30°$, $r = 100 - 75\dfrac{\sqrt3}{2} = 35.0$ mm, $\dot r = 75\dfrac{7\pi}{3}\dfrac{1}{2} = 275$ mm/s

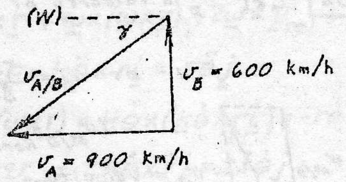
$\ddot r = 75\left(\dfrac{7\pi}{3}\right)^2\dfrac{\sqrt3}{2} = 3490$ mm/s²

$a_r = 3490 - 35.0\left(\dfrac{4\pi}{3}\right)^2 = 2875$ mm/s²

$a_\theta = 35.0(0) + 2(275)\left(\dfrac{4\pi}{3}\right) = 2303$ mm/s²

$a = \sqrt{2875^2 + 2303^2} = 3680$ mm/s² $= \underline{3.68 \text{ m/s}^2}$

2/127 $v_A = v_B + v_{A/B}$



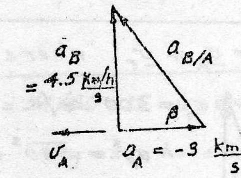
$v_{A/B} = \sqrt{600^2 + 900^2}$

$\underline{= 1082 \text{ km/h}}$

$\gamma = \tan^{-1}\dfrac{600}{900} = \underline{33.7°}$

<u>south of west</u>

2/128 $a_B = a_A + a_{B/A}$



$a_{B/A} = \sqrt{4.5^2 + 3^2}$

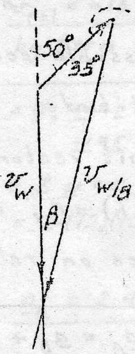
$= 5.41$ (km/h)/s

or $a_{B/A} = \dfrac{5.41}{3.6}$

$= \underline{1.502 \text{ m/s}^2}$

$\beta = \tan^{-1}\dfrac{4.5}{3} = \underline{56.3° \text{ north of west}}$

2/129 $v_W = v_B + v_{W/B}$



$\beta = 50° - 35° = 15°$

$\dfrac{v_W}{\sin 35°} = \dfrac{6.5}{\sin 15°}$

$v_W = \dfrac{6.5(0.5736)}{0.2588}$

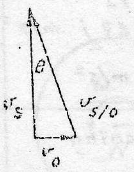
$= \underline{14.4 \text{ knots}}$

2/130 Let S = satellite, O = observer

$v_S = v_O + v_{S/O}$: $v_O = 6378(0.729)(10^{-4})(3600)$

$= 1674$ km/h (East)

$v_S = 27\,940$ km/h (North)



$\theta = \tan^{-1}\dfrac{1674}{27\,940} = 3.43°$

Satellite appears to travel <u>3.43°</u>

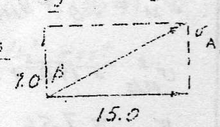
<u>west of north</u>

2/131 $v_{A/B} = v_{A/C} + v_{C/B}$; $v_{A/C} = v_{A/B} - v_{C/B}$

$v_{A/C} = (6.8i - 3.6j) - (-8.2i - 10.6j) = 15.0i + 7.0j$

$v_{A/C} = \sqrt{15.0^2 + 7.0^2} = \underline{16.55 \text{ knots}}$

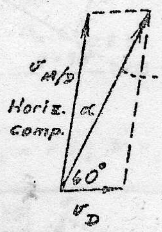
$\beta = \tan^{-1}\dfrac{15.0}{7.0} = \underline{65.0°}$



2/132 Let v_D = velocity of destroyer = 30 (1.852) = 55.6 km/h

or $v_D = \frac{55.6}{3.6} = 15.43$ m/s

$(v_{M/D})_{Horiz.} = 75 \cos 30° = 65.0$ m/s

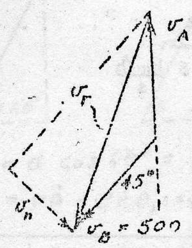


v_M = velocity of missile

$\sin\alpha = \frac{v_D}{v_{M/D}} \sin 60° = \frac{15.43}{65.0}\frac{\sqrt{3}}{2} = 0.2058$

$\underline{\alpha = 11.88°}$

2/133 $\underline{v}_B = \underline{v}_A + \underline{v}_r$ where $\underline{v}_r = \underline{v}_{B/A}$



$v_r^2 = \overline{500}^2 + \overline{500}^2 + 2(500)(500)\cos 45°$

$= 854\ 000\ (km/h)^2$

$\underline{v_r = 924 \text{ km/h}}$

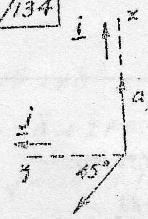
$v_n = 500 \cos 45° = \underline{354 \text{ km/h}}$

Let h = possible constant difference in altitude

$\underline{k}$ = unit vector in vertical direction

Since $\frac{d}{dt}(\underline{k}h) = \underline{0}$, vertical separation has no influence on relative-velocity equations.

2/134 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$



$|\underline{a}_A| = \frac{45}{3.6} = 12.5$ m/s²

$|\underline{a}_B| = \frac{30}{3.6} = 8.33$ m/s²

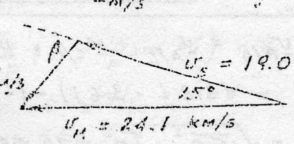
$\underline{a}_{A/B} = 12.5\underline{i} - 8.33\left(-\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{j}}{\sqrt{2}}\right)$

$= 18.39\underline{i} - 5.89\underline{j}$ m/s²

or $\underline{a_{A/B} = 19.31(0.952\underline{i} - 0.305\underline{j}) \text{ m/s}^2}$

2/135 Mars will appear to be approaching spacecraft head on when $\underline{v}_{M/S}$ is along line of sight M-S.

$\underline{v}_M = \underline{v}_S + \underline{v}_{M/S}$



$v_{M/S}^2 = \overline{19.0}^2 + \overline{24.1}^2 - 2(19.0)(24.1)\cos 15° = 57.2\ (km/s)^2$

$v_{M/S} = 7.56$ km/s, $\frac{24.1}{\sin(\pi-\beta)} = \frac{7.56}{\sin 15°}$

$\sin(\pi-\beta) = \sin\beta = 0.8246$, $\underline{\beta = 55.6°}$

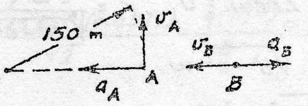
2/136 $\underline{a}_A = \underline{a}_B = 0$ so $\underline{a}_{A/B} = 0$

In polar coordinates relative to B,

$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 = 0$ so $\underline{\ddot{r} = r\dot{\theta}^2}$

$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$ so $\underline{\ddot{\theta} = -2\dot{r}\dot{\theta}/r}$

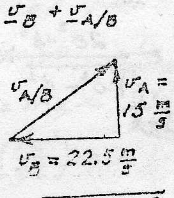
2/137 $v_A = 54/3.6 = 15$ m/s, $v_B = 81/3.6 = 22.5$ m/s



$a_A = \frac{v_A^2}{\rho} = \frac{15^2}{150} = 1.5$ m/

$a_B = 3$ m/s²

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$



$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

$a_{A/B}$

$a_A = 1.5$ m/s² $a_B = 3$ m/s²

$v_{A/B} = \sqrt{\overline{22.5}^2 + \overline{15}^2} = \underline{27.0 \text{ m/s}}$

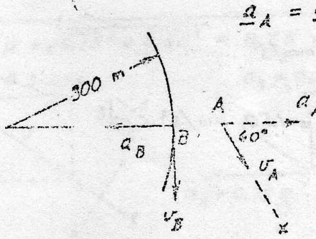
$\underline{\text{in dir. shown}}$

$a_{A/B} = \underline{4.5 \text{ m/s}^2}$

$\underline{\text{in dir. shown}}$

2/138 $a_B = v_B^2/r = (90/3.6)^2/300 = 2.08$ m/s²

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_A = 0$, so $\underline{a}_{A/B}$



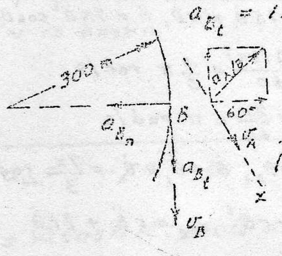
$a_{A/B} = 2.08$ m/s²

$(a_{A/B})_x = 2.08 \cos 60°$

$= \underline{1.042 \text{ m/s}^2}$

2/139 $a_{B_n} = 2.08$ m/s² (Prob. 2/138)

$a_{B_t} = 12/3.6 = 3.33$ m/s²



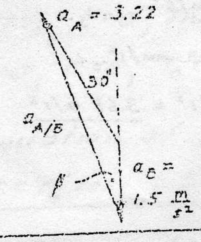
$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $\underline{a}_A =$

$\underline{a}_{A/B} = -\underline{a}_{B_n} - \underline{a}_{B_t}$

$(a_{A/B})_x = 2.08 \cos 60° - 3.33$

$= \underline{-1.245 \text{ m/s}^2}$

2/140 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_A = v_A^2/\rho = (50/3.6)^2/60 = 3$



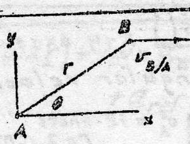
$a_{A/B} = \sqrt{(3.22\frac{\sqrt{3}}{2} + 1.5)^2 + (3.22$

$= \underline{4.53 \text{ m/s}^2}$

$\beta = \tan^{-1}\frac{1.608}{4.28} = \tan^{-1} 0$

$= \underline{20.6° \text{ west of north}}$

2/141 $\underline{v}_B = \underline{v}_A + (\underline{v}_{B/A})_r + (\underline{v}_{B/A})_\theta$



$v_{B/A} = v_B - v_A = (1500-1000)/3.6 = 138.9 \ \frac{m}{s}$

$(v_{B/A})_r = v_{B/A} \cos\theta = \dot{r}$

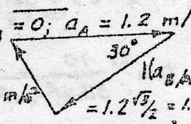
so $\dot{r} = 138.9(0.866) = 120.3$ m/s

$(v_{B/A})_\theta = -v_{B/A} \sin\theta = r\dot{\theta}$

so $\dot{\theta} = \dfrac{-138.9(0.5)}{(18-12)(10^3)/0.5} = -5.79$ mrad/s

$a_B = 0; \ a_A = 1.2$ m/s²

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_r + (\underline{a}_{B/A})_\theta$



$(a_{B/A})_\theta = 0.6$ m/s² $\quad = 1.2\frac{\sqrt{3}}{2} = 1.039 \ \frac{m}{s^2}$

$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2; \ -1.039 = \ddot{r} - 12(10^3)(-5.79)^2(10^{-3})^2$

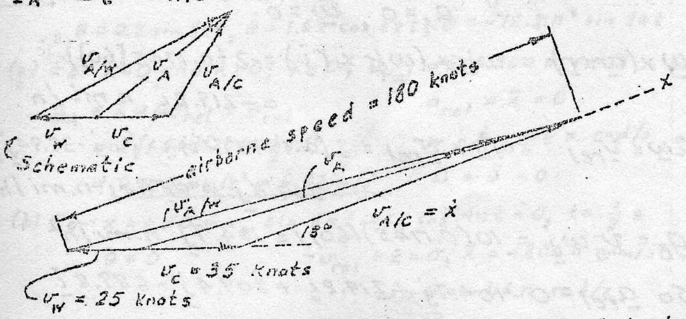
$\underline{\ddot{r} = -0.637 \ \text{m/s}^2}$

$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}; \ 0.600 = 12(10^3)\ddot{\theta} + 2(120.3)(-5.79)(10^{-3})$

$\underline{\ddot{\theta} = 0.1660 \ \text{mrad/s}^2}$

2/142 A = aircraft, C = carrier, W = wind

$\underline{v}_A = \underline{v}_C + \underline{v}_{A/C}, \quad \underline{v}_A = \underline{v}_W + \underline{v}_{A/W}$



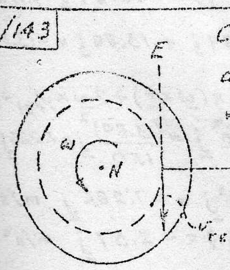
At takeoff $v_{A/C} = 180 - (25+35)\cos 18° = 122.9$ knots

or $\dot{x} = 122.9(1.852)/3.6 = 63.2$ m/s

$\int_0^{\dot{x}} \dot{x}\,d\dot{x} = \int_0^x \ddot{x}\,dx, \ \frac{1}{2}\dot{x}^2 = \ddot{x}x$ for $\ddot{x}$ constant

so $\ddot{x} = \dot{x}^2/(2x) = \dfrac{(63.2)^2}{2(150)} = \underline{13.33 \ \text{m/s}^2}$

2/143



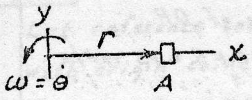
Coriolis component does not depend on latitude for E-W rel. velocity.

$|2\underline{\omega} \times \underline{v}_{rel}| = a_c = 2(0.729)(10^{-4})(2500)/3.6$

$= \underline{0.1012 \ \text{m/s}^2}$

away from earth normal to polar axis

2/144 Eq. 23, $\underline{a}_A = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



$\dot{\underline{\omega}} \times \underline{r} = \underline{k}\ddot{\theta} \times r\underline{i} = r\ddot{\theta}\underline{j}$

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{k}\dot{\theta} \times (\underline{k}\dot{\theta} \times r\underline{i}) = -r\dot{\theta}^2\underline{i}$

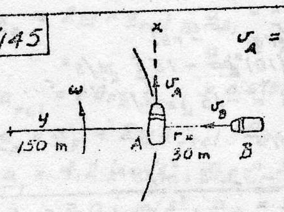
$2\underline{\omega} \times \underline{v}_{rel} = 2\underline{k}\dot{\theta} \times \dot{r}\underline{i} = 2\dot{r}\dot{\theta}\underline{j}$

$\underline{a}_{rel} = \underline{i}\ddot{r}$

So $(a_A)_{r \text{ or } x} = \ddot{r} - r\dot{\theta}^2, \ (a_A)_{\theta \text{ or } y} = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

which agree with Eq. 18 for a_r & a_θ

2/145



$v_A = 54/3.6 = 15$ m/s, $v_B = \frac{81}{3.6} = 22.5$ m/s

x-y axes have angular velocity of

$\omega = \frac{v_A}{r} = \frac{15}{150} = 0.10$ rad/s

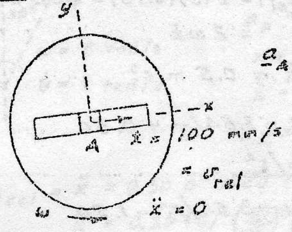
$\underline{v}_B = \underline{v}_A + \underline{\omega} \times \underline{r} + \underline{v}_{rel}; \quad \underline{\omega} \times \underline{r} = 0.10(30)\underline{i} = 3\underline{i}$ m/s

$22.5\underline{j} = 15\underline{i} + 3\underline{i} + \underline{v}_{rel}, \ \underline{\underline{v}_{rel} = -18\underline{i} + 22.5\underline{j}}$ m/s; No

$\underline{v}_{rel}$ differs from $-\underline{v}_{A/B}$ by term $\underline{\omega} \times \underline{r}$

2/146 For A at origin, $\underline{r} = \underline{0}, \ \underline{\omega} \times \underline{r} = \underline{0}, \ \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{0}$

$\dot{\underline{\omega}} \times \underline{r} = 0$



$\underline{a}_A = 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$= 2(12)(100)\underline{j} + \underline{0}$ mm/s²

$\dot{x} = 100$ mm/s $= v_{rel}$

$\underline{\underline{a}_A = 2.40\,\underline{j} \ \text{m/s}^2}$

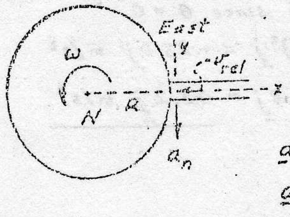
$\ddot{x} = 0$

$\omega = 12$ rad/s

$\dot{\omega} = 15$ rad/s²

2/147 For constant accel. g, $v_{rel}^2 = 2gh$

$v_{rel} = \sqrt{2(9.781)(25)} = 22.1$ m/s



take origin B at particle so $\underline{r} = \underline{0}$

$\underline{a}_A = \underline{a}_B + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

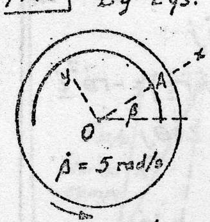
$\underline{a}_{rel} = -\underline{i}g + \underline{i}R\omega^2 - 2\omega\sqrt{2gh}\,\underline{j}$

Rel. accel. normal to pipe is

$a_n = 2\omega\sqrt{2gh}$ west $= 2(0.729)(10^{-4})(22.1)$

$= \underline{3.22 \ \text{mm/s}^2 \ \text{west}}$

2/148 By Eqs. 18; Let $\dot\theta$ = absolute angular velocity of OP



$= \omega + \dot\beta = 10 + 5 = 15$ rad/s

$\ddot\theta = 0$

$a_r = \ddot r - r\dot\theta^2 = 0 - 0.15(15)^2 = -33.8\ m/s^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 0 = 0$

$\omega = 10$ rad/s const.

By Eqs. 23; x-y axes attached to disk with x-axis passing through position of A.

$\underline{a}_A = \underline{\dot\omega}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

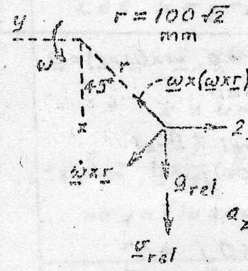
$\underline{\dot\omega}\times\underline{r} = \underline{0}$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -0.15(10)^2\underline{i} = -15\underline{i}\ m/s^2$

$2\underline{\omega}\times\underline{v}_{rel} = 2(10\underline{k})\times(0.15)(5)\underline{j} = -15\underline{i}\ m/s^2$

$\underline{a}_{rel} = -0.15\dot\beta^2\underline{i} = -3.75\underline{i}\ m/s^2$

Thus $a = a_r = -15 - 15 - 3.75 = \underline{-33.8\ m/s^2}$

2/149 $\underline{a}_P = \underline{\dot\omega}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$



$r = 100\sqrt2$ mm

$|\underline{\dot\omega}\times\underline{r}| = 10(0.1\sqrt2) = 1.414\ m/s^2$

$|\underline{\omega}\times(\underline{\omega}\times\underline{r})| = 0.1\sqrt2(5)^2 = 3.54\ m/s^2$

$|2\underline{\omega}\times\underline{v}_{rel}| = 2(5)(0.15) = 1.50\ m/s^2$

$= 2\omega\dot x$

$|\underline{a}_{rel}| = \ddot x = 0.5\ m/s^2$

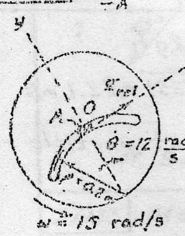
$a_x = 1.414/\sqrt2 - 3.54/\sqrt2 + 0.50$

$= \underline{-1.00\ m/s^2}$

$a_y = 1.414/\sqrt2 + 3.54/\sqrt2 - 1.50$

$= \underline{2.00\ m/s^2}$

2/150 $\underline{a}_A = 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$; $\underline{r} = \overline{OA} = \underline{0}$, $\underline{a}_O = \underline{0}$



$\underline{v}_{rel} = \rho\dot\theta\underline{i} = 0.2(12)\underline{i} = 2.4\underline{i}\ m/s$

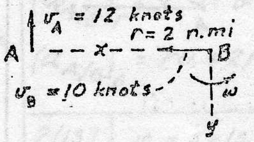
$2\underline{\omega}\times\underline{v}_{rel} = 2(15)(2.4)\underline{j} = 72\underline{j}\ m/s^2$

$\underline{a}_{rel} = -\rho\dot\theta^2\underline{j}$ since $\theta = 0$

$= -0.2(12)^2\underline{j} = -28.8\underline{j}\ m/s^2$

$\underline{a}_A = 72\underline{j} - 28.8\underline{j} = \underline{43.2\underline{j}\ m/s^2}$

2/151 x-y axes attached to B have the angular velocity $\omega = 10\frac{\pi}{180} = 0.1745$ rad/min



$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

$\underline{\omega}\times\underline{r} = 0.1745(60)(2)\underline{j}$

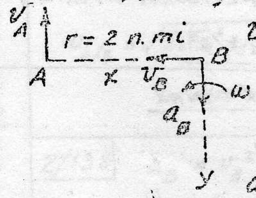
$= 20.94\underline{j}$ knots

$\underline{v}_{rel} = -12\underline{j} - 10\underline{i} - 20.94\underline{j} = -10\underline{i} - 32.94\underline{j}$ knots

$|\underline{v}_{rel}| = \sqrt{10^2 + 32.94^2} = \underline{34.4\text{ knots}}$

$\beta = \tan^{-1}\frac{10}{32.94} = \underline{16.89°}$

2/152 x-y axes attached to B with angular velocity $\omega = 0.1745$ rad/min (Prob. 2/151)



$\underline{v}_{rel} = -10\underline{i} - 32.94\underline{j}$ knots (" ")

$\underline{a}_A = \underline{a}_B + \underline{\dot\omega}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r})$

$+ 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_A = 0$, $\underline{\dot\omega} = 0$

$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = \omega\underline{k}\times(\omega\underline{k}\times r\underline{i}) = -2(0.1745[60])^2\underline{i}$

$= -219.2\underline{i}$ n.mi/h²

$2\underline{\omega}\times\underline{v}_{rel} = 2\omega\underline{k}\times\underline{v}_{rel} = 2(0.1745)(60)\underline{k}\times(-10\underline{i} - 32.94\underline{j})$

$= -209.4\underline{j} + 689.8\underline{i}$ n.mi/h²

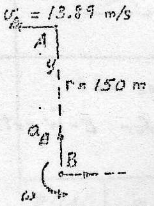
$\underline{a}_B = v_B\omega\underline{j} = 10(0.1745)(60)\underline{j} = 104.7\underline{j}$ n.mi/h²

So $\underline{a}_{rel} = 0 - 104.7\underline{j} + 219.2\underline{i} + 209.4\underline{j} - 689.8\underline{i}$

$= \underline{-471\underline{i} + 105\underline{j}}$ n.mi/h²

2/153 $\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$; $\omega = \frac{v_B}{r}\underline{k} = \frac{50/3.6}{150}\underline{k}$

$= 0.0926\underline{k}$ rad/s



$\underline{\omega}\times\underline{r} = 0.0926(150)(-\underline{i}) = -13.89\underline{i}$ m/s

$\underline{v}_{rel} = -13.89\underline{i} - 13.89\underline{i} + 13.89\underline{i} = -13.89\underline{i}\ \frac{m}{s}$

$\underline{a}_A = \underline{a}_B + \underline{\dot\omega}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = \underline{0}$; $\underline{a}_B = \frac{v_B^2}{r}\underline{j} = \frac{(13.89)^2}{150}\underline{j} = 1.286\underline{j}\ \frac{m}{s^2}$

$\underline{\dot\omega} = \underline{0}$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -150(0.0926)^2\underline{j} = -1.286\underline{j}\ m/s^2$

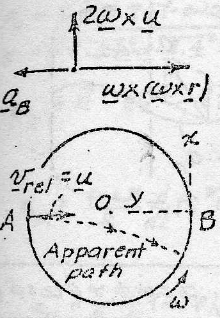
$2\underline{\omega}\times\underline{v}_{rel} = 2(0.0926)\underline{k}\times(-13.89\underline{i}) = -2.57\underline{j}\ m/s^2$

Thus $\underline{0} = 1.286\underline{j} + \underline{0} - 1.286\underline{j} - 2.57\underline{j} + \underline{a}_{rel}$,

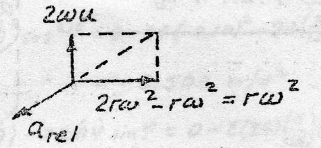
$\underline{a}_{rel} = 2.57\underline{j}\ m/s^2$; $\underline{a_{rel} = 2.57\ m/s^2 \text{ from B to A}}$

$\boxed{2/154}$ (Accel. of ball)$_{xy} = \underline{a} = \underline{a}_B + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times \underline{v}_{rel} + \underline{a}_{rel}$

Once ball leaves A its horiz. accel. is zero, $a = 0$



$a_B = r\omega^2$; $|\underline{\omega}\times(\underline{\omega}\times\underline{r}'')| = 2r\omega^2$ ($\underline{r}''$ = vector $\overline{BA}$)



$$\underline{a_{rel} = \omega\sqrt{4u^2 + r^2\omega^2}}$$

(Analysis is simplified slightly by attaching origin of axes to A.)

$\boxed{2/155}$ $x = 50 \sin 4\pi t$, $\dot{x} = 200\pi \cos 4\pi t$, $\ddot{x} = -800\pi^2 \sin 4\pi t$

$\theta = 0.2 \sin 8\pi t$, $\dot{\theta} = 1.6\pi \cos 8\pi t$, $\ddot{\theta} = -12.8\pi^2 \sin 8\pi t$

(a) For $x = 0$ and $\dot{x} > 0$; $t = 0$, $v_{rel} = \dot{x} = 200\pi$ mm/s

$\underline{a}_A = 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$ $\qquad a_{rel} = \ddot{x} = 0$

$= 2(1.6\pi)(200\pi)\underline{j} + \underline{0}$ $\qquad \omega = \dot{\theta} = 1.6\pi$ rad/s

$= \underline{6320\underline{j}\ \text{mm/s}^2 = 6.32\underline{j}\ \text{m/s}^2}$ $\qquad \dot{\omega} = \ddot{\theta} = 0$

(b) For $x = +50$ mm, $\sin 4\pi t = 1$, $\cos 4\pi t = 0$, $t = \frac{1}{8}$ s

$\theta = 0$ $\qquad v_{rel} = \dot{x} = 0$, $\ddot{x} = -800\pi$ mm/s^2

$\omega = \dot{\theta} = -1.6\pi$ rad/s

$\dot{\omega} = \ddot{\theta} = 0$

$\underline{a}_A = \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\dot{\underline{\omega}}\times\underline{r} = \underline{0}$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -50(1.6\pi)^2\underline{i} = -128\pi^2\underline{i}$ mm/s^2

$2\underline{\omega}\times\underline{v}_{rel} = \underline{0}$, $\underline{a}_{rel} = \ddot{x}\underline{i} = -800\pi^2\underline{i}$ mm/s^2

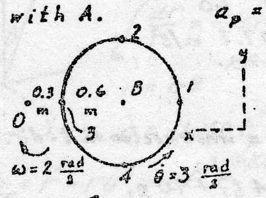
$\underline{a}_A = -128\pi^2\underline{i} - 800\pi^2\underline{i}$ mm/s^2 $= \underline{-9.16\ \underline{i}\ \text{m/s}^2}$

$\boxed{2/156}$ Let A be the point (1, 2, 3, or 4) in question. Let reference coordinates be fixed to OM with origin coincident with each point in turn. Thus $\underline{r} = \underline{0}$, $\dot{\underline{\omega}}\times\underline{r} = \underline{0}$, & $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = \underline{0}$. Let P be point fixed to rotating reference frame & coincident with A. $\quad a_p = r\omega^2$; $\underline{a}_{P_1} = 1.5(2)^2\underline{i} = 6\underline{i}$ m/s^2

$\underline{a}_{P_2} = (0.9\underline{i} - 0.6\underline{j})(2)^2$ m/s^2

$\underline{a}_{P_3} = 0.3(2)^2 = 1.2\underline{i}$ m/s^2

$\underline{a}_{P_4} = (0.9\underline{i} + 0.6\underline{j})(2)^2$ m/s^2



$v_{rel} = r\dot{\theta} = 0.6(3) = 1.8$ m/s^2

$\underline{a}_c = 2\underline{\omega}\times\underline{v}_{rel}$, $\underline{a}_{c_1} = -2(2)(1.8)\underline{i} = -7.2\underline{i}$ m/s^2,

$\underline{a}_{c_2} = 7.2\underline{j}$ m/s^2, $\underline{a}_{c_3} = 7.2\underline{i}$ m/s^2, $\underline{a}_{c_4} = -7.2\underline{j}\ \frac{m}{s^2}$

$a_{rel} = r\dot{\theta}^2$; $\underline{a}_{rel_1} = 0.6(3)^2\underline{i} = 5.4\underline{i}$, $\underline{a}_{rel_2} = -5.4\underline{j}$, $\underline{a}_{rel_3} = -5.4\underline{i}$

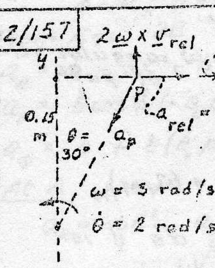
$\underline{a}_{rel_4} = 5.4\underline{j}$ m/s^2

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{\underline{a}_1 = 4.2\underline{i}\ \text{m/s}^2}$, $\underline{\underline{a}_2 = 3.6\underline{i} - 0.6\underline{j}\ \text{m/s}^2}$

$\underline{\underline{a}_3 = 3.0\underline{i}\ \text{m/s}^2}$, $\underline{\underline{a}_4 = 3.6\underline{i} + 0.6\underline{j}\ \text{m/s}^2}$

$\boxed{2/157}$



Let P = point on disk coincident with A

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_p = r\omega^2 = 0.15\frac{2}{\sqrt{3}}(3)^2 = \frac{2.7}{\sqrt{3}}$ m/s^2

$x = 0.15 \tan\theta$, $\dot{x} = 0.15\,\dot{\theta}\sec^2\theta$

$\dot{x} = 0.15(2)\left(\frac{2}{\sqrt{3}}\right)^2 = 0.4$ m/s

$2\underline{\omega}\times\underline{v}_{rel} = 2(3\underline{k})\times(0.4\underline{i}) = 2.4\underline{j}$ m/s^2

$a_{rel} = \ddot{x} = 0.30\,\dot{\theta}^2\sec^2\theta\tan\theta$, $\ddot{\theta} = 0$

$\ddot{x} = 0.30(2)^2\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{1}{\sqrt{3}}\right) = \frac{1.6}{\sqrt{3}}$ m/s^2

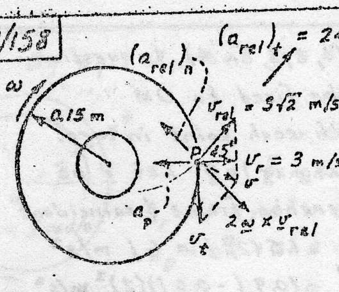
$\underline{a}_A = \frac{2.7}{\sqrt{3}}\left(\frac{-\underline{i}}{2}\right) + \frac{2.7}{\sqrt{3}}\left(-\underline{j}\frac{\sqrt{3}}{2}\right) + 2.4\underline{j} + \frac{1.6}{\sqrt{3}}\underline{i}$

$= \frac{0.25}{\sqrt{3}}\underline{i} + 1.05\underline{j}$ m/s^2

$\beta = \tan^{-1}\frac{1.05}{0.25/\sqrt{3}} = 82.2°$

$a_A = \sqrt{\frac{0.0625}{3} + 1.05^2} = \underline{1.060\ \text{m/s}^2}$

2/158



$(a_{rel})_t = 24\ m/s^2$; $\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_P = r\omega^2 = 0.15\left[\frac{200(2\pi)}{60}\right]^2$

$= 65.8\ m/s^2$

$|2\underline{\omega}\times\underline{v}_{rel}|$

$= 2\left(\frac{200(2\pi)}{60}\right)3\sqrt{2}$

$= 177.7\ m/s^2$

$(a_{rel})_n = \frac{v_{rel}^2}{\rho} = \frac{(3\sqrt{2})^2}{0.2} = 90\ m/s^2$

Let $\underline{n}_1$ = unit vector in n-dir., $\underline{t}_1$ = unit vector in t-dir.

$\underline{a} = \frac{65.8}{\sqrt{2}}\underline{n}_1 - \frac{65.8}{\sqrt{2}}\underline{t}_1 - 177.7\,\underline{n}_1 + 24\,\underline{t}_1 + 90\,\underline{n}_1$

$= -41.2\,\underline{n}_1 - 22.5\,\underline{t}_1\ m/s^2$

$a = \sqrt{41.2^2 + 22.5^2} = \underline{46.9\ m/s^2}$

2/159 Refer to Prob. figure & Fig 15

n-components of $a_{A/P}$ are due to $\omega ds + \dot{s}(d\theta + d\gamma)$

Divide by dt in the limit & get

$\omega\dot{s} + \dot{s}\dot{\theta} + \dot{s}\dot{\gamma}$; But $\dot{s} = v_{rel}$, $\dot{\theta} = \omega$, angular vel of path

$\dot{\gamma} = \dot{s}/\rho$

So n-components become

$2\omega v_{rel} + v_{rel}^2/\rho$ or $2\omega v_{rel} + (a_{rel})_n$

t-component of $a_{A/P}$ is due to $d\dot{s}$ & is

$\ddot{s} = (a_{rel})_t$

Thus $\underline{a}_A = \underline{a}_P + (\underline{a}_{A/P})_n + (\underline{a}_{A/P})_t$

$= \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$ which is Eq. 25b

Coriolis accel. comes from ωds term & $\dot{s}d\theta$ term, both due to rotation of the path

2/162 Vectors are perpendicular if

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\frac{3}{v}\frac{4}{a} + \frac{2}{v}\frac{3}{a} - \frac{9}{v}\frac{2}{a} = 0$ so v & a are perpendicular. In the osculating plane this means that the acceleration is normal to the curve & hence the tangential acceleration is zero.

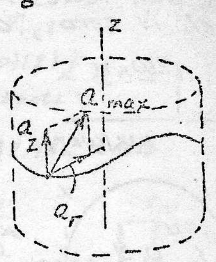
2/163 $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\omega^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi n t) = -4n^2\pi^2 z_0 \sin n\pi t$

$a = \sqrt{(r\omega^2)^2 + (4n^2\pi^2 z_0 \sin n\pi t)^2}$

$a_{max} = \sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}$



2/164 $d\underline{R}_1 = d\phi\,\underline{\phi}_1 + d\theta\cos\phi\,\underline{\theta}_1$, $\underline{\dot{R}_1 = \dot{\phi}\underline{\phi}_1 + \dot{\theta}\cos\phi\,\underline{\theta}_1}$

$d\underline{\theta}_1 = -d\theta\cos\phi\,\underline{R}_1 + d\theta\sin\phi\,\underline{\phi}_1$, $\underline{\dot{\underline{\theta}}_1 = -\dot{\theta}\cos\phi\,\underline{R}_1 + \dot{\theta}\sin\phi\,\underline{\phi}_1}$

$d\underline{\phi}_1 = -d\phi\,\underline{R}_1 - d\theta\sin\phi\,\underline{\theta}_1$, $\underline{\dot{\underline{\phi}}_1 = -\dot{\phi}\underline{R}_1 - \dot{\theta}\sin\phi\,\underline{\theta}_1}$

2/165 $\underline{R} = R\underline{R}_1$, $\dot{\underline{R}} = \dot{R}\underline{R}_1 + R(\dot{\phi}\underline{\phi}_1 + \dot{\theta}\cos\phi\,\underline{\theta}_1)$

$\ddot{\underline{R}} = \ddot{R}\underline{R}_1 + \dot{R}(\dot{\phi}\underline{\phi}_1 + \dot{\theta}\cos\phi\,\underline{\theta}_1)$

$+ \dot{R}\dot{\phi}\underline{\phi}_1 + R\ddot{\phi}\underline{\phi}_1 + R\dot{\phi}(-\dot{\phi}\underline{R}_1 - \dot{\theta}\sin\phi\,\underline{\theta}_1)$

$+ \dot{R}\dot{\theta}\cos\phi\,\underline{\theta}_1 + R\ddot{\theta}\cos\phi\,\underline{\theta}_1 - R\dot{\theta}\dot{\phi}\sin\phi\,\underline{\theta}_1$

$+ R\dot{\theta}\cos\phi(-\dot{\theta}\cos\phi\,\underline{R}_1 + \dot{\theta}\sin\phi\,\underline{\phi}_1)$

Collecting terms and combining give

$\ddot{\underline{R}} = (\ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi)\underline{R}_1$

$+ \left(\frac{\cos\phi}{R}\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\phi}\sin\phi\right)\underline{\theta}_1$

$+ \left(\frac{1}{R}\frac{d}{dt}(R^2\dot{\phi}) + R\dot{\theta}^2\sin\phi\cos\phi\right)\underline{\phi}_1$

2/166 Distance along arc $= ds = r\,d\theta/\cos\gamma$

$v\,dv = a_t\,ds = g\sin\gamma\,\frac{r\,d\theta}{\cos\gamma}$, $v^2 = 2gr(\tan\gamma)\theta$

$= 2gh$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - \frac{(r\dot{\theta})^2}{r}$ but $r\dot{\theta} = v\cos\gamma$

so $a_r = -\frac{v^2\cos^2\gamma}{r} = -\frac{2gh\cos^2\gamma}{r} = -2g(2\pi\tan\gamma)\cos^2\gamma$

$= \underline{-2\pi g\sin 2\gamma}$

2/167 $R = 24$ m const., $\dot\theta = \omega = \frac{2(2\pi)}{60} = \frac{\pi}{15}$ rad/s, $\ddot\theta = 0$

$\beta = 30°$, $\varphi = \pi/2 - \beta$, $\dot\varphi = -\dot\beta = -0.10$ rad/s, $\ddot\varphi = -\ddot\beta = 0$

$v_R = \dot R = 0$, $v_\theta = R\dot\theta\cos\varphi = 24\left(\frac{\pi}{15}\right)\frac{1}{2} = 2.51$ m/s

$v_\varphi = R\dot\varphi = 24(-0.10) = -2.4$ m/s

$v = \sqrt{2.51^2 + 2.4^2} = \underline{3.48 \text{ m/s}}$

$$a_R = \ddot R - R\dot\varphi^2 - R\dot\theta^2\cos^2\varphi = 0 - 24(-0.10)^2 - 24\left(\frac{\pi}{15}\right)^2\left(\frac{1}{2}\right)^2$$
$$= -0.503 \text{ m/s}^2$$

$$a_\theta = \frac{\cos\varphi}{R}\frac{d}{dt}(R^2\dot\theta) - 2R\dot\theta\dot\varphi\sin\varphi = 0 - 2(24)\left(\frac{\pi}{15}\right)(-0.10)\frac{\sqrt3}{2}$$
$$= 0.871 \text{ m/s}^2$$

$$a_\varphi = \frac{1}{R}\frac{d}{dt}(R^2\dot\varphi) + R\dot\theta^2\sin\varphi\cos\varphi = 0 + 24\left(\frac{\pi}{15}\right)^2\frac{\sqrt3}{2}\,\frac{1}{2}$$
$$= 0.456 \text{ m/s}^2$$

$$a = \sqrt{(-0.503)^2 + (0.871)^2 + (0.456)^2} = \underline{1.104 \text{ m/s}^2}$$

2/168 $R = 200 + 50\sin 4\pi t$, $\dot R = 200\pi\cos 4\pi t$

$\ddot R = -800\pi^2\sin 4\pi t$

$\dot\theta = \omega = 4\pi$ rad/s, $\ddot\theta = 0$, $\varphi = \pi/2 - \beta$, $\dot\varphi = \ddot\varphi = 0$

For $\dot R$ maximum, $\cos 4\pi t = 1$ & $\sin 4\pi t = 0$

so $a_R = \ddot R - R\dot\varphi^2 - R\dot\theta^2\cos^2\varphi$

$= 0 - 200(0) - 200(4\pi)^2\left(\frac{1}{2}\right)^2 = -800\pi^2$ mm/s²

$a_\theta = \frac{\cos\varphi}{R}\frac{d}{dt}(R^2\dot\theta) - 2R\dot\theta\dot\varphi\sin\varphi$

$= 2\dot R\dot\theta\cos\varphi - 2R\dot\theta\dot\varphi\sin\varphi = 2(200\pi)(4\pi)\frac{1}{2} = 800\pi^2 \frac{\text{mm}}{\text{s}^2}$

$a_\varphi = \frac{1}{R}\frac{d}{dt}(R^2\dot\varphi) + R\dot\theta^2\sin\varphi\cos\varphi$

$= 0 + 200(4\pi)^2\frac{\sqrt3}{2}\,\frac{1}{2} = 800\pi^2\sqrt3$ mm/s²

$a = 800\pi^2\sqrt{(-1)^2 + 1^2 + (\sqrt3)^2} = 800\pi^2\sqrt5 = 17\,660 \frac{\text{mm}}{\text{s}^2} = \underline{17.66 \frac{\text{m}}{\text{s}^2}}$

2/169 For $\ddot R$ = max. & directed toward O, from solution to Prob. 2/168, $\sin 4\pi t = 1$, $\cos 4\pi t = 0$, & $R = 250$ mm, $\dot R = 0$, $\ddot R = -800\pi^2$ mm/s² and with $\dot\theta = 4\pi$ rad/s, $\ddot\theta = 0$, $\varphi = 60$ deg, $\dot\varphi = \ddot\varphi = 0$, acceleration components from solution to Prob. 2/168 become

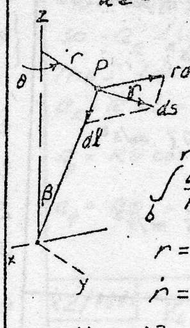
$a_R = -800\pi^2 - 0 - 250(4\pi)^2\left(\frac{1}{2}\right)^2 = -1800\pi^2$ mm/s²

$a_\theta = 0 - 0 = 0$

$a_\varphi = 0 + 250(4\pi)^2\frac{\sqrt3}{2}\,\frac{1}{2} = 1000\pi^2\sqrt3$ mm/s²

$a = 0.2\pi^2\sqrt{(-9)^2 + (5\sqrt3)^2} = 0.2\pi^2\sqrt{156} = \underline{24.7 \text{ m/s}^2}$

2/170 ds = differential distance along cure
dℓ = " " in direction of cone element



$$r = \frac{b}{h}z, \quad \tan\beta = b/h$$

$$d\ell = r\,d\theta\tan\gamma$$

$$dr = -d\ell\sin\beta = -r\,d\theta\tan\gamma\sin\beta$$

$$\int_b^r \frac{dr}{r} = -\tan\gamma\sin\beta\int_0^\theta d\theta$$

$$r = be^{-K\theta} \quad \text{where } K = \tan\gamma\sin\beta$$

$$\dot r = -bK\dot\theta e^{-K\theta}, \quad \ddot r = bK^2\dot\theta^2 e^{-K\theta}, \quad \ddot\theta = 0$$

$$a_r = \ddot r - r\dot\theta^2 = b\dot\theta^2 e^{-K\theta}(K^2 - 1)$$

or $\underline{a_r = b\dot\theta^2 e^{-\theta\tan\gamma\sin\beta}(\tan^2\gamma\sin^2\beta - 1)}$

$\underline{\text{where } \beta = \tan^{-1}\frac{b}{h}}$

2/171 $\theta = \theta_0\cos pt$, $\dot\theta = -\theta_0 p\sin pt$, $\ddot\theta = -\theta_0 p^2\cos pt$

$\dot\phi = K$, $\ddot\phi = 0$; $R = b$, $\dot R = \ddot R = 0$

From Eqs. 2/8

$a_R = 0 - bK^2 - b\theta_0^2 p^2\sin^2 pt\cos^2\phi$

$a_\theta = b\cos\phi(-\theta_0 p^2\cos pt) - 2b(-\theta_0 p\sin pt)K\sin\phi$

$a_\phi = 0 + b(\theta_0 p\sin pt)^2\sin\phi\cos\phi$

<u>At A</u>, $\cos pt = -1$, $\sin pt = 0$

$a_R = -bK^2$, $a_\theta = bp^2\theta_0\cos\phi$, $a_\phi = 0$

so $a = b\sqrt{K^4 + p^4\theta_0^2\cos^2\phi}$

<u>At B</u>, $\cos pt = 0$, $\sin pt = 1$, $\phi = \pi/2$

$a_R = -bK^2$, $a_\theta = 2bpK\theta_0$, $a_\phi = 0$

so $\underline{a = bK\sqrt{K^2 + 4p^2\theta_0^2}}$

2/172 Use Eqs. 2/8 where $\dot\varphi = -\dot\beta$, $R = L$, $\dot\theta = \omega$

$a_R = 0 - 1.2\left(-\frac{3}{2}\right)^2 - 1.2(2)^2\frac{1}{2} = \underline{-5.10 \text{ m/s}^2}$

$a_\theta = \frac{\sin\beta}{L}(2L\dot L\omega + 0) + 2L\omega\dot\beta\cos\beta = 2\omega(\dot L\sin\beta + L\dot\beta\cos\beta)$

$= 2(2)\left(0.9\frac{1}{\sqrt2} + 1.2\left(\frac{3}{2}\right)\frac{1}{\sqrt2}\right) = \frac{10.8}{\sqrt2} = \underline{7.64 \text{ m/s}^2}$

$a_\varphi = -2\dot L\dot\beta + L\omega^2\cos\beta\sin\beta = -2(0.9)\left(\frac{3}{2}\right) + 1.2(2)^2\frac{1}{\sqrt2}\frac{1}{\sqrt2}$

$= -2.7 + 2.4 = \underline{-0.3 \text{ m/s}^2}$

2/173 Use Eqs. 28 where $\dot{\varphi} = -\dot{\beta}$, $R = L$,

$\dot{\theta} = 4\pi(0.12)\cos 4\pi t = 0.48\pi$ rad/s, $\ddot{\theta} = -1.92\pi^2 \sin 4\pi t = 0$

$a_R = 0 - 1.2\left(-\frac{3}{2}\right)^2 - 1.2(0.48\pi)^2\left(\frac{\sqrt{3}}{2}\right)^2 = -4.75$ m/s²

$a_\theta = \cos\varphi(2\dot{R}\dot{\theta} + R\ddot{\theta}) - 2R\dot{\theta}\dot{\varphi}\sin\varphi$

$= \frac{\sqrt{3}}{2} 2(0.9)(0.48\pi) - 2(1.2)(0.48\pi)\left(-\frac{3}{2}\right)\frac{1}{2} = 5.07$ m/s²

$a_\varphi = 2\dot{R}\dot{\varphi} + R\ddot{\varphi} + R\dot{\theta}^2 \sin\varphi\cos\varphi$, $\ddot{\varphi} = 0$

$= 2(0.9)\left(-\frac{3}{2}\right) + 0 + 1.2(0.48\pi)^2 \frac{1}{2}\frac{\sqrt{3}}{2} = -1.518$ m/s²

$a = \sqrt{(-4.75)^2 + (5.07)^2 + (-1.518)^2} = \underline{7.11 \text{ m/s}^2}$

2/174 $\{s_{r\theta z}\} = [T_\theta]\{s_{xyz}\}$

where $s_x = 7 - 2 = 5$ m, $s_y = 4 - 2 = 2$ m, $s_z = 6 - 3 = 3$ m

$$s_{r\theta z} = \begin{Bmatrix} s_r \\ s_\theta \\ s_z \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 5 \\ 2 \\ 3 \end{Bmatrix} \text{ where } \tan\theta = \frac{2}{5},\ \sin\theta = \frac{2}{\sqrt{29}},\ \cos\theta = \frac{5}{\sqrt{29}}$$

so $s_r = 5\cos\theta + 2\sin\theta = \frac{1}{\sqrt{29}}(25 + 4) = \underline{\sqrt{29} \text{ m}}$

$s_\theta = -5\sin\theta + 2\cos\theta = \frac{1}{\sqrt{29}}(-10 + 10) = \underline{0}$

$s_z = \underline{3 \text{ m}}$

2/175 $\{v_{xyz}\} = [T_\theta]^{-1}[T_\varphi]^{-1}\{v_{R\theta\varphi}\}$

$$= \begin{bmatrix} \cos\varphi\cos\theta & -\sin\theta & -\sin\varphi\cos\theta \\ \cos\varphi\sin\theta & \cos\theta & -\sin\varphi\sin\theta \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{Bmatrix} 3 \\ -4 \\ 2 \end{Bmatrix}$$

where $\sin\theta = \sqrt{3}/2$, $\cos\theta = 1/2$, $\sin\varphi = 1/2$, $\cos\varphi = \sqrt{3}/2$

so $v_x = \frac{\sqrt{3}}{2}\frac{1}{2}(3) + \frac{\sqrt{3}}{2}(4) - \frac{1}{2}\frac{1}{2}(2) = \underline{4.26 \text{ m/s}}$

$v_y = \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}(3) + \frac{1}{2}(-4) - \frac{1}{2}\frac{\sqrt{3}}{2}(2) = \underline{-0.616 \text{ m/s}}$

$v_z = \frac{1}{2}(3) + 0 + \frac{\sqrt{3}}{2}(2) = \underline{3.23 \text{ m/s}}$

2/176 $\{v_{R\theta\phi}\} = [T_\phi][T_\theta]\{v_{xyz}\}$

$$\{v_{R\theta\phi}\} = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{bmatrix} \begin{Bmatrix} u \\ 0 \\ 0 \end{Bmatrix}$$

$$\left.\begin{aligned} v_R &= u\cos\phi\cos\theta \\ v_\theta &= -u\sin\theta \\ v_\phi &= -u\sin\phi\cos\theta \end{aligned}\right\} \text{Ans}$$

2/177 $\{v_{R\theta\phi}\} = [T_\phi][T_\theta]\{v_{xyz}\}$

$$\{v_{R\theta\phi}\} = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{bmatrix} \begin{Bmatrix} 0 \\ \dot{y} \\ \dot{z} \end{Bmatrix}$$

$v_\theta = R\dot{\theta}\cos\phi = 0 + \dot{y}\cos\theta$

$v_\phi = R\dot{\phi} = -\dot{y}\sin\phi\sin\theta + \dot{z}\cos\phi$

But $\dot{y} = u\cos\beta$, $\dot{z} = u\sin\beta - gt$

So
$$\begin{cases} \dot{\theta} = \dfrac{\dot{y}\cos\theta}{R\cos\phi} = \dfrac{u\cos\beta\cos\theta}{R\cos\phi} \\[2ex] \dot{\phi} = \dfrac{-u\cos\beta\sin\phi\sin\theta}{R} + \dfrac{(u\sin\beta - gt)\cos\phi}{R} \end{cases}$$

where $R = \sqrt{b^2 + (ut\cos\beta)^2 + (ut\sin\beta - \frac{1}{2}gt^2)^2}$

2/178 $\{a_{xyz}\} = [T_\theta]^{-1}[T_\varphi]^{-1}\{a_{R\theta\varphi}\}$

$$= \begin{bmatrix} \cos\varphi\cos\theta & -\sin\theta & -\sin\varphi\cos\theta \\ \cos\varphi\sin\theta & \cos\theta & -\sin\varphi\sin\theta \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \{a_{R\theta\varphi}\}$$

$$\begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} = \begin{bmatrix} 1/2 & -1/\sqrt{2} & -1/2 \\ 1/2 & 1/\sqrt{2} & -1/2 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{Bmatrix} -5.10 \\ 7.64 \\ -0.3 \end{Bmatrix};$$

$\sin\varphi = \cos\varphi = 1/\sqrt{2}$
$\sin\theta = \cos\theta = 1/\sqrt{2}$
$a_R = -5.10$ m/s²
$a_\theta = 7.64$ m/s²
$a_\varphi = -0.3$ m/s²

$a_x = \frac{1}{2}(-5.10) - \frac{1}{\sqrt{2}}(7.64) - \frac{1}{2}(-0.3) = \underline{-7.80 \text{ m/s}^2}$

$a_y = \frac{1}{2}(-5.10) + \frac{1}{\sqrt{2}}(7.64) - \frac{1}{2}(-0.3) = \underline{3.00 \text{ m/s}^2}$

$a_z = \frac{1}{\sqrt{2}}(-5.10) + 0 + \frac{1}{\sqrt{2}}(-0.3) = \underline{-3.82 \text{ m/s}^2}$

2/181 In x-y-z, $\mathbf{a}_{A/C} = \mathbf{a}_n + \mathbf{a}_t$, $a_n = r\omega^2$; $a_t = r$[illegible]

$\omega = 0.4t$, $\alpha = 0.4$ rad/s²; $\omega_{t=2s} = 0.8$ rad/s

For $t = 2$ s, $a_n = 0.9(0.8)^2 = 0.576$ m/s²

$a_t = 0.9(0.4) = 0.360$ m/s²

$a_{A/C} = \sqrt{0.576^2 + 0.360^2} = \underline{0.679 \text{ m/s}^2}$

$\mathbf{a}_{A/O} = \mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$; $a_{C_{t=2s}} = 9.81(0.1 + 0.05[2]^2) = 1.962$ m/s²

$a_{A/O} = \sqrt{1.962^2 + 0.679^2} = \underline{2.08 \text{ m/s}^2}$

2/182 $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$, $\mathbf{v}_{A/B} = 150\mathbf{i} - 35$[illegible]

$= 150\mathbf{i} - 64.8\mathbf{j}$ km/h

From Eq. 35, $|\dot{R}| = [v_{A/B}]_R = (v_{A/B})_x \cos\varphi \cos$[illegible]

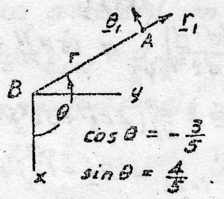
$+ (v_{A/B})_y \cos\varphi \sin$[illegible]

where $\cos\theta = -\frac{6.5}{7.5}$, $\sin\theta = \frac{6}{7.5}$, $\cos\varphi =$ [illegible]

$\dot{R} = 150(0.8575)(-0.4) - 64.8(0.8575)(0.$[illegible]$) = \underline{-13\text{[illegible] km/h}}$

2/183 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$; $\underline{a}_A = \underline{a}_B = \underline{0}$

$\underline{a}_{A/B} = (\ddot{r} - r\dot{\theta}^2)\underline{r}_1 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{\theta}_1 + \ddot{h}\underline{k}$



$\cos\theta = -\frac{3}{5}$, $\sin\theta = \frac{4}{5}$

But from Prob. 2/182

$(v_{A/B})_x = 150$ km/h

$(v_{A/B})_y = -64.8$ km/h

so $v_r = \dot{r} = (-\frac{3}{5})150 + (\frac{4}{5})(-64.8) = -141.9$ km/h

$v_\theta = r\dot{\theta} = -(\frac{4}{5})150 + (-\frac{3}{5})(-64.8) = -81.1$ km/h

Since $\ddot{h} = 0$ and $\underline{a}_{A/B} = \underline{0}$, $\ddot{r} - r\dot{\theta}^2 = 0$ & $r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$\ddot{r} = r\dot{\theta}^2 = (-81.1)^2/7.5 = \underline{877 \text{ km/h}^2}$

$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} = -\frac{2(-141.9)(-81.1)}{(7.5)^2} = \underline{-409 \text{ rad/h}^2}$

2/184 $\underline{v}_O = \frac{2\pi a}{\tau}(-\underline{i}) = -\frac{2\pi(149.6)(10^6)}{365.25(24)}\underline{i} = -107\,200\,\underline{i}$ km/h

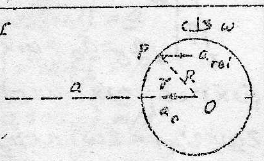
Satellite vel. rel. to O is $v_r = (R+h)\frac{2\pi}{\tau} = \frac{6370+480}{1.57}2\pi = 27\,400$ km/h

At A, $\underline{v}_r = 27\,400\,\underline{i}$, $\underline{v}_A = (27\,400 - 107\,200)\underline{i} = \underline{-79\,800\,\underline{i} \text{ km/h}}$

At B, $\underline{v}_r = 27\,400\,\underline{j}$, $\underline{v}_B = \underline{-107\,200\,\underline{i} + 27\,400\,\underline{j} \text{ km/h}}$

At C, $\underline{v}_r = -27\,400\,\underline{i}$, $\underline{v}_C = (-27\,400 - 107\,200)\underline{i} = \underline{-134\,600\,\underline{i} \text{ km/h}}$

2/185 Point P has zero accel. if



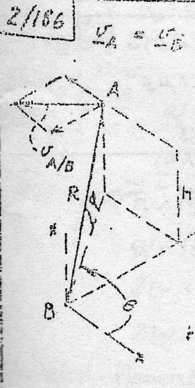
$|a_{rel}| = |a_o|$

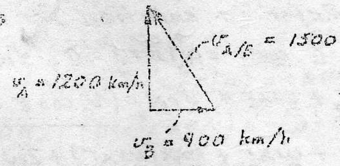
$a_o = v_o^2/a = \frac{(107\,200)^2}{149.6(10^6)} = 76.9$ km/h²

$a_{rel} = R\cos\gamma\,\omega^2 = 6370(0.729)^2(10^{-4})^2\cos\gamma\,(3600)^2 = 439\cos\gamma$ km/h²

$\therefore \cos\gamma = 76.9/439 = 0.1752$, $\underline{\gamma = 79.91°}$

2/186 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$





$(v_{A/B})_R = \dot{R} = 0$ when $\underline{v}_{A/B} \perp \underline{R}$

$\underline{v}_{A/B} = -1200\,\underline{i} - 900\,\underline{j}$

$\underline{R} = R\cos\phi\cos\theta\,\underline{i} + R\cos\phi\sin\theta\,\underline{j} + h\underline{k}$

Vectors are perpendicular when

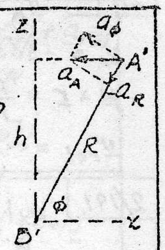
$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$ or $\underline{v}_{A/B}\cdot\underline{R} = 0$, so

$(-1200)R\cos\phi\cos\theta + (-900)R\cos\phi\sin\theta + 0(h) = 0$

$\tan\theta = -\frac{1200}{900} = -1.333$, $\underline{\theta = 126.9°}$

irrespective of h.

2/187 In positions A' & B', $\dot{\theta} = 0$, $\dot{\phi} = 0$



$\underline{a}_A = \underline{a}_{A/B}$ since $\underline{a}_B = 0$, where $a_A = u^2/r$

so $-a_R = (u^2/r)\cos\phi$, $a_\phi = (u^2/r)\sin\phi$, $a_\theta = 0$

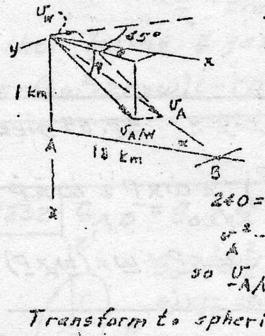
Thus from Eqs. 28

$a_R = \ddot{R} - 0 - 0$, $\underline{\ddot{R} = -\frac{u^2}{r}\cos\phi = -\frac{u^2}{\sqrt{h^2+r^2}}}$

$a_\theta = R\ddot{\theta}\cos\phi + 0 - 0$, $\underline{\ddot{\theta} = 0}$

$a_\phi = R\ddot{\phi} + 0 + 0$, $\ddot{\phi} = \frac{u^2}{rR}\frac{h}{R} = \underline{\frac{hu^2}{r(r^2+h^2)}}$

2/188 $\underline{v}_A$ = true velocity of airplane; $\underline{v}_W$ = velocity of wind



$\underline{v}_{A/W}$ = velocity of airplane rel. to wind

x-y-z moves with wind

$v_{A/W} = 240$ km/h, $v_W = 50$ km/h

$\underline{v}_{A/W} = \underline{v}_A - \underline{v}_W$

$= (0.9985\,v_A\underline{i} + 0.0555\,v_A\underline{k}) - 50(0.707)(\underline{i} - \underline{j})$

$240 = \sqrt{(0.9985\,v_A - 35.4)^2 + (35.4)^2 + (0.0555\,v_A)^2}$

$v_A^2 - 70.6\,v_A - 55\,100 = 0$, $v_A = 273$ km/h

so $\underline{v}_{A/W} = 237\,\underline{i} + 35.4\,\underline{j} + 15.13\,\underline{k}$

Transform to spherical coords (Eq. 35)

$$\{v_{R\theta\phi}\} = [T_\phi][T_\theta]\{v_{xyz}\};\ \begin{Bmatrix} v_{A/W} \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{bmatrix} \begin{Bmatrix} 237 \\ 35.4 \\ 15.13 \end{Bmatrix}$$

The 2nd Eq. is $0 = -237\sin\theta + 35.4\cos\theta$, $\underline{\theta = 8.49°}$

" 3rd " " $0 = -237(0.9890)\sin\phi - 35.4(0.1476)\sin\phi + 15.13\cos\phi$

$\tan\phi = \frac{15.13}{240}$, $\underline{\phi = 3.61°}$

2/189 Attach x-y-z to missile whose coordinates are

$X = 45(\cos 12°)(\cos 60°) = 22.0$ km

$Y = 45(\cos 12°)(\sin 60°) = 38.1$ km

$Z = 45\sin 12° = 9.36$ km

$r = \overline{AB} = \sqrt{(30-22.0)^2 + (75-38.1)^2 + (15-9.36)^2} = 38.2$ km

Dir. cosines of $\overline{AB}$ are

$l = 7.99/38.2 = 0.2094$, $m = 36.9/38.2 = 0.9666$, $n = 5.64/38.2 = 0.1479$

Velocity of missile $= 3750(0.2094\,\underline{i} + 0.9666\,\underline{j} + 0.1479\,\underline{k})$ km/h

$= 785\,\underline{i} + 3620\,\underline{j} + 555\,\underline{k}$ km/h

$\underline{v}_A$ = velocity of aircraft, $\underline{v}_{A/M}$ = vel. of aircraft rel. to missile

$\underline{v}_{A/M} = \underline{v}_A - \underline{v}_M = 1200\underline{j} - 785\,\underline{i} - 3620\underline{j} - 555\,\underline{k}$

$= -785\,\underline{i} - 2420\,\underline{j} - 555\,\underline{k}$ km/h

From Eq. 35, & angles θ', φ' in x, y, z

$$\begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \cos\phi'\cos\theta' & \cos\phi'\sin\theta' & \sin\phi' \\ -\sin\theta' & \cos\theta' & 0 \\ -\sin\phi'\cos\theta' & -\sin\phi'\sin\theta' & \cos\phi' \end{bmatrix} \begin{Bmatrix} 7.99 \\ 36.9 \\ 5.64 \end{Bmatrix}$$

This gives $\cos\theta' = 0.2118$, $\sin\theta' = 0.9773$, $\cos\phi' = 0.9890$, $\sin\phi' = 0.1479$

Then from Eq. 35 with $\underline{v}_{A/M}$

$$\begin{Bmatrix} (v_{A/M})_r \\ (v_{A/M})_{\theta'} \\ (v_{A/M})_{\phi'} \end{Bmatrix} = \begin{bmatrix} 0.2094 & 0.9666 & 0.1479 \\ -0.9773 & 0.2118 & 0 \\ -0.0313 & -0.1446 & 0.9890 \end{bmatrix} \begin{Bmatrix} -785 \\ -2420 \\ -555 \end{Bmatrix}$$

1st Eq. gives $-(v_{A/M})_r = -\dot{r} = 2590$ km/h

2/190 $\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

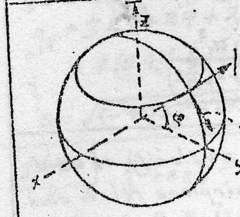
$\underline{v}_A = -300\,\underline{i}$ km/h, $\underline{v}_B = 210\,\underline{j}$ km/h

$\underline{\omega}\times\underline{r} = \frac{1.0\pi}{180}(3600)\,\underline{i}\times 9\,\underline{j} = 565\,\underline{k}$ km/h

$\underline{v}_{rel} = -300\,\underline{i} - 210\,\underline{j} - 565\,\underline{k}$ km/h

---

2/191 $|\omega| = 0.729\,(10^{-4})$ rad/s



$|2\,\underline{\omega}\times\underline{v}_{rel}| = 2\,(0.729)(10^{-4})\frac{100}{3.6}\sin 40°$

$= 2.60$ mm/s² East

$|\underline{v}_{rel}| = \frac{100}{3.6}$ m/s

2/192 With $\underline{P}$ invariant in x-y-z, Eq. 39 gives

$\left(\frac{d\underline{P}}{dt}\right)_{XYZ} = \underline{\omega}\times\underline{P}$ Thus $\left(\frac{d^2\underline{P}}{dt^2}\right)_{XYZ} = \dot{\underline{\omega}}\times\underline{P} + \underline{\omega}\times\dot{\underline{P}}$

so $\left(\frac{d^2\underline{P}}{dt^2}\right)_{XYZ} = \dot{\underline{\omega}}\times\underline{P} + \underline{\omega}\times(\underline{\omega}\times\underline{P})$

2/193 $\left(\frac{d\underline{H}}{dt}\right)_{XYZ} = \underline{\omega}\times\underline{H} + \left(\frac{d\underline{H}}{dt}\right)_{xyz}$

where $\underline{H} = 6\,\underline{i} + 3\,\underline{j} + 5\,\underline{k}$ kg·m²/s

$\underline{\omega} = 2t\,\underline{i} + 3t^2\,\underline{j} + 4t^3\,\underline{k}$ rad/s

For $t = 2$ s, $\underline{\omega} = 4\,\underline{i} + 12\,\underline{j} + 32\,\underline{k}$ rad/s

$\left(\frac{d\underline{H}}{dt}\right)_{xyz} = \underline{0}$ since $\underline{H}$ is fixed in x-y-z

so $\left(\frac{d\underline{H}}{dt}\right)_{XYZ} = (4\,\underline{i} + 12\,\underline{j} + 32\,\underline{k})\times(6\,\underline{i} + 3\,\underline{j} + 5\,\underline{k})$

$= -36\,\underline{i} + 172\,\underline{j} - 60\,\underline{k}$ kg·m²/s²

2/194 $\left(\frac{d}{dt}\left[\frac{d\underline{V}}{dt}\right]_{XYZ}\right)_{XYZ} = \underline{\omega}\times\left[\underline{\omega}\times\underline{V} + \left(\frac{d\underline{V}}{dt}\right)_{xyz}\right]$

$+ \left(\frac{d}{dt}\left[\underline{\omega}\times\underline{V} + \left(\frac{d\underline{V}}{dt}\right)_{xyz}\right]\right)_{xyz}$

$\left(\frac{d^2\underline{V}}{dt^2}\right)_{XYZ} = \underline{\omega}\times(\underline{\omega}\times\underline{V}) + \underline{\omega}\times\left(\frac{d\underline{V}}{dt}\right)_{xyz} + \dot{\underline{\omega}}\times\underline{V} + \underline{\omega}\times\left(\frac{d\underline{V}}{dt}\right)_{xyz} + \left(\frac{d^2\underline{V}}{dt^2}\right)_{xyz}$

Note that $\left(\frac{d\underline{\omega}}{dt}\right)_{xyz} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \dot{\underline{\omega}}$ since $\underline{\omega}\times\underline{\omega} = \underline{0}$

$\underline{\omega}$ being absolute & the same in both systems

so $\left(\frac{d^2\underline{V}}{dt^2}\right)_{XYZ} = \dot{\underline{\omega}}\times\underline{V} + \underline{\omega}\times(\underline{\omega}\times\underline{V}) + 2\underline{\omega}\times\left(\frac{d\underline{V}}{dt}\right)_{xyz} + \left(\frac{d^2\underline{V}}{dt^2}\right)_{xyz}$

2/195 $\left(\frac{d\underline{H}}{dt}\right)_{XYZ} = \underline{\omega}\times\underline{H}$, $\underline{H}$ invariant in xyz

$\left(\frac{d^2\underline{H}}{dt^2}\right)_{XYZ} = \dot{\underline{\omega}}\times\underline{H} + \underline{\omega}\times\dot{\underline{H}} = \dot{\underline{\omega}}\times\underline{H} + \underline{\omega}\times(\underline{\omega}\times\underline{H})$

$\dot{\underline{\omega}} = \frac{d}{dt}(2t\,\underline{i} + 3t^2\,\underline{j} + 4t^3\,\underline{k}) = (2\,\underline{i} + 6t\,\underline{j} + 12t^2\,\underline{k}) + \underline{\omega}\times\underline{\omega}$

$\dot{\underline{\omega}}_{t=2s} = 2\,\underline{i} + 12\,\underline{j} + 48\,\underline{k}$ rad/s²

$\dot{\underline{\omega}}\times\underline{H} = (2\,\underline{i} + 12\,\underline{j} + 48\,\underline{k})\times(6\,\underline{i} + 3\,\underline{j} + 5\,\underline{k})$

$= -84\,\underline{i} + 278\,\underline{j} - 66\,\underline{k}$ kg·m²/s³

$\underline{\omega}\times\underline{H} = (4\,\underline{i} + 12\,\underline{j} + 32\,\underline{k})\times(6\,\underline{i} + 3\,\underline{j} + 5\,\underline{k})$

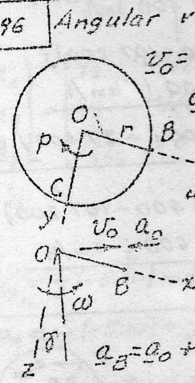
$= -36\,\underline{i} + 172\,\underline{j} - 60\,\underline{k}$ kg·m²/s²

$\underline{\omega}\times(\underline{\omega}\times\underline{H}) = (4\,\underline{i} + 12\,\underline{j} + 32\,\underline{k})\times(-36\,\underline{i} + 172\,\underline{j} - 60\,\underline{k})$

$= -6224\,\underline{i} - 912\,\underline{j} + 1120\,\underline{k}$ kg·m²/s³

Thus $\left(\frac{d^2\underline{H}}{dt^2}\right)_{XYZ} = -6308\,\underline{i} - 634\,\underline{j} + 1054\,\underline{k}$ kg·m²/s³

2/196 Angular velocity of axes is $\underline{\omega} = -\dot{\gamma}\,\underline{j} = \omega\,\underline{j}$, $\dot{\underline{\omega}} = \underline{0}$



$\underline{v}_0 = v_0(\underline{i}\cos\gamma - \underline{k}\sin\gamma)$

$\underline{a}_0 = a_0(-\underline{i}\cos\gamma + \underline{k}\sin\gamma)$

$\underline{v}_B = \underline{v}_0 + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

where $\underline{\omega}\times\underline{r} = \omega\underline{j}\times r\underline{i} = -r\omega\underline{k}$

$\underline{v}_{rel} = rp\,\underline{j}$

so $\underline{v}_B = \underline{i}\,v_0\cos\gamma + rp\,\underline{j} - (v_0\sin\gamma + r\omega)\,\underline{k}$

$\underline{a}_B = \underline{a}_0 + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\dot{\underline{\omega}}\times\underline{r} = 0$; $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = \omega\underline{j}\times(\omega\underline{j}\times r\underline{i}) = -r\omega^2\underline{i}$

$2\underline{\omega}\times\underline{v}_{rel} = 2\omega\underline{j}\times rp\,\underline{j} = \underline{0}$

$\underline{a}_{rel} = -rp^2\,\underline{i}$

Thus $\underline{a}_B = -(a_0\cos\gamma + r\omega^2 + rp^2)\,\underline{i} + \underline{k}\,a_0\sin\gamma$

2/197 Refer to solution for Prob. 2/196 for $\underline{\omega}$, $\underline{v}_0$,

For pt. C, $\underline{\omega}\times\underline{r} = \omega\underline{j}\times r\underline{j} = \underline{0}$, $\underline{v}_{rel} = -rp\,\underline{i}$

$\underline{v}_C = \underline{v}_0 + \underline{\omega}\times\underline{r} + \underline{v}_{rel} = (v_0\cos\gamma - rp)\,\underline{i} - \underline{k}\,v_0\sin\gamma$

$\underline{a}_C = \underline{a}_0 + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\dot{\underline{\omega}}\times\underline{r} = \underline{0}$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = \underline{0}$

$2\underline{\omega}\times\underline{v}_{rel} = 2\omega\underline{j}\times(-rp)\,\underline{i} = 2rp\omega\,\underline{k}$

$\underline{a}_{rel} = -rp^2\,\underline{j}$

$\underline{a}_C = -\underline{i}\,a_0\cos\gamma - rp^2\,\underline{j} + (a_0\sin\gamma + 2rp\omega)\,\underline{k}$

2/198 With origin fixed & $\dot{\omega}=0$, acceleration of A is $\underline{a}_A=\underline{\omega}\times(\underline{\omega}\times\underline{r})+2\underline{\omega}\times\underline{v}_{rel}+\underline{a}_{rel}$

$\underline{\omega}=\underline{k}\omega$; $\underline{r}=s(\underline{j}\cos\beta-\underline{k}\sin\beta)$; $\underline{\omega}\times\underline{r}=-(\omega s\cos\beta)\underline{i}$

$\underline{\omega}\times(\underline{\omega}\times\underline{r})=-(s\omega^2\cos\beta)\underline{j}$; $s=s_o\sin 2\pi nt$

$\underline{v}_{rel}=\dot{s}(-\underline{k}\sin\beta+\underline{j}\cos\beta)$

$=2\pi n s_o\cos 2\pi nt(-\underline{k}\sin\beta+\underline{j}\cos\beta)$

$\underline{a}_{rel}=\ddot{s}(-\underline{k}\sin\beta+\underline{j}\cos\beta)$

$=-4\pi^2n^2s_o\sin 2\pi nt(-\underline{k}\sin\beta+\underline{j}\cos\beta)$

$2\underline{\omega}\times\underline{v}_{rel}=-(4\pi n s_o\omega\cos 2\pi nt\cos\beta)\underline{i}$

For $s=s_o$ with $\dot{s}$ negative, $\sin 2\pi nt=+1$, $\cos 2\pi nt=0$

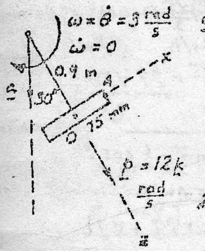
(a) $\underline{a}_A=-(s_o\omega^2\cos\beta)\underline{j}-4\pi^2n^2s_o(-\underline{k}\sin\beta+\underline{j}\cos\beta)$

$=s_o[-\underline{j}(\omega^2+4\pi^2n^2)\cos\beta+\underline{k}(4\pi^2n^2\sin\beta)]$

For $s=0$ & $\dot{s}$ positive, $\sin 2\pi nt=0$, $\cos 2\pi nt=+1$

(b) $\underline{a}_A=-(4\pi n\omega s_o\cos\beta)\underline{i}$

2/199 $\underline{a}_A=\underline{a}_o+\dot{\underline{\omega}}\times\underline{r}+\underline{\omega}\times(\underline{\omega}\times\underline{r})+2\underline{\omega}\times\underline{v}_{rel}+\underline{a}_{rel}$



$\underline{a}_o=\bar{\underline{a}}+\underline{a}_{o/G}$

$=9(-\underline{i}\sin 30°+\underline{k}\cos 30°)+0.9(3^2)(-\underline{k})$

$=-4.5\underline{i}-0.306\underline{k}$ m/s²

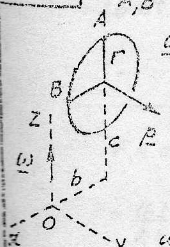
$\underline{\omega}\times(\underline{\omega}\times\underline{r})=-3\underline{j}\times(-3\underline{j}\times0.075\underline{i})=-0.675\underline{i}$ m/s²

$2\underline{\omega}\times\underline{v}_{rel}=2(-3\underline{j})\times0.075(12)\underline{j}=\underline{0}$

$\underline{a}_{rel}=0.075(12^2)(-\underline{i})=-10.8\underline{i}$ m/s²

Collect terms & get $\underline{a}_A=-(15.98\underline{i}+0.306\underline{k})$ m/s²

2/200 $\underline{a}_{A,B}=\underline{a}_o+\dot{\underline{\omega}}\times\underline{r}_{A,B}+\underline{\omega}\times(\underline{\omega}\times\underline{r}_{A,B})+2\underline{\omega}\times\underline{v}_{rel}+\underline{a}_{rel}$



$\underline{a}_o=\underline{0}$, $\dot{\underline{\omega}}=\underline{0}$, $\underline{v}_{rel_A}=rp\underline{i}$, $\underline{v}_{rel_B}=-rp\underline{k}$

$\underline{a}_{rel_A}=-rp^2\underline{k}$, $\underline{a}_{rel_B}=-rp^2\underline{i}$

$\underline{\omega}\times\underline{r}_A=\omega\underline{k}\times(-b\underline{i}+[c+r]\underline{k})=-b\omega\underline{j}$

$\underline{\omega}\times\underline{r}_B=\omega\underline{k}\times(-[b-r]\underline{i}+c\underline{k})=-(b-r)\omega\underline{j}$

$\underline{\omega}\times(\underline{\omega}\times\underline{r}_A)=\omega\underline{k}\times(-b\omega\underline{j})=b\omega^2\underline{i}$

$\underline{\omega}\times(\underline{\omega}\times\underline{r}_B)=\omega\underline{k}\times(-[b-r]\omega\underline{j})=(b-r)\omega^2\underline{i}$

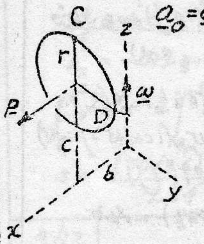
$2\underline{\omega}\times\underline{v}_{rel_A}=2\omega\underline{k}\times rp\underline{i}=2rp\omega\underline{j}$

$2\underline{\omega}\times\underline{v}_{rel_B}=2\omega\underline{k}\times(-rp\underline{k})=\underline{0}$

Collect terms & get

$\underline{a}_A=b\omega^2\underline{i}+2rp\omega\underline{j}-rp^2\underline{k}$, $\underline{a}_B=[(b-r)\omega^2-rp^2]\underline{i}$

2/201 $\underline{a}_{C,D}=\underline{a}_o+\dot{\underline{\omega}}\times\underline{r}_{C,D}+\underline{\omega}\times(\underline{\omega}\times\underline{r}_{C,D})+2\underline{\omega}\times\underline{v}_{rel}+\underline{a}_{rel}$



$\underline{a}_o=\underline{0}$, $\dot{\underline{\omega}}=\underline{0}$, $\underline{v}_{rel_C}=-rp\underline{j}$, $\underline{v}_{rel_D}=rp\underline{k}$

$\underline{a}_{rel_C}=-rp^2\underline{k}$, $\underline{a}_{rel_D}=-rp^2\underline{j}$

$\underline{\omega}\times\underline{r}_C=\omega\underline{k}\times[b\underline{i}+(c+r)\underline{k}]=b\omega\underline{j}$

$\underline{\omega}\times\underline{r}_D=\omega\underline{k}\times[b\underline{i}+r\underline{j}+c\underline{k}]=-r\omega\underline{i}+b\omega\underline{j}$

$\underline{\omega}\times(\underline{\omega}\times\underline{r}_C)=\omega\underline{k}\times b\omega\underline{j}=-b\omega^2\underline{i}$

$\underline{\omega}\times(\underline{\omega}\times\underline{r}_D)=\omega\underline{k}\times[-r\omega\underline{i}+b\omega\underline{j}]=-b\omega^2\underline{i}-r\omega^2\underline{j}$

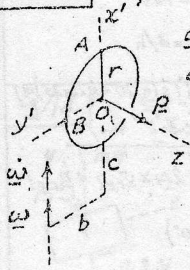
$2\underline{\omega}\times\underline{v}_{rel_C}=2\omega\underline{k}\times(-rp\underline{j})=2rp\omega\underline{i}$

$2\underline{\omega}\times\underline{v}_{rel_D}=2\omega\underline{k}\times rp\underline{k}=\underline{0}$

Collect terms & get

$\underline{a}_C=-(b\omega^2-2rp\omega)\underline{i}-rp^2\underline{k}$, $\underline{a}_D=-b\omega^2\underline{i}-(r\omega^2+rp^2)\underline{j}$

2/202 $\underline{a}_{A,B}=\underline{a}_o+\dot{\underline{\omega}}\times\underline{r}_{A,B}+\underline{\omega}\times(\underline{\omega}\times\underline{r}_{A,B})+2\underline{\omega}\times\underline{v}_{rel}+\underline{a}_{rel}$



$\underline{a}_o=b\omega^2\underline{j}-b\dot{\omega}\underline{k}$

$\dot{\underline{\omega}}\times\underline{r}_A=\dot{\omega}\underline{i}\times r\underline{i}=\underline{0}$, $\dot{\underline{\omega}}\times\underline{r}_B=\dot{\omega}\underline{i}\times r\underline{j}=r\dot{\omega}\underline{k}$

$\underline{\omega}\times\underline{r}_A=\omega\underline{i}\times r\underline{i}=\underline{0}$, $\underline{\omega}\times\underline{r}_B=\omega\underline{i}\times r\underline{j}=r\omega\underline{k}$

$\underline{\omega}\times(\underline{\omega}\times\underline{r}_A)=\underline{0}$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}_B)=\omega\underline{i}\times r\omega\underline{k}=-r\omega^2\underline{j}$

$2\underline{\omega}\times\underline{v}_{rel_A}=2\omega\underline{i}\times rp\underline{j}=2rp\omega\underline{k}$

$2\underline{\omega}\times\underline{v}_{rel_B}=2\omega\underline{i}\times(-rp\underline{i})=\underline{0}$

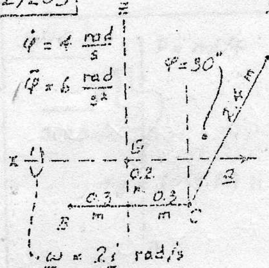
$\underline{a}_{rel_A}=-rp^2\underline{i}$, $\underline{a}_{rel_B}=-rp^2\underline{j}$

Collect terms & get

$\underline{a}_A=-rp^2\underline{i}+b\omega^2\underline{j}+(2rp\omega-b\dot{\omega})\underline{k}$

$\underline{a}_B=(b\omega^2-r\omega^2-rp^2)\underline{j}+(r\dot{\omega}-b\dot{\omega})\underline{k}$

2/203 $\underline{a}_A=\bar{\underline{a}}+\dot{\underline{\omega}}\times\underline{r}+\underline{\omega}\times(\underline{\omega}\times\underline{r})+2\underline{\omega}\times\underline{v}_{rel}+\underline{a}_{rel}$



$\bar{\underline{a}}=-6\underline{i}$ m/s², $\dot{\underline{\omega}}=0$

$\underline{\omega}\times\underline{r}=2\underline{i}\times(-1.5\underline{i}+[2.4\frac{\sqrt{3}}{2}-0.2]\underline{k})$

$=-3.76\underline{j}$ m/s

$\underline{\omega}\times(\underline{\omega}\times\underline{r})=2\underline{i}\times(-3.76\underline{j})=-7.52\underline{k}$ m/s²

$\underline{v}_{rel}=2.4(4)(-\underline{i}\frac{\sqrt{3}}{2}-\underline{k}\frac{1}{2})=-8.32\underline{i}-4.8\underline{k}$ m/s

$2\underline{\omega}\times\underline{v}_{rel}=2(2\underline{i})\times(-8.32\underline{i}-4.8\underline{k})=19.2\underline{j}$ m/s²

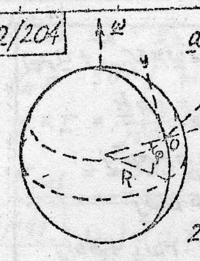
$\underline{a}_{rel}=2.4(4^2)(\underline{i}\frac{1}{2}-\underline{k}\frac{\sqrt{3}}{2})+2.4(6)(-\underline{i}\frac{\sqrt{3}}{2}-\underline{k}\frac{1}{2})$

$=6.73\underline{i}-40.5\underline{k}$ m/s²

Collect terms & get

$\underline{a}_A=0.729\underline{i}+19.2\underline{j}-48.0\underline{k}$ m/s²

2/204



$$\mathbf{a} = \mathbf{a}_o + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$$

$$\mathbf{r}=0,\ \mathbf{a}_o = R\omega^2\cos\varphi(\mathbf{j}\sin\varphi - \mathbf{k}\cos\varphi)$$

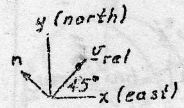
$$= 6370(10^3)(0.729)^2(10^{-4})^2\cos 30^\circ(\mathbf{j}\sin 30^\circ - \mathbf{k}\cos 30^\circ)$$

$$= 0.0293(0.5\mathbf{j} - 0.866\mathbf{k})\ \text{m/s}^2$$

$$2\boldsymbol{\omega}\times\mathbf{v}_{rel} = 2\omega(\mathbf{j}\cos\varphi + \mathbf{k}\sin\varphi)\times v_{rel}(\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ)$$

$$= 2(0.729)(10^{-4})(0.866\mathbf{j} + 0.5\mathbf{k})\times\frac{600}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$$

$$= 0.0619(-0.5\mathbf{i} + 0.5\mathbf{j} - 0.866\mathbf{k})\ \text{m/s}^2$$



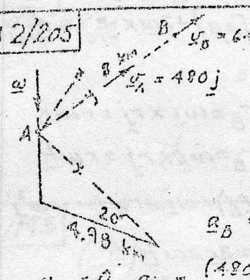
Collecting x-y terms gives

$$\mathbf{a}_{xy} = -0.0309\,\mathbf{i} + 0.0456\,\mathbf{j}\ \text{m/s}^2$$

$$a_n = \frac{0.0309}{\sqrt{2}} + \frac{0.0456}{\sqrt{2}} = \underline{0.0541\ \text{m/s}^2\ (\text{north-west})}$$

Note: $\mathbf{a}_{rel}$ would have no n-component if it existed due to vertical curvature of rails or due to accel. along rails.

2/205



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r} + \mathbf{v}_{rel}$$

$$\omega = \frac{480}{4.98} = 96.4\ \text{rad/h}$$

$$\mathbf{v}_{rel} = (640-480)\mathbf{j} - 96.4(\mathbf{i}\sin 20^\circ - \mathbf{k}\cos 20^\circ)\times 8\mathbf{j}$$

$$= \underline{-725\,\mathbf{i} + 160\,\mathbf{j} - 264\,\mathbf{k}\ \text{km/h}}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$$

$$\mathbf{a}_B = \mathbf{0};\ \mathbf{a}_A = \frac{(480/3.6)^2}{4.98(10^3)}(\mathbf{i}\cos 20^\circ + \mathbf{k}\sin 20^\circ)$$

$$= 3.57(0.940\,\mathbf{i} + 0.342\,\mathbf{k}) = 3.35\,\mathbf{i} + 1.221\,\mathbf{k}\ \text{m/s}^2$$

$$\dot{\boldsymbol{\omega}} = \mathbf{0};\ \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) = -r\omega^2\mathbf{j} = -8(10^3)(96.4/3600)^2\mathbf{j} = -5.73\,\mathbf{j}\ \text{m/s}^2$$

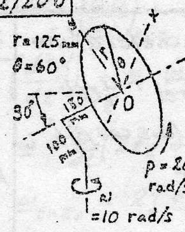
$$2\boldsymbol{\omega}\times\mathbf{v}_{rel} = 2\,\frac{96.4}{3600}(\mathbf{i}\sin 20^\circ - \mathbf{k}\cos 20^\circ)\times(-725\,\mathbf{i} + 160\,\mathbf{j} - 264\,\mathbf{k})/3.6$$

$$= 2.24\,\mathbf{i} + 11.46\,\mathbf{j} + 0.814\,\mathbf{k}\ \text{m/s}^2$$

$$\mathbf{a}_{rel} = \mathbf{0} - (3.35\,\mathbf{i} + 1.221\,\mathbf{k}) + 5.73\,\mathbf{j} - (2.24\,\mathbf{i} + 11.46\,\mathbf{j} + 0.814\,\mathbf{k})$$

$$= \underline{-5.59\,\mathbf{i} - 5.73\,\mathbf{j} - 2.03\,\mathbf{k}\ \text{m/s}^2}$$

2/206



$$\mathbf{v}_P = \mathbf{v}_O + \boldsymbol{\omega}\times\mathbf{r} + \mathbf{v}_{rel};\ \boldsymbol{\omega} = 8.66\,\mathbf{j} + 5.0\,\mathbf{k}\ \text{rad/s}$$

$$\mathbf{v}_O = (150\cos 30^\circ - 100\sin 30^\circ)10\,\mathbf{i} = 799\,\mathbf{i}\ \text{mm/s}$$

$$\boldsymbol{\omega}\times\mathbf{r} = (8.66\,\mathbf{j} + 5\,\mathbf{k})\times 125(\mathbf{i}\cos 60^\circ + \mathbf{j}\sin 60^\circ)$$

$$= -541\,\mathbf{i} + 312\,\mathbf{j} - 541\,\mathbf{k}\ \text{mm/s}$$

$$\mathbf{v}_{rel} = 125(20)(-\mathbf{i}\sin 60^\circ + \mathbf{j}\cos 60^\circ)$$

$$= -2170\,\mathbf{i} + 1250\,\mathbf{j}\ \text{mm/s}$$

So $\underline{\mathbf{v}_P = -1.907\,\mathbf{i} + 1.562\,\mathbf{j} - 0.541\,\mathbf{k}\ \text{m/s}}$

$$\mathbf{a}_P = \mathbf{a}_O + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel};\ \dot{\boldsymbol{\omega}} = 0$$

$$\mathbf{a}_O = (0.15\cos 30^\circ - 0.1\sin 30^\circ)10^2(\mathbf{j}\sin 30^\circ - \mathbf{k}\cos 30^\circ) = 4.00\,\mathbf{j} - 6.92\,\mathbf{k}\ \text{m/s}^2$$

$$\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) = (8.66\,\mathbf{j} + 5.00\,\mathbf{k})\times(-541\,\mathbf{i} + 312\,\mathbf{j} - 541\,\mathbf{k})(10^{-3})$$

$$= -6.25\,\mathbf{i} - 2.71\,\mathbf{j} + 4.69\,\mathbf{k}\ \text{m/s}^2$$

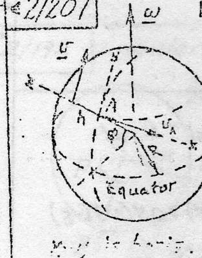
$$2\boldsymbol{\omega}\times\mathbf{v}_{rel} = 2(8.66\,\mathbf{j} + 5.0\,\mathbf{k})\times(-2.17\,\mathbf{i} + 1.25\,\mathbf{j}) = -12.5\,\mathbf{i} - 21.7\,\mathbf{j} + 37.5\,\mathbf{k}\ \text{m/s}^2$$

$$\mathbf{a}_{rel} = 0.125(20)^2(-\mathbf{i}\cos 60^\circ - \mathbf{j}\sin 60^\circ) = -25.0\,\mathbf{i} - 43.3\,\mathbf{j}\ \text{m/s}^2$$

Collect terms & get

$$\underline{\mathbf{a}_P = -43.75\,\mathbf{i} - 63.7\,\mathbf{j} + 35.3\,\mathbf{k}\ \text{m/s}^2}$$

2/207



Velocity of satellite $\mathbf{v} = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r} + \mathbf{v}_{rel}$

Accel. " " $\mathbf{a} = \mathbf{a}_A + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$

$$\boldsymbol{\omega}\times\mathbf{r} = \boldsymbol{\omega}\times h\mathbf{k} = h\omega\cos\varphi\,\mathbf{i},\ \mathbf{v}_A = R\omega\cos\varphi\,\mathbf{i}$$

$$\mathbf{v} = v\mathbf{j}\ \text{so}\ \mathbf{v}_{rel} = -(h+R)\omega\cos\varphi\,\mathbf{i} + v\mathbf{j}$$

$$h\omega\cos\varphi = 640(0.729)(10^{-4})\frac{\sqrt{3}}{2}(3600) = 145.5\ \frac{\text{km}}{\text{h}}$$

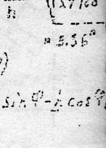
$$R\omega\cos\varphi = 6370(0.729)(10^{-4})\frac{\sqrt{3}}{2}(3600) = 1446\ \text{km/h}$$

$$\mathbf{v}_{rel} = -1593\,\mathbf{i} + 27\,100\,\mathbf{j}\ \text{km/h}$$

x-y is horiz. plane at A.

$$v_{rel} = \sqrt{(1593)^2 + (27\,100)^2} = \underline{27\,150\ \frac{\text{km}}{\text{h}}}$$

$$\theta = 360^\circ - 3.36^\circ = \underline{356.6^\circ}$$



$$\mathbf{a} = (v^2/[R+h])(-\mathbf{k});\ \mathbf{a}_A = R\omega^2\cos\varphi(\mathbf{j}\sin\varphi - \mathbf{k}\cos\varphi)$$

$$\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) = \omega(\mathbf{j}\cos\varphi + \mathbf{k}\sin\varphi)\times h\omega\cos\varphi\,\mathbf{i} = h\omega^2\cos\varphi(\mathbf{j}\sin\varphi - \mathbf{k}\cos\varphi)$$

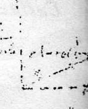
$$2\boldsymbol{\omega}\times\mathbf{v}_{rel} = 2\omega(\mathbf{j}\cos\varphi + \mathbf{k}\sin\varphi)\times(-[h+R]\omega\cos\varphi\,\mathbf{i} + v\mathbf{j})$$

$$= 2\omega(-\mathbf{i}\,v\sin\varphi - \mathbf{j}[h+R]\omega\sin\varphi\cos\varphi + \mathbf{k}[h+R]\omega\cos^2\varphi)$$

Thus $(\mathbf{a}_{rel})_{xy} = \omega\sin\varphi[2v\mathbf{i} + (h+R)\omega\cos\varphi\,\mathbf{j}]$

$$= 0.729(10^{-4})\tfrac{1}{2}[2(27\,100)\,\mathbf{i} + (640+6370)(0.729)(10^{-4})\tfrac{\sqrt{3}}{2}3600\,\mathbf{j}]/3.6$$

$$= 0.549\,\mathbf{i} + 0.0161\,\mathbf{j}\ \text{m/s}^2$$

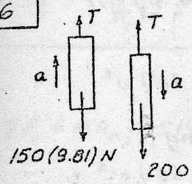


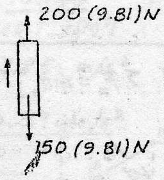
$$(a_{rel})_{xy} = \sqrt{(0.549)^2 + (0.0161)^2} = \underline{0.549\ \text{m/s}^2};\ \theta = \tan^{-1}\frac{0.549}{0.0161} = \underline{88.3^\circ}$$

CHAPTER THREE

KINETICS OF PARTICLES

3/6





(a) $\Sigma F = ma$

$150\text{ kg}: \; T - 1472 = 150a$

$200\text{ kg}: \; 1962 - T = 200a$

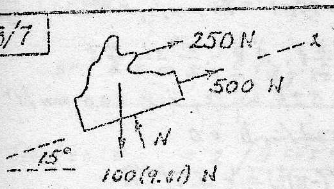
Solve & get $\underline{a = 1.40\text{ m/s}^2}$

(b) $\Sigma F = ma$

$1962 - 1472 = 150a$

$\underline{a = 3.27\text{ m/s}^2}$

3/7

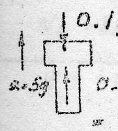


$\Sigma F_x = ma_x$

$500 + 250 - 981\sin 15° = 100a$

$\underline{a = 4.96\text{ m/s}^2}$

3/8



$\Sigma F = ma; \quad 0.1g + 0.005k - 0.1g = 0.1(5g)$

$k = \dfrac{0.5}{0.005}g = \underline{981\text{ N/m}}$

3/9

$F = ma; \quad 2.5 = 70(10^3)a, \quad a = 35.7\ \mu\text{m/s}^2$

$\Delta v = \int a\,dt = at; \quad t = \dfrac{(65-40)(10^6)}{35.7(10^{-6})(3600)^2(24)} = 2251$ days

$= \underline{6.16\text{ years}}$

$\Delta s = \int v\,dt = v_{av}t = \dfrac{65\,000 + 40\,000}{2}(2251)(24)$

$= \underline{2.84(10^9)\text{ km}}$

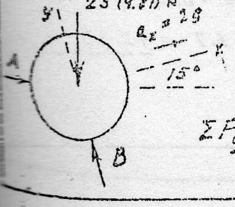
3/10

$F = ma_y; \quad 16t = 3.5\ddot{y}$

$d(\dot{y}) = \dfrac{16t}{3.5}dt, \quad \int_{-3}^{\dot{y}} d(\dot{y}) = \dfrac{16}{3.5}\int_0^t t\,dt$

$\dot{y} + 3 = \dfrac{8}{3.5}t^2, \quad \Delta y = \int_0^3\left(\dfrac{8}{3.5}t^2 - 3\right)dt = \underline{11.57\text{ m}}$

3/11



$\Sigma F_x = ma_x$

$A\cos 15° - 25(9.81)\sin 15° = 25(2)(9.81)$

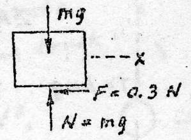
$A = \dfrac{25}{0.9659}(2 + 0.2588)9.81 = \underline{574\text{ N}}$

$\Sigma F_y = 0; \quad B - 25(9.81)\cos 15° - 574\sin 15° = 0$

$\underline{B = 385\text{ N}}$

3/12

Let m = mass of crate



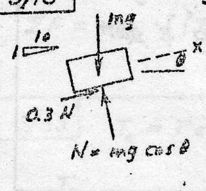
$\Sigma F_x = ma_x; \quad -0.3mg = ma_x$

$a_x = -0.3g = -2.94\text{ m/s}^2$

$\int_v^0 v\,dv = \int_0^s a_x\,dx, \quad -\dfrac{v^2}{2} = a_x s$

$s = \dfrac{(70/3.6)^2/2}{2.94} = \underline{64.3\text{ m}}$

3/13



$\sin\theta = 0.0995, \quad \cos\theta = 0.9950$

$\Sigma F_x = ma_x; \quad 0.3(mg\cos\theta) - mg\sin\theta = ma_x$

$a_x = 9.81(0.2985 - 0.0995)$

$= 1.952\text{ m/s}^2$

$\int_0^v v\,dv = \int_0^x a_x\,dx, \quad v^2 = 2ax$

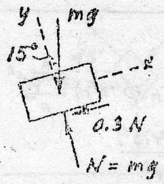
$= 2(1.952)50$

$= 195.2\ (\text{m/s})^2$

$v = 13.97\text{ m/s}$

$= \underline{50.3\text{ km/h}}$

3/14



$\Sigma F_x = ma_x; \quad -mg\sin 15° - 0.3(mg\cos 15°) = ma_x$

$a_x = -9.81(0.3[0.9659] + 0.2588)$

$= -5.38\text{ m/s}^2$

$\int_9^6 v\,dv = -5.38\int_0^s dx$

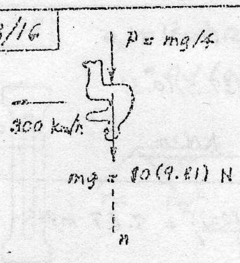
$\frac{1}{2}(6^2 - 9^2) = -5.38(s - 0), \quad \underline{s = 4.18\text{ m}}$

3/15

$\Sigma F_n = ma_n; \quad R + 2(9.81) = 2\dfrac{(4.8)^2}{0.5}$

$R = 92.2 - 19.62 = \underline{72.5\text{ N}}$ Down

3/16



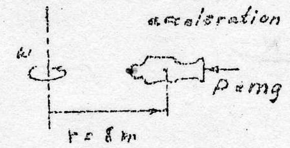
$\Sigma F_n = ma_n; \quad a_n = v^2/\rho$

$mg + mg/4 = mv^2/\rho, \quad \rho = \dfrac{4v^2}{5g}$

so $\rho = \dfrac{4(300/3.6)^2}{5(9.81)} = \underline{566\text{ m}}$

3/17

g = surface gravitational acceleration on earth



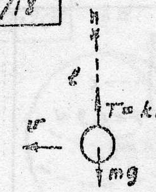
$\Sigma F_n = ma_n$

$mg = mr\omega^2, \quad \omega = \sqrt{\dfrac{g}{r}}$

$\omega = \sqrt{\dfrac{9.81}{8}} = 1.107\text{ rad/s}$

or $\underline{N = 10.57\text{ rev/min}}$

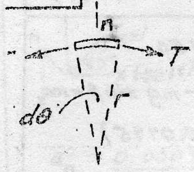
3/18



$\Sigma F_n = ma_n;\ kmg - mg = m\dfrac{v^2}{\ell}$

$v^2 = g\ell(k-1),\quad \underline{v = \sqrt{g\ell(k-1)}}$

3/19



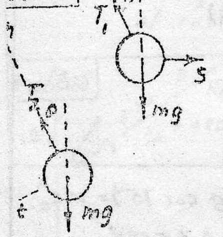
$\Sigma F_n = ma_n;\ 2T\sin\dfrac{d\theta}{2} = \rho r\,d\theta\,\dfrac{v^2}{r}$

where ρ = mass per unit length of rim

Simplify for limit $d\theta \to 0$ & get

$\underline{T = \rho v^2}$

3/20



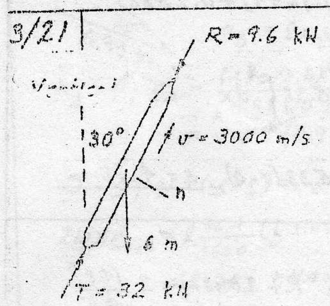
Before: $\Sigma F_y = 0;\ T_1\cos\theta = mg$

After: $\Sigma F_n = ma_n$ where $a_n = \dfrac{v^2}{r} = 0$

$T_2 = m\cos\theta$

Thus $k = T_2/T_1 = \dfrac{m\cos\theta}{m/\cos\theta} = \underline{\cos^2\theta}$

3/21

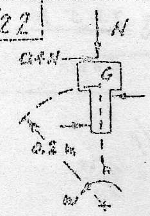


$\Sigma F_n = ma_n;\ 6m\sin 30° = m\dfrac{(3000)^2}{\rho}$

$6\left(\tfrac{1}{2}\right) = \dfrac{9(10^6)}{\rho}$

$\rho = 3(10^6)\text{ m} = \underline{3000\text{ km}}$

3/22



$\omega = \dfrac{3000(2\pi)}{60} = 314.2\text{ rad/s}$

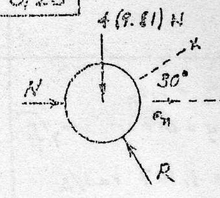
$\Sigma F_n = ma_n;\ N = 2(0.2)(314.2)^2$

$= 39.5(10^3)\text{ N}$

$M = 4fNr = 4(0.4)(39.5)(0.3)$

$= \underline{18.96\text{ kN·m}}$

3/23

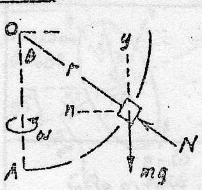


$a_n = r\omega^2 = 0.25\left(\dfrac{60[2\pi]}{60}\right)^2 = 9.87\text{ m/s}^2$

$\Sigma F_x = ma_x;\ N\cos 30° - 39.2\sin 30° = 4(9.87)\cos 30°$

$\underline{N = 62.1\text{ N}}$

3/24



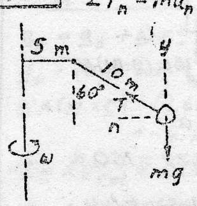
$\Sigma F_y = 0;\ N\cos\theta = mg$

$\Sigma F_n = ma_n;\ N\sin\theta = m(r\sin\theta)\omega^2$

Combine & get

$\underline{\theta = \cos^{-1}\dfrac{g}{r\omega^2},\ \omega \geq \sqrt{g/r}}$

3/25



$\Sigma F_n = ma_n;\ T\sin 60° = m(5 + 10\sin 60°)\omega^2$

$\Sigma F_y = 0;\ T\cos 60° = mg$

$\tan 60° = \dfrac{(5 + 10\sin 60°)\omega^2}{9.81}$

$\omega^2 = 1.244\ (\text{rad/s})^2,\ \omega = 1.115\ \dfrac{\text{rad}}{\text{s}}$

or $N = \dfrac{1.115(60)}{2\pi} = \underline{10.65\text{ rev/min}}$

3/26

$r = 50t^3 - 15t + 100\text{ mm};\ \dot r = 150t^2 - 75\text{ mm/s},\ \ddot r = 300t\ \dfrac{\text{mm}}{\text{s}^2}$

$\theta = 2\sin\dfrac{\pi}{2}t,\ \dot\theta = \pi\cos\dfrac{\pi}{2}t,\ \ddot\theta = -\dfrac{\pi^2}{2}\sin\dfrac{\pi}{2}t$

For $t = 2$ s, $r = 350$ mm, $\dot r = 525$ mm/s, $\ddot r = 600\text{ mm/s}^2$

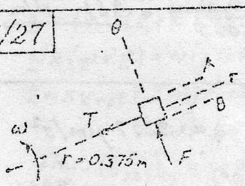
$\theta = 0,\ \dot\theta = -\pi\text{ rad/s},\ \ddot\theta = 0$

$F_r = m(\ddot r - r\dot\theta^2) = 2(0.6 - 0.35(-\pi)^2) = -5.71\text{ N}$

$F_\theta = m(r\ddot\theta + 2\dot r\dot\theta) = 2[0.35(0) + 2(0.525)(-\pi)] = -6.60\text{ N}$

$F = \sqrt{F_r^2 + F_\theta^2} = \sqrt{(5.71)^2 + (6.60)^2} = 8.72\text{ N}$

3/27



$\dot\theta = \omega = 3\text{ rad/s}$

$\ddot\theta = \dot\omega = -2\text{ rad/s}^2$

$\dot r = 0.1\text{ m/s},\ \ddot r = 0$

$r = 0.375\text{ m}$

$\Sigma F_\theta = m(r\ddot\theta + 2\dot r\dot\theta);\ F = 2(0.375[-2] + 2[0.1][3])$

$= -0.300\text{ N}$

Thus contact is on $\underline{\text{side } A}$ & $\underline{F = 0.300\text{ N}}$

$\Sigma F_r = m(\ddot r - r\dot\theta^2);\ -T = 2(0 - 0.375[3]^2)$

$\underline{T = 6.75\text{ N}}$

3/28

$r = 100 + 10\sin 6(12)t\ \text{mm}$

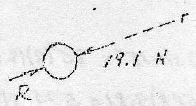
$\dot r = 720\cos 72t\ \text{mm/s},\ \ddot r = -51.84\sin 72t\ \text{m/s}^2$

For r maximum, $\sin 72t = 1$, $\cos 72t = 0$

& $a_r = \ddot r - r\omega^2 = -51.84 - 0.110(12)^2 = -67.7\text{ m/s}^2$

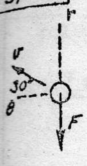
$\Sigma F_r = ma_r;\ R - 19.1 = 0.1(-67.7)$

$= -6.77\text{ N}$



$R = 19.1 - 6.77 = \underline{12.33\text{ N}}$

3/29



$v = 26\ 730$ km/h $= 7425$ m/s, $F = 519$ N

$v_r = \dot r = 7425 \sin 30° = 3712$ m/s

$v_\theta = r\dot\theta = 7425 \cos 30° = 6430$ m/s

$$a_r = \ddot r - r\dot\theta^2 = \ddot r - \frac{(r\dot\theta)^2}{r} = \ddot r - \frac{(6430)^2}{10.729(10^6)}$$

$$= \ddot r - 3.85 \text{ m/s}^2$$

$\Sigma F_r = ma_r$; $-519 = 150(\ddot r - 3.85)$, $\underline{\ddot r = 0.39 \text{ m/s}^2}$

3/30 From Prob. 2/115, $a_r = -\frac{5}{2}R\dot\theta^2 \sin\frac{\theta}{2}$

$\Sigma F_r = ma_r$, $T - mg\sin\theta = m\left(-\frac{5}{2}R\dot\theta^2 \sin\frac{\theta}{2}\right)$

$T = m\left(g\sin\theta - \frac{5}{2}R\dot\theta^2\sin\frac{\theta}{2}\right)$

or $\underline{T = m\sin\frac{\theta}{2}\left(2g\cos\frac{\theta}{2} - \frac{5}{2}R\dot\theta^2\right)}$

(Note: $\dot\theta_{max} = 2\sqrt{\frac{g\cos\theta/2}{5R}}$ for T to be positive)

3/31 $\Sigma F_y = ma_y$; $0.3 = 0.2a_y$, $a_y = 1.5$ m/s²

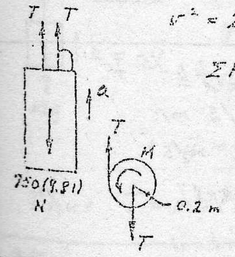
$\Delta v = \int_0^2 a\,dt$; $v_y - 0 = 1.5(2) = 3.0$ m/s²

No change in v_x or v_z, so final velocity is

$\underline{v} = 1.0\underline{i} + 3.0\underline{j} + 4.5\underline{k}$, $v = \sqrt{1.0^2 + 3.0^2 + 4.5^2}$

$= \underline{5.5 \text{ m/s}}$

3/32 a is constant for constant M, so



$v^2 = 2ay$; $a = \frac{3^2}{2(4)} = 1.125$ m/s²

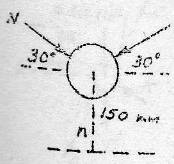
$\Sigma F = ma$; $2T - 750(9.81) = 750(1.125)$

$T = 4100$ N

$\Sigma M = 0$; $0.2(4100) - M = 0$

$\underline{M = 820 \text{ N·m}}$

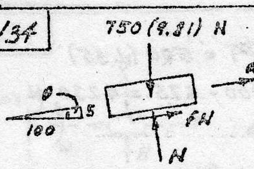
3/33 $\Sigma F_n = ma_n$; $2N\sin 30° = 2.5(0.15)\left(\frac{600(2\pi)}{60}\right)^2$



$N = 1480$ N

$F = 4N\cos 30° = \underline{5130 \text{ N}}$

3/34



$15 - 3 = 12$ m crate

15 m truck

Since $s = \frac{1}{2}at^2$,

$a = \frac{12}{15}a_{truck}$

$\cos\theta = 0.9988$

$\sin\theta = 0.0499$

$= 0.8\frac{v^2}{2s} = 0.8\frac{(40/3.6)^2}{2(15)} = 3.29\ \frac{m}{s^2}$

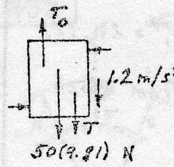
$N = 750(9.81)0.9988$

$= 7350$ N

$\Sigma F = ma$; $f(7350) - 750(9.81)(0.0499) = 750(3.29)$

$\underline{f = 0.386}$

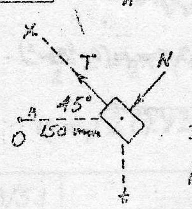
3/35 $T_0 = Te^{f\beta} = Te^{0.2(\pi+\pi)} = 3.51\,T$



$\Sigma F = ma$; $490 + T - 3.51T = 50(1.2)$

$\underline{T = 171.3 \text{ N}}$

3/36 $a_n = r\omega^2 = 0.15\left(300\frac{2\pi}{60}\right)^2 = 148.0$ m/s²

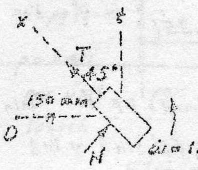


$\Sigma F_x = ma_x$; $T = 3(148.0\cos 45°)$

$= \underline{314 \text{ N}}$

Direction of rotation does not change accel., hence has no influence on T or N.

3/37 $a_t = r\alpha = 0.15(120) = 18$ m/s²



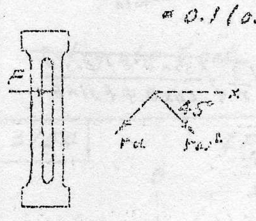
$\Sigma F_x = ma_x$; $T = 3(18\cos 45°) = \underline{38.2 \text{ N}}$

$\Sigma F_n = ma_n = 0$; $N\frac{1}{\sqrt2} - \frac{38.2}{\sqrt2} = 0$

$\underline{N = 38.2 \text{ N}}$

3/38 $a_x = r\omega^2\cos 45° - r\alpha\sin 45°$

$= 0.1(0.7071)(9^2 - 24) = 4.03$ m/s²

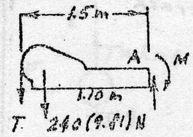


$\Sigma F_x = ma_x$; $F = 10(4.03)$

$= \underline{40.3 \text{ N}}$

contacts slot on right side

3/39



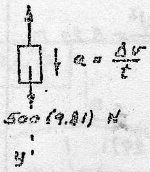
$\Sigma F_y = ma_y;\ 4900 - T = 500(1.35)$

$T = 4900 - 675 = 4230$ N

Beam: $\Sigma M_A = 0;$

$M = 240(9.81)(1.10) + 4230(1.5)$

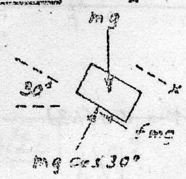
$= \underline{8930 \text{ N}\cdot\text{m}}$



$a = \frac{\Delta v}{t} = \frac{6-0.6}{4} = 1.35 \text{ m/s}^2$

3/40

$\Sigma F_x = ma_x;\ mg \sin 30° - fmg \cos 30° = m a_x$



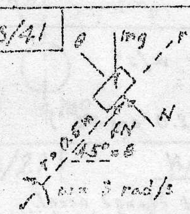
$a_x = 9.81\left(0.5 - f\frac{\sqrt{3}}{2}\right)$

$\int_{v_1}^{v_2} v\,dv = \int_0^2 a_x\,dx,\ v_2^2 - v_1^2 = 2a_x(2)$

$1^2 - 0.4^2 = 2(9.81)(0.5 - f\frac{\sqrt{3}}{2})2$ from which

$\underline{f = 0.553}$

3/41



$\Sigma F_\theta = ma_\theta;\ N - mg\cos\theta = m(0)$

$N = mg \cos\theta$

$\Sigma F_r = ma_r;\ fN - mg\sin\theta = m(\ddot{r} - r\omega^2)$

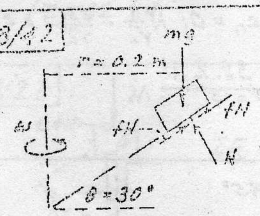
where $\ddot{r} = 0$ up to the point of slipping.

Thus $fmg\cos\theta - mg\sin\theta = -mr\omega^2$

& $f = \tan\theta - \frac{r\omega^2}{g\cos\theta} = 1 - \frac{0.5(3)^2}{9.81(1/\sqrt{2})}$

$= 1 - 0.649 = \underline{0.351}$

3/42



fN is down for ω_{max}, up for ω_{min}

$\Sigma F_y = 0;\ N\cos\theta \mp fN\sin\theta = mg$

$\Sigma F_n = ma_n;\ N\sin\theta \pm fN\cos\theta = mr\omega^2$

upper sign for ω_{max}
lower " " ω_{min}

Combine & get

$\frac{\sin\theta \pm f\cos\theta}{\cos\theta \mp f\sin\theta} = \frac{r\omega^2}{g},\ \omega = \sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta \pm f\cos\theta}{\cos\theta \mp f\sin\theta}}$

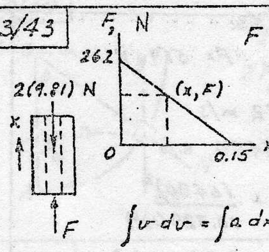
So $\omega = \sqrt{\frac{9.81}{0.2}}\sqrt{\frac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$

Upper sign $\omega_{max} = 7.21$ rad/s

lower sign $\omega_{min} = 3.41$ rad/s

or $\underline{32.5 < N < 68.9 \text{ rev/min}}$

3/43



$F = 262 - 1750x = 1750(0.15 - x)$, x in m

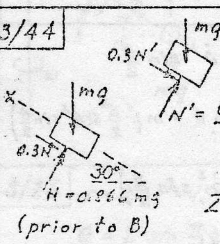
$\Sigma F_x = ma_x;\ 1750(0.15 - x) - 19.62 = 2a$

$\underline{a = 121.4 - 875x \text{ m/s}^2}$

$\int v\,dv = \int a\,dx;\ \int_0^v v\,dv = \int_0^{0.15}(121.4 - 875x)\,dx$

$\frac{v^2}{2} = 121.4x - 875\frac{x^2}{2}\Big|_0^{0.15} = 8.37,\ \underline{v = 4.09 \text{ m/s}}$

3/44



mg (just past B)

$\Sigma F_n = ma_n;\ 0.866\,mg - 0.866\,mg/2$

$= m\frac{v_B^2}{2}$

$v_B^2 = 0.866\,g$

(prior to B) $\quad \Sigma F_x = ma_x;\ -0.5\,mg - 0.3(0.866\,mg) = ma_x$

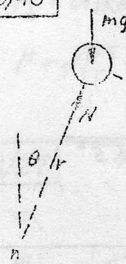
$a_x = -0.760\,g$

$v_B^2 = u^2 + 2a_x x;\ u^2 = 0.866\,g - 2(-0.760\,g)2\sqrt{3}$

$= 6.13(9.81) = 60.1\ (\text{m/s})^2$

$\underline{u = 7.75 \text{ m/s}}$

3/45



$\Sigma F_t = ma_t;\ mg\sin\theta = ma_t,\ a_t = g\sin\theta$

$\omega\,d\omega = \alpha\,d\theta;\ \int_0^\omega \omega\,d\omega = \int_0^\theta \frac{a_t}{r}\,d\theta.$

$\frac{\omega^2}{2} = \frac{g}{r}(-\cos\theta)\Big|_0^\theta,\ \omega^2 = \frac{2g}{r}(1 - \cos\theta)$

$\Sigma F_n = ma_n;\ mg\cos\theta - N = mr\omega^2$

so $N = mg\cos\theta - 2mg(1 - \cos\theta) = mg(3\cos\theta - 2)$

$N = 0$ when $3\cos\theta - 2 = 0,\ \theta = \cos^{-1}\frac{2}{3} = \underline{48.19°}$

3/46

$\underline{r} = \underline{i}\sin 2\pi t - 16\underline{j}(1 - e^{-t/4}) + \frac{1}{4}\underline{k}t^2$ m

$\underline{\dot{r}} = \underline{i}(2\pi\cos 2\pi t) - \underline{j}(4e^{-t/4}) + \underline{k}t/2$ m/s

$\underline{\ddot{r}} = -\underline{i}(4\pi^2\sin 2\pi t) + \underline{j}e^{-t/4} + \underline{k}/2$ m/s^2

$\Sigma F_x = R_x = m(\ddot{r})_x;\ R_x = -2(4\pi^2\sin 2\pi t)$

$\underline{R_x = -79.0\sin 2\pi t \text{ N}}$

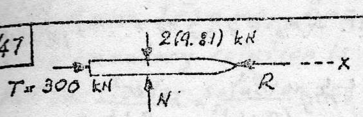
$t = 4$ s; $\underline{\ddot{r}} = -\underline{i}(0) + \underline{j}e^{-1} + \underline{k}/2$

$= 0.368\,\underline{j} + 0.5\,\underline{k}$ m/s^2

$|\ddot{r}| = \sqrt{0.368^2 + 0.5^2} = 0.621$ m/s^2

$\underline{R} = m\underline{\ddot{r}};\ |R| = 2(0.621) = \underline{1.242 \text{ N}}$

3/47



$R = 0.5\,v$ kN

$\Sigma F_x = ma_x;\ 300 - 0.5v = 2a$

or $(300-0.5v)\,dx = 2v\,dv$

$$\int_0^x dx = 2\int_0^v \frac{v\,dv}{300-0.5v}$$

$$x = 2\frac{1}{(0.5)^2}\Big[300 - 0.5v - 300\ln(300-0.5v)\Big]_0^{400}$$

$$= 8.00\Big[-200 - 300\ln\frac{100}{300}\Big]$$

$$= 8.00\,[-200 + 300(1.099)]$$

$$= \underline{1037\ \text{m}}$$

3/48

$\Sigma F = ma;\ 2F\cos\theta = m(-\ddot{x}),\ \frac{2K}{m}\frac{l-b}{b}\frac{x}{l} = -\ddot{x}$

but $\dot{x}\,d\dot{x} = \ddot{x}\,dx$, so $\frac{1}{2}\int_v^0 d\dot{x}^2 = -2\frac{K}{m}\int_0^x\left(\frac{1}{b}-\frac{1}{l}\right)x\,dx$

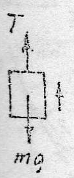
& $l = \sqrt{x^2+b^2}$, hence

$$\frac{v^2}{2} = 2\frac{K}{m}\int_0^x\left(\frac{x}{b} - \frac{x}{\sqrt{x^2+b^2}}\right)dx = 2\frac{K}{m}\left[\frac{x^2}{2b} - \sqrt{x^2+b^2} + b\right]$$

$$v^2 = \frac{2K}{mb}\left(\sqrt{b^2+x^2} - b\right)^2;\quad v = \left(\sqrt{b^2+x^2} - b\right)\sqrt{\frac{2K}{mb}}$$

3/49

$\Sigma F_y = ma_y;\ T - mg = m(-\ddot{y})$



$y^2 + b^2 = s^2$ where $s = \overline{CB}$

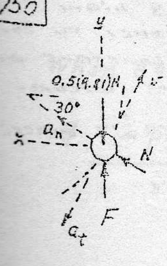
$y\dot{y} + 0 = s\dot{s}$, but $2\dot{s} = \frac{d}{2}\omega$

so $y\dot{y} = s\frac{\omega d}{4};\ y\ddot{y} + \dot{y}^2 = \left(\frac{\omega d}{4}\right)^2 + 0$

$$\ddot{y} = \frac{1}{y}\left[\left(\frac{\omega d}{4}\right)^2 - \left(\frac{s}{y}\frac{\omega d}{4}\right)^2\right] = \frac{1}{y}\frac{\omega^2 d^2}{16}\left(1 - \frac{s^2}{y^2}\right)$$

Thus $T = mg(1 - \ddot{y}/g) = \underline{mg\left[1 + \frac{\omega^2 b^2 d^2}{16 g y^3}\right]}$

3/50



$v = v_y/\cos 30° = 2/0.866 = 2.31$ m/s

$a_n = v^2/r = \overline{2.31}^2/0.25 = 21.3$ m/s²

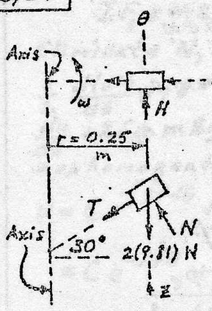
$\ddot{v}_y = 0$ so $a_t(0.866) = 21.3(0.5)$

$a_t = 12.32$ m/s²

$\Sigma F_x = ma_x;\ 0.866N = 0.5(21.3(0.866) + 12.32[0.5])$

$\underline{N = 14.22\ \text{N}}$

3/51



$a_r = \ddot{r} - r\dot{\theta}^2$

$= 0 - 0.25(3\pi)^2 = -22.2$ m/s²

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= 0 + 2(0.1)\frac{\sqrt{3}}{2}(3\pi) = 0.3\sqrt{3}\,\pi$ m/s²

$a_z = 0;\ a_N = -a_r\sin 30°$

$= 22.2(0.5) = 11.1$ m/s²

$\Sigma F_N = ma_N;\ N - 19.62\cos 30° = 2(11.1)$

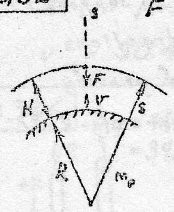
$\underline{N = 39.2\ \text{N}}$

$\omega = \frac{90(2\pi)}{60} = 3\pi$ rad/s

$\Sigma F_\theta = ma_\theta;\ H = 2(0.3)\sqrt{3}\pi = \underline{3.26\ \text{N}}$

3/52

$F = \gamma\frac{mm_e}{s^2}$ where m_e = mass of earth



When $s = R$, $F = mg$, so $\gamma m_e = gR^2$

Thus $F = gm\frac{R^2}{s^2}$

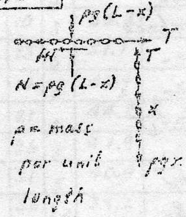
$\Sigma F_s = ma_s;\ -gm\frac{R^2}{s^2} = ma_s,\ a_s = -gR^2/s^2$

$v\,dv = a\,ds;\ \int_v^0 v\,dv = -gR^2\int_R^\infty\frac{ds}{s^2}$

$-\frac{v^2}{2} = -gR^2\left(0 + \frac{1}{R}\right),\ v = \sqrt{2gR}$

$v = \sqrt{2(9.821)(10^{-3})(6370)} = 11.19$ km/s

3/53



Initial value of x for impending slipping given by equil.

$f\rho g(L-b) = \rho g b,\ b = \frac{f}{1+f}L$

$\Sigma F = ma;\ T - f\rho g(L-x) = \rho(L-x)a$

$\rho g x - T = \rho x a$

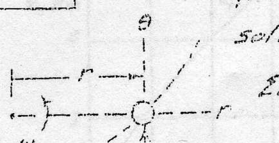
Eliminate T & get $a = \frac{g}{L}[x(1+f) - fL]$

$v\,dv = a\,dx;\ \int_0^v v\,dv = \int_{\frac{fL}{1+f}}^{L}\frac{g}{L}[x(1+f) - fL]\,dx$

Integrate & get $\underline{v = \sqrt{\frac{gL}{1+f}}}$

3/54

$\Sigma F_r = ma_r;\ 0 = m(\ddot{r} - r\omega^2)$



Sol. is $r = r_0\cosh\omega t,\ \dot{r} = r_0\omega\sinh\omega t$

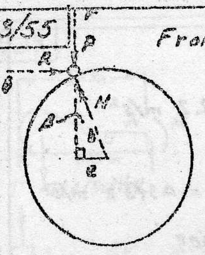
$\Sigma F_\theta = ma_\theta;\ N = m(0 + 2\dot{r}\dot{\theta}),\ \dot{\theta} = \omega$

$= 2mr_0\omega^2\sinh\omega t$

But $\cosh^2\omega t - \sinh^2\omega t = 1$, $\sinh\omega t = \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$

So $\underline{N = 2m\omega^2\sqrt{r^2 - r_0^2}}$

3/55



From sol. to Prob. 2/121

$a_r = \dfrac{2e^2-b^2}{\sqrt{b^2-e^2}}\,\omega^2, \quad a_\theta = -2e\omega^2$

$\Sigma F_r = ma_r;\ N\cos\beta - P = m\,a_r$

$\Sigma F_\theta = ma_\theta;\ N\sin\beta - R = m\,a_\theta$

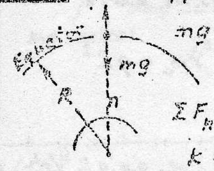
Eliminate N & get

$-R = m(a_\theta - a_r\tan\beta) - P\tan\beta$

so $R = m\left(+2e + \dfrac{2e^2-b^2}{\sqrt{b^2-e^2}}\,\dfrac{e}{\sqrt{b^2-e^2}}\right)\omega^2 + \dfrac{Pe}{\sqrt{b^2-e^2}}$

$\underline{R = me\omega^2\,\dfrac{b^2}{b^2-e^2} + \dfrac{Pe}{\sqrt{b^2-e^2}}}$

3/56



W' = apparent gravity force (force to support)

mg = true gravitational attraction

where g = absolute accel. of gravity

$\Sigma F_n = ma_n;\ mg - W' = mR\omega^2,\ W' = m(g - R\omega^2)$

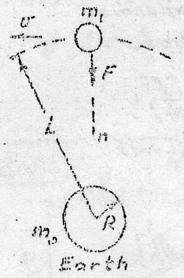
$k = \dfrac{W'}{mg} = \underline{1 - \dfrac{R\omega^2}{g}}$

For values given

$W' = 100\left(1 - \dfrac{6378(10^3)(7.729)^2(10^{-8})}{9.815}\right)$

$= 100(1 - 0.00345) = \underline{99.655\ \text{N}}$

3/57



$F = K\dfrac{m_e m_1}{L^2}$ But $Km_e = gR^2$ so $F = \dfrac{gR^2 m_1}{L^2}$

where g = gravitational accel. at earth's surface

For moon, $\Sigma F_n = ma_n;$

$\dfrac{gR^2 m_1}{L^2} = m_1\dfrac{v^2}{L}, \quad v^2 = \dfrac{gR^2}{L}$

Period $\tau = \dfrac{2\pi L}{v} = \dfrac{2\pi L}{R}\sqrt{\dfrac{L}{g}}$

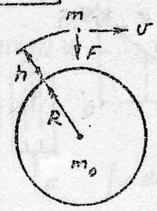
$L = 384\ 398\ (10^3)\ \text{m}$

$R = 6370\ (10^3)\ \text{m}$

$\tau = \dfrac{2\pi\,(384\ 398)}{6370}\sqrt{\dfrac{384\ 398\ (10^3)}{9.824}} = 23.72\ (10^5)\ \text{s}$

or $\tau = \dfrac{23.72\ (10^5)}{(3600)(23.93)} = \underline{27.5\ \text{days}}$

3/58



$F = K\dfrac{mm_0}{(R+h)^2}$, but $Km_0 = gR^2$, so $F = mg\dfrac{R^2}{(R+h)^2}$

$\Sigma F = ma_n;\ mg\dfrac{R^2}{(R+h)^2} = m\dfrac{v^2}{R+h}$

so $v = R\sqrt{\dfrac{g}{R+h}}$

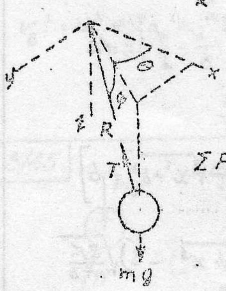
$\tau = \dfrac{2\pi(R+h)}{v} = \dfrac{2\pi(R+h)}{R\sqrt{g/(R+h)}} = 2\pi\,\dfrac{(R+h)^{3/2}}{R\sqrt{g}}$

For $h = 640$ km, $v = 6370\sqrt{\dfrac{9.824(10^{-3})}{(6370+640)}}\,(3600)$

$= \underline{27\ 147\ \text{km/h}}$ or $v = 7541$ m/s

& $\tau = 2\pi\,\dfrac{(6370+640)^{3/2}}{6370\sqrt{9.824(10^{-3})}} = 5842$ s or $\underline{\tau = 1\ \text{h}\ 37\ \text{min}\ 22\ \text{s}}$

3/59



$\Sigma F_R = ma_R;\ \underline{mg\sin\varphi - T = -mR(\dot\varphi^2 + \dot\theta^2\cos^2\varphi)}$

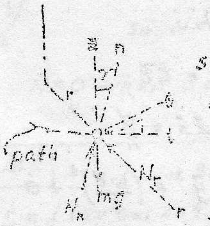
$\Sigma F_\theta = ma_\theta;\ \underline{0 = \ddot\theta\cos\varphi - 2\dot\varphi\dot\theta\sin\varphi}$

$\Sigma F_\varphi = ma_\varphi;\ \underline{mg\cos\varphi = mR(\ddot\varphi + \dot\theta^2\sin\varphi\cos\varphi)}$

where a_R, a_θ, a_φ come from

Eqs. 26 with R constant

3/60



$\Sigma F_t = ma_t;\ mg\sin\gamma = ma_t,\ a_t = g\sin\gamma$

$\Sigma F_n = ma_n;\ N_n - mg\cos\gamma = ma_n = 0$

since $a_n = \ddot z\cos\gamma + r\ddot\theta\sin\gamma$ where

$z = z_0 - \dfrac{h}{2\pi r}\,r\theta = z_0 - r\theta\tan\gamma;\ \ddot z = -r\ddot\theta\tan\gamma$

$\Sigma F_r = ma_r;\ N_r = m(2\pi g\sin 2\gamma)$

where a_r is obtained as in Prob. 2/16_

so $\underline{N = mg\sqrt{\cos^2\gamma + 4\pi^2\sin^2 2\gamma}}$

3/61 Force on ion is $e\mathbf{v}\times\mathbf{H}$ which is always in x-y plane and normal to $\mathbf{v}$. Hence curvilinear motion in x-y plane with tangential acceleration equal to zero. Thus magnitude of $\mathbf{v}$ remains constant and is therefore v_0. Motion must be circular in x-y. At A, $\Sigma F_x = ma_x$; $ev_0H = m\dfrac{v_0^2}{r}$ where r is the radius. So $r = mv_0/eH$

Center of circle lies at

$\left(\dfrac{mv_0}{eH},\ 0,\ 0\right)$

3/62 From $\Sigma F_r = ma_r$, $\ddot{r} - r\omega^2 = g \sin\omega t$, $\theta = \omega t$

Complementary solution is $r = A\cosh\omega t + B\sinh\omega t$

For particular solution try $r = K\sin\omega t$ which gives

$K = -\frac{g}{2\omega^2}$; thus $r = A\cosh\omega t + B\sinh\omega t - \frac{g}{2\omega^2}\sin\omega t$

When $t=0$, $r=0$ so $0 = A + 0 - 0$, $A = 0$

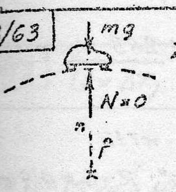
" " , $\dot{r} = 0$ so $0 = B\omega - \frac{g}{2\omega}$, $B = \frac{g}{2\omega^2}$

Thus $\underline{r = \frac{g}{2\omega^2}(\sinh\theta - \sin\theta)}$

$\Sigma F_\theta = ma_\theta$ gives $N = mg\left(\cos\theta - \frac{2\omega}{g}\dot{r}\right)$

$\underline{N = mg\,(2\cos\theta - \cosh\theta)}$

3/63



$\Sigma F_n = ma_n$; $mg = m\frac{v^2}{\rho}$, $v^2 = g\rho$, $v = \sqrt{g\rho}$

At top of hump $dy/dx = 0$

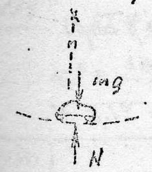
& radius of curvature is

$\rho = (d^2y/dx^2)^{-1}$ where $y = b\sin\frac{2\pi x}{L}$

$\frac{dy}{dx} = \frac{2\pi b}{L}\cos\frac{2\pi x}{L}$, $\frac{d^2y}{dx^2} = -\frac{4\pi^2}{L^2}b\sin\frac{2\pi x}{L}$

At top of hump $x = \frac{L}{4}$, $\sin\frac{2\pi x}{L} = 1$

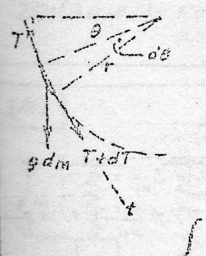
& $\left|\frac{d^2y}{dx^2}\right| = \frac{1}{\rho} = \frac{4\pi^2 b}{L^2}$; thus $\underline{v = \frac{L}{2\pi}\sqrt{g/b}}$



At bottom of dip $\Sigma F_n = ma_n$

$N - mg = m\frac{v^2}{\rho}$, $N = mg(1+1) = \underline{2mg}$

3/64



$g\,dm = \rho g r\,d\theta$; $\Sigma F_t = ma_t$;

$(T+dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} + \rho g r\,d\theta\,(\cos\theta)$

$= \rho r\,d\theta\,(a_t)$

$dT + \rho g r\cos\theta\,d\theta = \rho r a_t\,d\theta$

$dT = \rho r(a_t - g\cos\theta)\,d\theta$

$\int_0^T dT = \rho r\int_0^\theta (a_t - g\cos\theta)\,d\theta$

$T = \rho r(a_t\theta - g\sin\theta)$

When $\theta = \frac{\pi}{2}$, $T = 0$, so $0 = \rho r(a_t\frac{\pi}{2} - g)$; $\underline{a_t = \frac{2g}{\pi}}$

& $\underline{T = \rho g r\left[\frac{2\theta}{\pi} - \sin\theta\right]}$

3/65 $\Sigma F_n = ma_n$; $N - mg\sin\theta = m\frac{v^2}{r}$

$\Sigma F_t = ma_t$; $mg\cos\theta - fN = m\dot{v}$

Eliminate N, substitute $v\,dv = \dot{v}r\,d\theta$ & get

$\frac{1}{2}\frac{d(v^2)}{d\theta} + fv^2 = gr(\cos\theta - f\sin\theta)$ or let $u = v^2$ so that

$\frac{du}{d\theta} + 2fu = 2gr(\cos\theta - f\sin\theta)$ which is a linear

non homogeneous diff. eq. with known solution of

$u = Ce^{-\int 2f\,d\theta} + e^{-\int 2f\,d\theta}\int 2gr(\cos\theta - f\sin\theta)e^{\int 2f\,d\theta}\,d\theta$

$= Ce^{-2f\theta} + e^{-2f\theta}\frac{2gre^{2f\theta}}{1+4f^2}(2f\cos\theta + \sin\theta - 2f^2\sin\theta + f\cos\theta)$

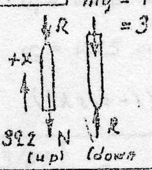
$u = v^2 = Ce^{-2f\theta} + \frac{2gr}{1+4f^2}\left(3f\cos\theta + [1-2f^2]\sin\theta\right)$

When $\theta = 0$, $v = 0$ & $C = -\frac{2gr}{1+4f^2}3f$

$v^2_{\theta=\frac{\pi}{2}} = \frac{2gr}{1+4f^2}\left(1 - 2f^2 - 3fe^{-f\pi}\right)$; for $f = \frac{1}{3}$, $r = 3$ m,

$v^2 = 30.4\ (\text{m/s})^2$, $\underline{v = 5.52\ \text{m/s}}$

3/66



$mg = 4(9.81)$
$= 39.2$ N

$\Sigma F_x = ma_x$

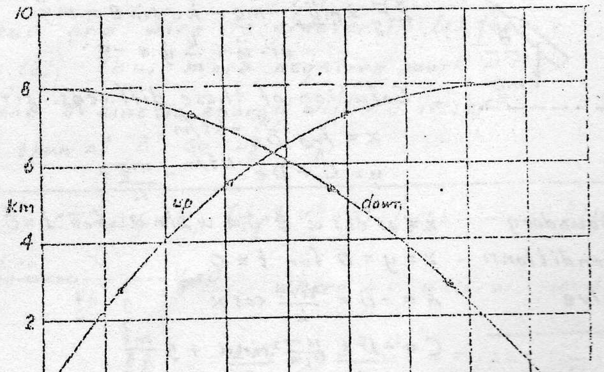
(up) $-(R+39.2) \cong 4\frac{\Delta\dot{x}}{\Delta t}$, $\Delta\dot{x} \approx -\frac{1}{4}(R+39.2)\Delta t$

(down) $R - 39.2 \approx 4\frac{\Delta\dot{x}}{\Delta t}$, $\Delta\dot{x} \approx \frac{1}{4}(R-39.2)\Delta t$

& $\Delta x = \dot{x}_{AV}\Delta t$

UP						DOWN					
t, s	Δt, s	$R+39.2$, N	$\Delta\dot{x}$, m/s	$\dot{x}$, m/s	Δx, m	t, s	Δt, s	$R-39.2$, N	$\Delta\dot{x}$, m/s	$\dot{x}$, m/s	Δx, m
0	2	95.4	-47	500	952	0	5	-39.2	-49	0	-123
2	2	80.8	-40	452	864	5	5	-38.3	-48	-49	-365
4	3	71.7	-54	412	1154	10	5	-37.3	-47	-97	-602
7	3	61.5	-46	358	1004	15	5	-35.6	-45	-144	-830
10	3	55.3	-42	312	873	20	5	-33.4	-42	-186	-1046
13	3	51.1	-38	270	754	25	5	-30.8	-39	-230	-1246
16	5	47.9	-60	232	1010	30	5	-27.3	-34	-269	-1428
21	5	44.2	-55	172	722	35	5	-24.2	-30	-303	-1599
26	5	41.9	-52	117	453	40	5	-20.3	-25	-333	-1728
31	5	40.4	-51	64	196	45				-358	
36	1.4	39.2	-14	14	10						
37.4				0							

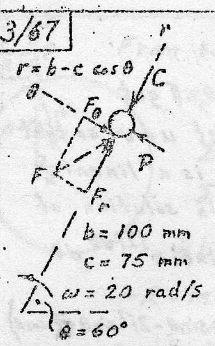
$x = \Sigma\Delta x = 7490$ m



t = total time $37.4 + 42.2 = \underline{80\ \text{s}}$

x = Max. altitude $\approx \underline{8.00\ \text{km}}$

3/67



From Prob. 2/86,

$a_r = (2c\cos\theta - b)\omega^2 = (0.15[0.5] - 0.1)(20)^2 = -10.00\ m/s^2$

$a_\theta = 2c\omega^2\sin\theta = 0.15(20)^2\sqrt{3}/2 = 52.0\ m/s^2$

$C = k\Delta r = 5.4([100-75(\frac{1}{2})] - [100-75(1)]) = 202\ N$

$\Sigma F_r = ma_r;\ F_r - 202 = 0.5(-10)$

$F_r = -5 + 202 = 197\ N$

Since F is normal to smooth cam surface,

$F_\theta / F_r = dr/rd\theta,\quad F_\theta = \frac{dr}{rd\theta}F_r$

Thus $F_\theta = \frac{c}{r}\sin\theta\,(F_r) = \frac{75}{100-75(1/2)}\frac{\sqrt{3}}{2}(197) = 205\ N$

$\Sigma F_\theta = ma_\theta;\ P - 205 = 0.5(52.0)$

$\underline{P = 205 + 26.0 = 231\ N}$

3/68

$\Sigma F_x = m\ddot{x};\ T\sin\theta = m\ddot{x},\ T\sin kt = m\ddot{x}$

$\dot{x} = -\frac{T}{km}\cos kt + C_1$

$t = 0,\ \dot{x} = 0,\ C_1 = \frac{T}{km},\ \dot{x} = \frac{T}{mk}(1-\cos kt)$

$\underline{\&\ x = \frac{T}{mk}(t - \frac{1}{k}\sin kt)}$

$\Sigma F_y = m\ddot{y};\ T\cos\theta - mg = m\ddot{y}$

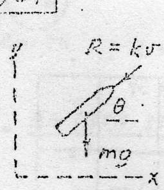
$\ddot{y} = \frac{T}{m}\cos kt - g;\quad \dot{y} = \frac{T}{mk}\sin kt - gt + C_2$

$u = C_2$ so $\dot{y} = \frac{T}{mk}\sin kt - gt + u$

$\&\ y = -\frac{T}{mk^2}\cos kt - \frac{1}{2}gt^2 + ut + C_3,\quad C_3 = \frac{T}{mk^2}$

$\underline{y = \frac{T}{mk^2}(1-\cos kt) - \frac{1}{2}gt^2 + ut}$

3/69



$\Sigma F_x = m\ddot{x};\ -kv\cos\theta = m\ddot{x}$

or $\ddot{x} + \frac{k}{m}\dot{x} = 0$

$\Sigma F_y = m\ddot{y};\ -mg - kv\sin\theta = m\ddot{y}$

or $\ddot{y} + \frac{k}{m}\dot{y} = -g$

Solution of these diff. eqs. gives

$x = A + Be^{-kt/m}$

$y = C + De^{-kt/m} - \frac{mg}{k}t$

Boundary Conditions give: $\dot{x} = u\cos\alpha\ \&\ \dot{y} = u\sin\alpha$ for $t = 0$

$x = y = 0$ for $t = 0$

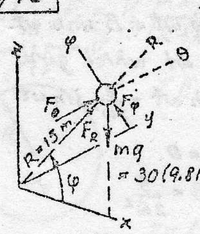
$A = -D = \frac{um}{k}\cos\alpha$

$C = -D = \frac{um}{k}\sin\alpha + g\frac{m^2}{k^2}$

so $x = \frac{mu}{k}\cos\alpha\,(1 - e^{-kt/m}),\quad x_{max} = \frac{mu}{k}\cos\alpha$

$\underline{y = \frac{m}{k}\left(u\sin\alpha + \frac{gm}{k}\right)(1 - e^{-kt/m}) - \frac{gm}{k}t}$

3/70



For R = const., $\varphi = 60°$, $\dot{\varphi} = 0.3\frac{rad}{s}$, $\dot{\theta} = \omega = 0.2$ rad/s

$a_R = -R(\dot{\varphi}^2 + \dot{\theta}^2\cos^2\varphi)$

$= -15(\overline{0.3}^2 + \overline{0.2}^2[\frac{1}{2}]^2) = -1.5\ m/s^2$

$a_\theta = R\cos\varphi\,\ddot{\theta} - 2R\dot{\varphi}\dot{\theta}\sin\varphi$

$= 0 - 2(15)(0.3)(0.2)\frac{\sqrt{3}}{2} = -0.9\sqrt{3}\ m/s^2$

$a_\varphi = R\ddot{\varphi} + R\dot{\theta}^2\sin\varphi\cos\varphi$

$= 0 + 15(0.2)^2\frac{\sqrt{3}}{2}\frac{1}{2} = 0.15\sqrt{3}\ m/s^2$

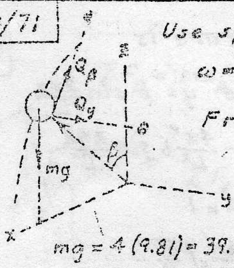
$\Sigma F_R = ma_R;\ F_R - 294\frac{\sqrt{3}}{2} = 30(-1.5),\ F_R = 209.9\ N$

$\Sigma F_\theta = ma_\theta;\ F_\theta = 30(-0.9\sqrt{3}) = -46.8\ N$

$\Sigma F_\varphi = ma_\varphi;\ F_\varphi - 294\frac{1}{2} = 30(+0.15\sqrt{3}),\ F_\varphi = 154.8\ N$

$F = \sqrt{\overline{209.9}^2 + \overline{46.8}^2 + \overline{154.8}^2} = \sqrt{70211} = \underline{265\ N}$

3/71



Use spherical coordinates with

$\omega = \dot{\theta},\ \beta = 90° - \varphi;\ \beta = 60°,\ R = L = 0.9\ m$

From Eq. 2B

$a_\varphi = 2\dot{R}\dot{\varphi} + R\dot{\theta}^2\sin\varphi\cos\varphi$

$= -2\dot{L}\dot{\beta} + L\omega^2\sin\beta\cos\beta$

$mg = 4(9.81) = 39.2\ N$

$= -2(0.6)(45\frac{\pi}{180}) + 0.9(60\frac{2\pi}{60})^2\frac{\sqrt{3}}{2}\frac{1}{2}$

$= 14.44\ m/s^2$

$a_\theta = 2\dot{R}\dot{\theta}\cos\varphi - 2R\dot{\theta}\dot{\varphi}\sin\varphi = 2\omega(\dot{L}\sin\beta + L\dot{\beta}\cos\beta)$

$= 2(2\pi)(0.6\frac{\sqrt{3}}{2} + 0.9\frac{45\pi}{180}\frac{1}{2}) = 10.97\ m/s^2$

$\Sigma F_\varphi = ma_\varphi;\ -39.2\frac{\sqrt{3}}{2} + Q_\beta = 4(14.44),\quad Q_\beta = 91.8\ N$

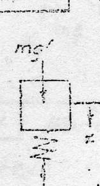
$\Sigma F_\theta = ma_\theta;\ Q_y = 4(10.97) = 43.9\ N$

Total shear $Q = \sqrt{\overline{91.8}^2 + \overline{43.9}^2} = \underline{101.7\ N}$

In space, omit gravity force, so

$Q = \sqrt{\overline{57.8}^2 + \overline{43.9}^2} = \underline{72.5\ N}$

3/72



x = spring deformation, m

$g' = 1.62\ m/s^2$ (moon gravity)

$\Sigma F_x = m\ddot{x};\ mg' - 3kx = m\ddot{x},\ \ddot{x} + 3\frac{k}{m}x = g'$

Sol. is $x = A\sin pt + B\cos pt + \frac{g'm}{3k},\ p^2 = \frac{3k}{m}$

$\dot{x} = Ap\cos pt - Bp\sin pt$

When $x = 0$, let $t = 0$; $0 = 0 + B + \frac{g'm}{3k},\ B = -\frac{g'm}{3k} = -\frac{g'}{p^2}$

$x = 0,\ \dot{x} = v_0 = 1.5\ m/s;\ v_0 = Ap,\ A = v_0/p$

"Jerk" $J = \dddot{x} = -\frac{3k}{m}\dot{x} = -p^2(v_0\cos pt + \frac{g'}{p}\sin pt)$

For max., $\frac{dJ}{dt} = -p^2(-v_0 p\sin pt + g'\cos pt) = 0$

so $\tan pt = \frac{g'}{v_0 p};\ \sin pt = \frac{g'}{\sqrt{v_0^2p^2 + g'^2}},\ \cos pt = \frac{v_0 p}{\sqrt{v_0^2p^2 + g'^2}}$

$\&\ J = -\frac{p}{\sqrt{v_0^2p^2 + g'^2}}(\overline{v_0 p}^2 + \overline{g'}^2) = -p\sqrt{v_0^2p^2 + g'^2}$

so $|J| = \sqrt{\frac{3(15)}{17.5}}\sqrt{(1.5)^2\frac{3(15)}{17.5} + 1.62^2}$ [illegible]

3/76

$\tan\theta = \frac{1}{10}$, $\cos\theta = 0.9950$, $\sin\theta = 0.0995$

$U = \Delta T;\ (mg\sin\theta - fmg\cos\theta)s = -\frac{1}{2}mv^2$

$(0.976 - 9.76f)15 = -\frac{1}{2}(50/3.6)^2$

$\underline{f = 0.759}$

3/77

$\Delta T + \Delta V_g = 0;\ -\frac{1}{2}m(1)^2 + m(9.81)(0.5)(1-\cos\theta) = 0$

$\cos\theta = 0.8981$, $\underline{\theta = 26.1°}$

3/78

$U = \Delta T;\ U = \int(\underline{F} + m\underline{g})\cdot d\underline{r}$

$= \int(F_x\,dx + F_y\,dy + [F_z - mg]\,dz)$

$= \int(-15\,dx + 10\,dy + [15 - 2(9.81)]dz) = -15(-0.4) + 10(0.8)$
$-4.62(-0.5)$
$= 19.31$ J

$\Delta T = \frac{1}{2}2(v_B^2 - 0) = v_B^2$

$19.31 = v_B^2$, $\underline{v_B = 4.39 \text{ m/s}}$

3/79

$\Delta T + \Delta V_g = 0;\ \frac{1}{2}m(v_C^2 - 0) - mg(0.6 - 0.9/2) = 0$

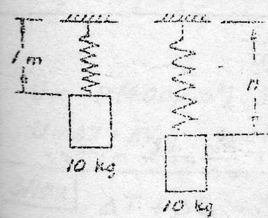
$v_C^2 = 2.94\ (\text{m/s})^2$, $\underline{v_C = 1.716 \text{ m/s}}$

3/80

$U = \Delta T + \Delta V_g;\ 250(\sqrt{2.4^2 + 1.8^2} - 0.6) = \frac{1}{2}10v^2$
$+10(9.81)2.4$

$250(3.0 - 0.6) = 5v^2 + 235$, $v^2 = 72.9\ (\text{m/s})^2$, $\underline{v = 8.54 \text{ m/s}}$

3/81



$\Delta T = 0$ so $\Delta V_g + \Delta V_e = 0$

$-10(9.81)h - 10(9.81)(-1) + \frac{1}{2}450(h-1)^2 = 0$

$22.5h^2 - 54.8h + 32.3 = 0$

$h = \frac{54.8}{45.0} \pm \frac{1}{45.0}\sqrt{(54.8)^2 - 4(22.5)(32.3)}$

$= \underline{1.436 \text{ m}}$ or 1 m

3/82

$\Delta T + \Delta V_g = 0;$ When $y = 0$, $x = 0$

$\frac{1}{2}mv_A^2 + mg(0 - \frac{0.9}{\sqrt{2}}) = 0$, $v_A^2 = 2(9.81)\frac{0.9}{\sqrt{2}}$

$\underline{v_A = 3.53 \text{ m/s}}$

3/83

Power $P = \dot{V}_g;\ 0.70P = 70(9.81)(6)(100)/60$
$= 6867$ W

$\underline{P = 9.81 \text{ kW}}$

3/84

$P = \underline{F}\cdot\underline{\dot{r}} = (60\underline{i} - 25\underline{j} - 40\underline{k})\cdot(1.2\underline{i} + 1.8t\underline{j} - 1.8t^2\underline{k})$

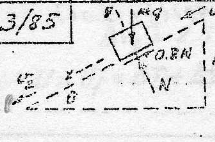
which for $t = 4$ s becomes

$P = (60\underline{i} - 25\underline{j} - 40\underline{k})\cdot(1.2\underline{i} + 7.2\underline{j} - 28.8\underline{k})$

$= 72 - 180 + 1152 = 1044$ W

or $\underline{P = 1.044 \text{ kW}}$

3/85



$\Sigma F_y = 0;\ N - mg\cos\theta = 0$

$U = \Delta T;\ (mg\sin\theta - 0.3mg\cos\theta)\frac{1.8}{\sin\theta}$

$= \frac{1}{2}m(\overline{0.15}^2 - \overline{0.45}^2)$

$v_1 = 0.45$ m/s

$v_2 = 0.15$ m/s

$1.8(9.81)(1 - \frac{0.3}{\tan\theta}) = \frac{-0.180}{2}$

$\tan\theta = 0.2985$, $\underline{\theta = 16.62°}$

3/86

For $x = 75$ mm, $U = \Delta T$ &

$\frac{1}{2}0.075R_m = \frac{1}{2}0.25(600)^2$, $R_m = 1.2$ MN

For $x = 25$ mm, $R = \frac{1}{3}1.2(10^6) = 0.4(10^6)$ N

$U = \Delta T;\ \frac{1}{2}(0.025)0.4(10^6) = \frac{1}{2}0.25(\overline{600}^2 - v^2)$

$v^2 = 320\,000\ (\text{m/s})^2$, $\underline{v = 566 \text{ m/s}}$

3/87

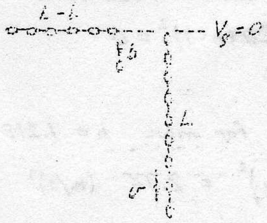
With no friction, $\Delta V_g + \Delta T = 0$,

$-mgh + \frac{1}{2}mv_B^2 = 0$, $v_B = v_{B'} = \sqrt{2gh}$

With some friction, the friction force will be greater in case (a) than in case (b), because the normal reaction between bead and wire is greater in (a) than in (b). Thus more negative work will be done & kinetic energy will be less at B than at B' so $v_B < v_{B'}$.

3/88

$\Delta T + \Delta V_g = 0;\ (\frac{1}{2}\rho Lv^2 - 0) + (-\rho gL\frac{L}{2} - [illegible]) = 0$



where ρ = mass per unit length

$\frac{Lv^2}{2} = g\frac{L^2 - l^2}{2}$, $\underline{v = \sqrt{g\frac{L^2 - l^2}{L}}}$

3/89 $U = \Delta T + \Delta V_g + \Delta V_e$, $\Delta T = 0$

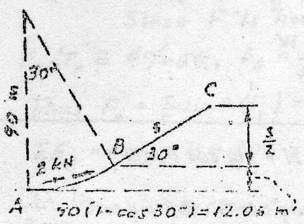
$U = 200(\sqrt{0.45^2 + 0.225^2} - 0.225) = 200(0.278) = 55.6 \text{ J}$

$\Delta V_g = 7(9.81)(0.45) = 30.9 \text{ J}$

$\Delta V_e = \frac{1}{2}k(0.075)^2 = 0.00281k \text{ J}$

so $55.6 = 30.9 + 0.00281\,k$, $\underline{k = 8.79 \text{ kN/m}}$

3/90 $\overline{AB} = r\theta = 90\frac{\pi}{6}$ $= 47.1 \text{ m}$



$U = \Delta T + \Delta V_g$

But $\Delta T = 0$ from A to B

$U = 2000(47.1) = 94.2(10^3) \text{ J}$

$\Delta V_g = 100(9.81)(\frac{s}{2} + 12.06)$

Thus,

$94.2(10^3) = 981(\frac{s}{2} + 12.06)$

$\underline{s = 168.0 \text{ m}}$

3/91 Neglect ΔV_g ; $\Delta V_e = 0 - 2(\frac{1}{2})k(l-b)^2$

$= -\frac{2}{2}k(\sqrt{b^2+x^2} - b)^2$

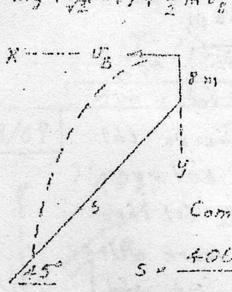
$\Delta T = \frac{1}{2}mv^2 - 0$; Also, stiffness of entire spring AB is $k = K/b$

Substitute in $\Delta V_e + \Delta T = 0$ & get

$mv^2 = \frac{2K}{b}(\sqrt{b^2+x^2} - b)^2$, $\underline{v = \sqrt{\frac{2K}{mb}}(\sqrt{b^2+x^2} - b)}$

3/92 From A to B, $\Delta V_g + \Delta T = 0$

$-mg(\frac{32}{\sqrt{2}} - 8) + \frac{1}{2}mv_B^2 = 0$, $v_B = 16.94 \text{ m/s}$



x-dir.: $a_x = 0$, $x = \dot{x}t$, so

$s/\sqrt{2} = 16.94\,t$

y-dir.: $a_y = g$, $y = \frac{1}{2}gt^2$, so

$s/\sqrt{2} + 8 = \frac{9.81}{2}t^2$

Combine & get $4.905^2 - 406s - 4590 = 0$

$s = \frac{406 \pm \sqrt{164\,700 + 90\,100}}{9.81} = \frac{406 \pm 505}{9.81}$

$= 92.8 \text{ m}$ or -10.08 m $\underline{s = 92.8 \text{ m}}$

3/93 $\Delta T + \Delta V_g + \Delta V_e = 0$

$\frac{1}{2}10v^2 - 10(9.81)(h-1) + \frac{1}{2}450(h-1)^2 = 0$

$v^2 = 9.81(2)(h-1) - 45(h-1)^2$

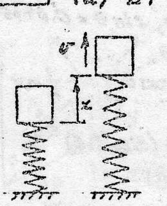
$\frac{d(v^2)}{dh} = 9.81(2) - 90(h-1) = 0$ for max. $h = 1.218 \text{ m}$

$v_{max}^2 = 9.81(2)(1.218-1) - 45(1.218-1)^2 = 2.14 \quad (\text{m/s})^2$

$v_{max} = \sqrt{2.14} = \underline{1.46 \text{ m/s}}$

3/94 (a) $\Delta T = 0$, $\Delta V_g + \Delta V_e = 0$; $2(9.81)h - \frac{1}{2}120(0.5)^2 = 0$

$\underline{h = 0.764 \text{ m}}$



(b) $\Delta T + \Delta V_g + \Delta V_e = 0$;

$\frac{1}{2}2v^2 + 2(9.81)x - \frac{1}{2}120[(0.5)^2 - \underbrace{(0.5-x)^2}_{=0}]$

$v^2 = 40.4x - 60x^2$ assuming $x \leq 0.5 \text{ m}$

$d(v^2)/dx = 40.4 - 120x = 0$ for max. v; $x = 0.336 \text{ m}$

& $v_{max} = \sqrt{(40.4 - 60[0.336])0.336} = \underline{2.61 \text{ m/s}}$

3/95 Let $V_g = 0$ in initial position $(x = 0)$

x = deformation of spring

$\Delta V_g + \Delta V_e + \Delta T = 0$; $\Delta V_g = -mgx$, $\Delta V_e = \frac{1}{2}kx^2$, $\Delta T = \frac{1}{2}mv^2$

$-mgx + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = 0$

For max. v, $\frac{1}{2}m\frac{d(v^2)}{dx} = mg - kx = 0$, $x = \frac{mg}{k}$

so $v^2 = \frac{2}{m}\left(\frac{(mg)^2}{k} - \frac{1}{2}k\left[\frac{mg}{k}\right]^2\right) = \frac{mg^2}{k}$, $\underline{v = g\sqrt{\frac{m}{k}}}$

Max δ & R occur for $\Delta T = 0$

$-mgx + \frac{1}{2}kx^2 = 0$, $x = \delta = \underline{\frac{2mg}{k}}$; $R = k\delta = \underline{2mg}$

3/96 $x^2 + y^2 = \overline{0.9}^2$, $x\dot{x} + y\dot{y} = 0$

$\Delta T + \Delta V_g = 0$; $\frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mg(y - \frac{0.9}{\sqrt{2}}) = 0$

$\dot{x}^2\left(1 + \frac{x^2}{y^2}\right) = 2(9.81)\left(\frac{0.9}{\sqrt{2}} - y\right)$, $\overline{0.9}^2\dot{x}^2 = 19.62\left(\frac{0.9}{\sqrt{2}}y^2 - y^3\right)$

For max. $\dot{x}$, $\frac{d(\dot{x}^2)}{dy} = \frac{19.62}{0.81}\left(\frac{1.8}{\sqrt{2}}y - 3y^2\right) = 0$

so $y = \frac{0.6}{\sqrt{2}} \text{ m}$ & $\dot{x}^2 = \frac{19.62}{0.81}\left(\frac{0.162}{\sqrt{2}} - \frac{0.108}{\sqrt{2}}\right) = \frac{19.62\sqrt{2}}{30}$

$v_{B_{max}} = \dot{x} = \sqrt{\frac{19.62\sqrt{2}}{30}} = \underline{0.962 \text{ m/s}}$

3/97 Driving force $F = m\frac{dv}{dt}$

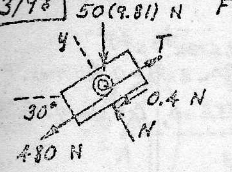
Power $P = Fv$ constant

Thus $\frac{P}{v} = m\frac{dv}{dt}$ or $P\,ds = mv\frac{dv}{dt}ds = mv^2dv$

& $\int_0^s P\,ds = \int_{v_1}^{v_2} m\,v^2dv$; $Ps = \frac{m}{3}(v_2^3 - v_1^3)$

& $\underline{s = \frac{m}{3P}(v_2^3 - v_1^3)}$

3/98



For 1.2-m displ. of block, P moves 1.2 m down incline. T does no work.

$\Sigma F_y = 0;\ N = 50(9.81)\frac{\sqrt{3}}{2} = 425$ N

$0.4N = 169.9$ N

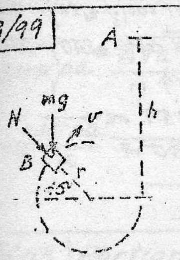
$U = \Delta T;$

$480(1.2) - 169.9(1.2) - 490(\tfrac{1}{2})(1.2) = \tfrac{1}{2}50v^2$

$v^2 = \frac{77.8}{25} = 3.11\ (m/s)^2$

$\underline{v = 1.764\ m/s}$

3/99



$\Delta T + \Delta V_g = 0;\ \tfrac{1}{2}mv^2 - mg(h - \frac{r}{\sqrt{2}}) = 0$

$v^2 = 2g(h - \frac{r}{\sqrt{2}})$

$\Sigma F_n = ma_n;\ N + \frac{mg}{\sqrt{2}} = m a_n$

where $a_n = v^2/r = 2g(\frac{h}{r} - \frac{1}{\sqrt{2}})$

So $N = mg\left[2(\frac{h}{r} - \frac{1}{\sqrt{2}}) - \frac{1}{\sqrt{2}}\right] = mg\left[2\frac{h}{r} - \frac{3}{\sqrt{2}}\right]$

For $m = 0.25$ kg, $r = 0.15$ m, $h = 0.6$ m,

$N = 0.25(9.81)\left[2\frac{0.60}{0.15} - \frac{3}{\sqrt{2}}\right] = \underline{14.42\ N}$

3/100

$\Delta T + \Delta V_g = 0,\qquad V_g = -\frac{mgR^2}{r}$

$\tfrac{1}{2}m(v_B^2 - [7100]^2) = 9.81(10^{-3})(3600)^2 m(6370)^2(\frac{1}{7200} - \frac{1}{16\,000})$

$\tfrac{1}{2}v_B^2 - 25.2(10^6) = 394(10^6)\ (km/h)^2$

$\underline{v_B = 29\,000\ km/h}$

3/101

$\Delta T = \tfrac{1}{2}m(v_B^2 - v_A^2) = \tfrac{1}{2}22(\overline{12\,600}^2 - \overline{13\,000}^2)(3.6)^{-2}$

$= -12.94(10^6)$ J

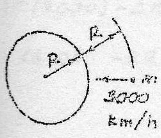
$\Delta V_g = -mgR^2(\frac{1}{r_B} - \frac{1}{r_A}) = -22(9.81)(6370)^2(\frac{1}{6450} - \frac{1}{6410})(10^3)$

$= 8.47(10^6)$ J

$U = -P(400)(10^3)$ J

$U = \Delta T + \Delta V_g;\ -400(10^3)P = (-12.94 + 8.47)(10^6),\ \underline{P = 11.16\ N}$

3/102



$\Delta T + \Delta V_g = 0,\ \Delta T = \tfrac{1}{2}m(v^2 - [3000/3.6]^2)$

$\Delta V_g = -mgR^2(\frac{1}{R} - \frac{1}{2R}) = -mgR/2$

$= -m\frac{1.62}{2}(1738)(10^3)$

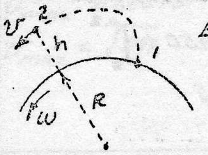
Thus $v^2 - \left(\frac{3000}{3.6}\right)^2 - 1.62(1738)(10^3) = 0$

$v^2 = 3.51(10^6)\ (m/s)^2$

$v = 1873$ m/s, or $v = 1873(3.6) = \underline{6740\ km/h}$

3/103

From Prob. 3/58 $v = R\sqrt{\frac{g}{R+h}}$



$E_2 = T_2 + V_2 = \tfrac{1}{2}mR^2\frac{g}{R+h} - \frac{gR^2m}{R+h}$

$E_1 = T_1 + V_1 = \tfrac{1}{2}mR^2\omega^2 - \frac{gR^2m}{R}$

$\underline{E = E_2 - E_1 = \tfrac{1}{2}mR\left[g\frac{R+2h}{R+h} - R\omega^2\right]}$

3/104

$\Delta T = \tfrac{1}{2}(2.5)(2.4^2 - 1.8^2) = 3.15$ J

$\Delta V_e = \tfrac{1}{2}30[(\sqrt{1.5^2 + 1.2^2} - 0.9)^2 - (\sqrt{0.9^2 + 1.2^2} - 0.9)^2]$

$= 10.23$ J

$\Delta V_g = 2.5(9.81)(-0.9) = -22.1$ J

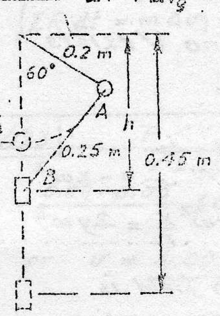
$U = U_f = \Delta T + \Delta V_e + \Delta V_g;\ U_f = 3.15 + 10.23 - 22.1$

$U_f = -8.69$ J

$|U_f| = F_{av}\Delta s;\ 8.69 = F_{av}\sqrt{0.9^2 + 1.5^2},\ \underline{F_{av} = 4.97\ N}$

3/105

$\Delta T + \Delta V_g = 0;\ \Delta V_g = mg(-0.45 + h) + mg(-0.2 + 0.2\tfrac{1}{2})$



where $h = 0.1 + \sqrt{0.25^2 - (0.2\sin 60°)^2}$

$= 0.280$ m

$\Delta V_g = -mg(0.45 - 0.280) - mg(0.2 - 0.1)$

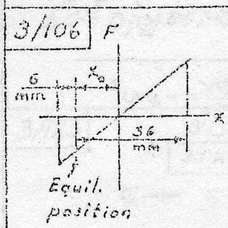
$= -0.1697mg + 0.1mg = -0.270mg$

$\Delta T = \tfrac{1}{2}m(v_A^2 - 0)$

Thus $\tfrac{1}{2}v_A^2 - 0.270(9.81) = 0$

$v_A^2 = 5.29\ (m/s)^2,\quad \underline{v_A = 2.30\ m/s}$

3/106



Let x = stretch of spring from position of zero force

At equil. position $\Sigma F = 0;\ 2.5(9.81) + kx_0 = 0$

$x_0 = -\frac{2.5(9.81)}{1.8} = -13.62$ mm

$\Delta V_g + \Delta V_e + \Delta T = 0$

$\Delta V_g = -mgh = -2.5(9.81)(0.042) = -1.030$ J

$\Delta V_e = \tfrac{1}{2}k(x_2^2 - x_1^2) = \frac{1.8}{2}[(x_0 - 6)^2 - (x_0 + 36)^2]$

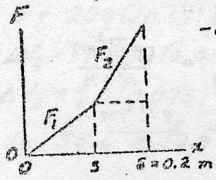
$= 0.9[-84x_0 - 1260] = 1030 - 1134 = -104$ mJ

$\Delta T = \tfrac{1}{2}mv^2 = \tfrac{1}{2}2.5v^2 = 1.25v^2$

Thus $1.25v^2 - 1.030 - 0.104 = 0,\ v^2 = 0.907\ (m/s)^2$

$\underline{v = 0.952\ m/s}$

3/107 $F_1 = 300x$, $F_2 = 300x + 150(x-s) = 450x - 150s$



$$-U = \frac{1}{2}300s^2 + \int_s^{0.2\,m}(450x - 150s)\,dx$$

$$= 150s^2 + 225x^2\Big]_s^{0.2} - 150sx\Big]_s^{0.2}$$

$$= 75s^2 - 30s + 9 \text{ J}$$

$U = \Delta T$;

$$75s^2 - 30s + 9 = \frac{1}{2}0.5(5)^2$$

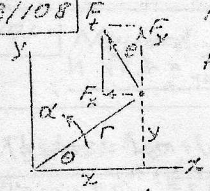
or $75s^2 - 30s + 2.75 = 0$

$$s = \frac{30 \pm \sqrt{30^2 - 4(75)(2.75)}}{2(75)} = \frac{30 \pm 8.66}{150}$$

$$= 0.1423 \text{ m or } 0.257 \text{ m}$$

0.257 m > 200 mm so impossible; Thus $\underline{s = 142.3 \text{ mm}}$

3/108



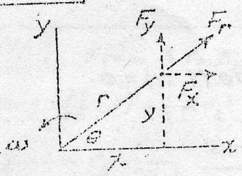
$F_x = -F_t \sin\theta = -r\alpha\frac{y}{r} = -\alpha y$

$F_y = F_t\cos\theta = r\alpha\frac{x}{r} = \alpha x$

$$\left(\frac{\partial F_x}{\partial y} = -\alpha\right) \neq \left(\frac{\partial F_y}{\partial x} = +\alpha\right)$$

so field is nonconservative & no V exists

3/109



$F_x = F\cos\theta = -r\omega^2\frac{x}{r} = -x\omega^2$

$F_y = F\sin\theta = -r\omega^2\frac{y}{r} = -y\omega^2$

$\frac{\partial F_x}{\partial y} = 0 = \frac{\partial F_y}{\partial x}$ so field is conservative

$dV = -\underline{F}\cdot d\underline{r} = -(-r\omega^2)dr$, $\quad V = \int_0^r r\omega^2 dr = \frac{1}{2}r^2\omega^2$

$V = \frac{1}{2}(x^2+y^2)\omega^2$, $\quad F_x = -\frac{\partial V}{\partial x} = -x\omega^2$, $\quad F_y = -\frac{\partial V}{\partial y} = -y\omega^2$

▲3/110 Power developed by spring is $P = Fv = 120(0.5-x)\dot{x}$

P_{max} occurs where P^2 is max. & from Prob. 3/94

$$P^2 = (120)^2(0.5-x)^2 60(0.673x - x^2)$$

$$= 0.864(10^6)(0.1682x - 0.923x^2 + 1.673x^3 - x^4)$$

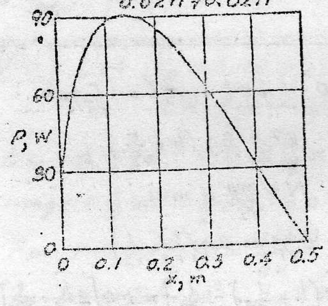
$$\frac{d(P^2)}{dx} = 0.864(10^6)(0.1682 - 1.846x + 5.02x^2 - 4x^3) = 0 \text{ for max. } P^2$$

To solve cubic $x^3 - \frac{5.02}{4}x^2 + \frac{1.846}{4}x - \frac{0.1682}{4} = 0$, let $x = y + \frac{5.02}{3(4)}$

which gives $y^3 = 0.0633y - 0.00463$

"p" $= 0.0211$, "q" $= -0.00231$; $q^2 - p^3$ is neg., so

$$\cos u = \frac{-0.00231}{0.0211\sqrt{0.0211}} = -0.755, \quad u = 139.0°$$



$y_1 = 2\sqrt{0.0211}\cos\frac{u}{3} = 0.201$

$y_2 = 2\sqrt{0.0211}\cos\left(\frac{u}{3} + 120°\right) = -0.282$

$y_3 = 2\sqrt{0.0211}\cos\left(\frac{u}{3} + 240°\right) = 0.0817$

so $x_1 = 0.619$ m

$\underline{x_2 = 0.1359 \text{ m}}$

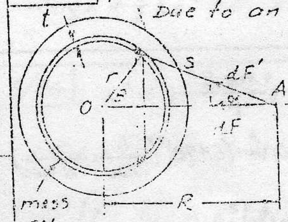
$x_3 = 0.500$ m

$\underline{P_{x_2} = 91.4 \text{ W}}$

▲3/111 ρ = mass density; φ = angle of rotation around OA

Due to an element of ring of mass dm',

$$dF' = \gamma\frac{dm'}{s^2} = \gamma\frac{\rho\, r d\theta\,(r\sin\theta)d\varphi\, t}{R^2 + r^2 - 2Rr\cos\theta}$$



Integrate over φ, 0 to 2π

$$dF = dF'\cos\alpha = 2\pi\rho\gamma r^2 t\frac{\sin\theta\cos\alpha}{R^2 + r^2 - 2Rr\cos\theta}$$

$$= 2\pi\rho\gamma r^3 t\frac{(R - r\cos\theta)\sin\theta\, d\theta}{(R^2 + r^2 - 2Rr\cos\theta)^{3/2}}$$

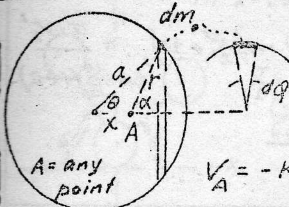
Let $u = R - r\cos\theta$, $du = r\sin\theta\, d\theta$

$$\& \quad F = 2\pi\rho\gamma r\, t\int_{\theta=0}^{\theta=\pi}\frac{u\,du}{(r^2 - R^2 + 2Ru)^{3/2}} = \pi\gamma\rho\frac{tr}{R^2}\left[\frac{2r(r - R\cos\theta)}{(r^2 + R^2 - 2Rr\cos\theta)}\right]$$

$$= \pi\gamma\rho t\frac{2r^2}{R^2}\left[\frac{R+r}{R+r} - \frac{r-R}{R-r}\right] = \frac{4\pi r^2\gamma\rho t}{R^2} = \frac{\gamma m'}{R^2}$$

Replace m' by dm mass of differential shell of solid sphere & F by dF & write $dF = \gamma dm/R^2$ which yields $F = \gamma m/R^2$ same as if m were concentrated at a point, distant R from A.

3/112 $dm_0 = (r\sin\alpha)\,d\varphi\; a\,d\theta\,\rho$, ρ = mass per unit area of shell



$dV_A = -K\dfrac{dm_0}{r}$ where $K\,dm_0$ is equivalent to gR^2 of Eq. 55

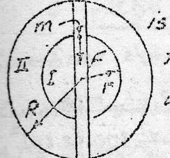
$$V_A = -K\rho a\int_0^{2\pi}\!\!\int_0^{\pi} \sin\alpha\, d\theta\, d\varphi = -2\pi K\rho a\int_0^{\pi}\sin\alpha\, d\theta$$

$$= -2\pi K\rho a\int_0^{\pi}\frac{a\sin\theta\, d\theta}{\sqrt{a^2+x^2-2ax\cos\theta}} = 2\pi K\rho a^2\int\frac{du}{\sqrt{C+Du}} \text{ where } \begin{cases} u=\cos\theta \\ C=a^2+x^2 \\ D=-2ax\end{cases}$$

$$= 2\pi K\rho a^2\, \frac{2\sqrt{a^2+x^2-2ax\cos\theta}}{-2ax}\Bigg]_0^{\pi} = 2\pi K\rho a^2\,\frac{(a+x)-(a-x)}{-ax}$$

$$= -K\frac{4\pi\rho a^2}{a} = \underline{-K\frac{m_0}{a}}$$ independent of x, so constant throughout

3/113 Portion II of earth consists of concentric shells within which potential, by Prob 3/112, is constant – hence no gradient & no force. Force is, then, $F = K\dfrac{m_I m}{r^2}$



where $m_I = \int_0^r 4\pi x^2\rho\, dx$ & m = particle mass

If $\rho = f(x)$ were known, then $U = \Delta T$ would give

$F(-dr) = d\left(\frac{1}{2}mv^2\right)$

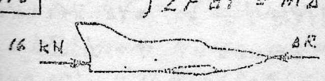
so $-\displaystyle\int_0^R\left[\frac{Km}{r^2}\int_0^r 4\pi x^2\rho\, dx\right]dr = \int_v^0 d\left(\tfrac{1}{2}mv^2\right)$

or $4\pi Km\displaystyle\int_0^R\frac{1}{r^2}\int_0^r \rho x^2\, dx\, dr = \frac{1}{2}mv^2$

where ρ = density of earth at radius x (concentric symmetry assumed)

v = velocity reached by m at center of earth

3/118 $\int\Sigma F\, dt = m\Delta v$;



$[2(8000) - \Delta R]9 = 10\,000\,(1050-1000)/3.6$

$16\,000 - \Delta R = 15\,432$; $\underline{\Delta R = 568 \text{ N}}$

3/119 Impulse $I = m\Delta v = mv_{max}$

v_{max} occurs where $\dot{v} = 0$ or $P = R = kv^2$

$v_{max} = \sqrt{P/k}$ so $\underline{I = m\sqrt{P/k}}$

3/120 For system of disk & projectile $\Delta G = 0$

so $180(3000) + 0 = 180(1500) + 960v'$, $\underline{v' = 281 \text{ m/s}}$

3/121 $\underline{F} = \underline{\dot{G}} = \dfrac{d}{dt}(6t^3\underline{i} + 15t^2\underline{j})$

$= 18t^2\underline{i} + 30t\,\underline{j}$

For $t = 3$ s, $\underline{F} = 18(3^2)\underline{i} + 30(3)\underline{j} = 162\underline{i} + 90\underline{j}$ N

& $F = \sqrt{162^2 + 90^2} = \underline{185.3 \text{ N}}$

3/122 6th rocket burns out after $2\frac{3}{4}$ s, so total impulse on sled during 3 s is $6(100)(10^3) - 3R$ N·s

$\int\Sigma F\, dt = m\Delta v$; $600(10^3) - 3R = 3(10^3)(150)$

$\underline{R = 50 \text{ kN}}$

3/123 $F \longleftarrow \bigcirc \xrightarrow{v} - - - x$ $\quad\int\Sigma F\, dt = m\Delta v$

$-\dfrac{0.09-0.05}{2}\,6000 = 5(v-20)$, $v = -4.00$ m/s

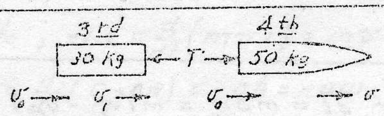
or $\underline{v = 4.00 \text{ m/s to the left.}}$

3/124 Impact velocity $v_0 = \sqrt{2gh} = \sqrt{2(9.81)(1.2)} = 4.85$ m/s

$\Delta G = 0$; $400(4.85) + 0 = (400+250)v$

$\underline{v = 2.99 \text{ m/s}}$

3/125



$\int\Sigma F\, dt = \Delta G$; $-T(0.5) = 30(v_1 - v_0)$

$T(0.5) = 50(v - v_0)$

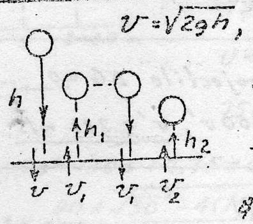
$0.5T\left(\frac{1}{30}+\frac{1}{50}\right) = (v - v_0) - (v_1 - v_0)$, $v - v_1 = 0.0267\,T$

But $v - v_1 = 10$ m/s, so $T = \dfrac{10}{0.0267} = \underline{375 \text{ N}}$

3/126 $v = \sqrt{2gh}$, $v' = \sqrt{2gh'}$, $e = \frac{v'}{v} = \underline{\sqrt{\frac{h'}{h}}}$

$n = \frac{mgh - mgh'}{mgh} = 1 - \frac{h'}{h}$, $\underline{n = 1 - e^2}$

3/127 $v = \sqrt{2gh}$, $v_1 = \sqrt{2gh_1}$, $v_2 = \sqrt{2gh_2}$

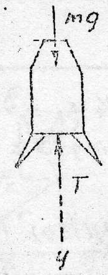


$e = \frac{v_1}{v}$ & $e = \frac{v_2}{v_1}$

so $v_1^2 = v v_2$

& $e = \frac{\sqrt{v v_2}}{v} = \sqrt{\frac{v_2}{v}} = \underline{\left(\frac{h_2}{h}\right)^{1/4}}$

3/128



$mg = 200(1.62) = 324$ N

$T = 18\,000/10 = 1800$ N

$\int \Sigma F_y\,dt = m\Delta v_y$

$(324 - 1800)t = 200(1.5 - 45)$

$\underline{t = 5.89\text{ s}}$

3/129 $\int \Sigma F_y\,dt = m\Delta v_y$; $0.4t = 0$ [illegible]

or $v_y = \frac{dy}{dt} = 0.8t - 5$, $y = \int_0^t (0.8t - 5)\,dt$

$= 0.4t^2 - 5t$

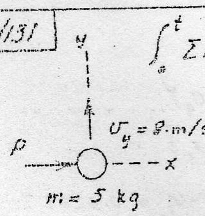
For $y = 0$, $(0.4t - 5)t = 0$, $\underline{t = 12.5\text{ s}}$

3/130 $T_y = 600\cos\theta$; $\dot\theta = \pi/10$ rad/s, so $dt = \frac{10}{\pi}d\theta$

$\int F_y\,dt = m\Delta v_y$; $\int_0^{\pi/2} 600\cos\theta\left(\frac{10}{\pi}d\theta\right) = 260(v_y - 0)$

$\frac{6000}{\pi}\sin\theta\Big|_0^{\pi/2} = 260 v_y$, $v_y = \frac{6000}{260\pi} = \underline{7.35\text{ m/s}}$

3/131



$\int_0^t \Sigma F_x\,dt = m\Delta v_x = m(v_{2_x} - v_{1_x})$

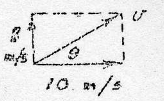
From graph:

$\int_0^2 \Sigma F_x\,dt = \frac{40}{2} + \frac{40}{2} + \frac{20}{2} = 50$ N·s

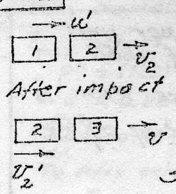
so $50 = 5(v_{2_x} - 0)$, $v_{2_x} = 10$ m/s

$v = \sqrt{10^2 + 8^2} = \underline{12.81\text{ m/s}}$,

$\theta = \tan^{-1}\frac{8}{10} = \underline{38.7°}$



3/132 $\Delta G = 0$; $mu = m(u' + v_2)$; $e = \frac{v_2 - u'}{u}$



Solve for $v_2 = \frac{u}{2}(1+e)$

After impact

$\Delta G = 0$; $m\frac{u}{2}(1+e) = m(v_2' + v)$; $e = \frac{v - v_2'}{\frac{u}{2}(1+e)}$

Solve for $\underline{v = \frac{u}{4}(1+e)^2}$

3/133 Let v_s & v_b stand for rebound velocities from steel & brass plates, respectively.

Impact velocity $= \sqrt{2gh} = \sqrt{19.62(0.15)} = 1.716$ m/s

$0.6 = \frac{v_s}{1.716}$, $v_s = 1.029$ m/s

$0.4 = \frac{v_b}{1.716}$, $v_b = 0.686$ m/s

$\omega = \frac{1.029 - 0.686}{0.60}$

$= \underline{0.572\text{ rad/s CCW}}$

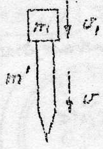
3/134 Final velocity of m is to be zero

so for $\Delta G = 0$, $mv_1 + 0 = 300v + 0$

$e = \frac{v - 0}{v_1 - 0}$, so $0.3 = \frac{m}{300}$, $\underline{m = 90\text{ kg}}$

Thus for $v_1 = \sqrt{19.62(4)} = 8.86$ m/s

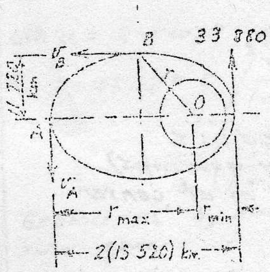
$v = ev_1 = 0.3(8.86) = \underline{2.66\text{ m/s}}$



$\Delta E = mgh - \frac{1}{2}m'v^2 = 90(9.81)(4) - \frac{1}{2}300(2.66)^2$

$\Delta E = 3530 - 1059 = \underline{2470\text{ J}}$

3/135 $\Sigma M_O = \dot H_O = 0$ so H_O = constant



$r_{min} = 6370 + 390 = 6760$ km

$r_{max} = 2(13\,520) - 6760$

$= 20\,280$ km

For H_O constant,

$6760(33\,880) = 11\,720\,v_B = 20\,280\,v_A$

$\underline{v_A = 11\,290\text{ km/h}}$

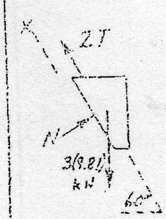
$\underline{v_B = 19\,540\text{ km/h}}$

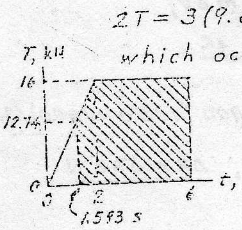
3/136 $\int \Sigma F_x\,dt = m\Delta v_x$; Skip begins to move when

$2T = 3(9.81)\frac{\sqrt{3}}{2}$, $T = 12.74$ kN

which occurs at $t = \frac{12.74}{16}2$

$= 1.593$ s



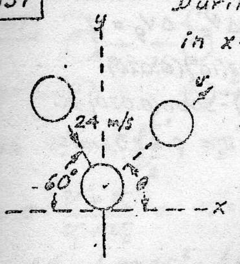


$\int \Sigma F_x\,dt = 2\left[\frac{16 + 12.74}{2}(2 - 1.593) + 16(6 - 2)\right] - 3(9.81)\frac{\sqrt{3}}{2}(6 - 1.593)$

$= 27.4$ kN·s

Thus $27.4 = 3(v - 0)$, $\underline{v = 9.13\text{ m/s}}$

3/137 During impact $\Sigma F_x = 0$ so no change in x-component of velocity



$v \cos\theta = 24\,(0.5) = 12$ m/s

In y-dir., $e = \dfrac{v \sin\theta}{24 \cos 30°} = 0.8$

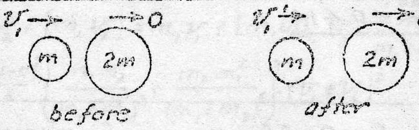
Combine & get

$\tan\theta = \dfrac{16.63}{12} = 1.3856$

$\underline{\theta = 54.2°}$

$v = \dfrac{12}{\cos 54.2°} = \dfrac{12}{0.5852} = \underline{20.5 \text{ m/s}}$

3/138



Let e = coefficient of restitution

$\Delta G = 0;\ m v_1' + 2m v_2' = m v_1$

& $v_2' - v_1' = e\,(v_1)$

Combine & get $v_1' = \dfrac{1-2e}{3} v_1,\quad v_2' = \dfrac{1+e}{3} v_1$

Kinetic energy loss is

$\Delta T = \frac{1}{2} m v_1^2 - \left[\frac{1}{2} m \left(\frac{1-2e}{3} v_1\right)^2 + \frac{1}{2} 2m \left(\frac{1+e}{3} v_1\right)^2\right]$

$= \frac{1}{3} m v_1^2 (1 - e^2),\quad \dfrac{d\Delta T}{de} = \frac{1}{3} m v_1^2 (-2e) = 0$

so $e = 0$

& $\underline{v_1' = \dfrac{v_1}{3}}\quad \underline{v_2' = \dfrac{v_1}{3}}$

3/139 Let v_r = ram velocity after impact

$\Delta G = 0;\ 400\sqrt{19.62(1.2)} + 0 = 400\,v_r + 250\,v$

& $e = \dfrac{v - v_r}{\sqrt{19.62(1.2)}}$

Combine & get $v_r = \dfrac{(400 - 250e)\sqrt{19.62(1.2)}}{400 + 250} = 0.373(8 - 5e)$

Max. transfer of energy occurs for v_r a minimum which occurs for $e = e_{max} = 1$

Thus $v_{r_{min}} = 0.373\,(8 - 5) = 1.120$ m/s

so $v = (8\sqrt{19.62(1.2)} - 8[1.120])/5 = \underline{5.97 \text{ m/s}}$

e=1 is larger than could be realised with steel on wood.

3/140 $H_{vertical}$ = constant; $m r_1^2 \omega_1 = m r_2^2 \omega_2$

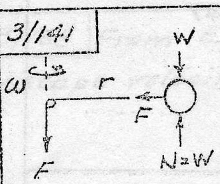
so $r_1^2 N_1 = r_2^2 N_2,\ \left(300\frac{\sqrt{3}}{2}\right)^2 (90) = \left(300\,\frac{1}{2}\right)^2 N,\ \underline{N = 270 \text{ rev/min}}$

$U = \Delta T + \Delta V_g;\ U = 2\,\frac{1}{2}\,4\left[\left(0.3\,\frac{1}{2}\ 270\,\frac{2\pi}{60}\right)^2 - \left(0.3\,\frac{\sqrt{3}}{2}\ 90\,\frac{2\pi}{60}\right)^2\right]$

$+ 2(4)(9.81)\left[0.3\,\frac{\sqrt{3}}{2} - 0.3\,\frac{1}{2}\right]$

$= 48.0 + 8.62 = \underline{56.6 \text{ J}}$

3/141 Angular momentum about central axis is conserved so.



$\dot{H} = \dfrac{d}{dt}(m r^2 \omega) = 0$

$m\,(2r\omega\,dr + r^2\,d\omega) = 0,\ \underline{\dfrac{d\omega}{dr} = \dfrac{-2\omega}{r}}$

$\Sigma F_n = m a_n;\ F = m r \omega^2$

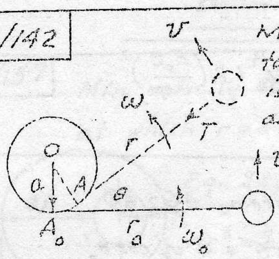
$dU = dT;\ -F\,dr = d\left(\frac{1}{2} m r^2 \omega^2\right),$

$-m r \omega^2 dr = m(r\omega^2 dr + r^2 \omega\, d\omega)$

$= m\left(r\omega^2 dr + r^2 \omega\left[\frac{-2\omega}{r} dr\right]\right)$

$= -m r \omega^2 dr$ (check)

3/142



Moment of tension T about tangency point A is zero, but A is a moving point. Also moment about fixed point O is not zero. Thus angular momentum is not conserved. Also $\Sigma F \neq 0$ on particle, so linear momentum is not conserved.

But energy is conserved so $\frac{1}{2} m (r_0 \omega_0)^2 = \frac{1}{2} m (r\omega)^2$

& $\omega = \dfrac{r_0}{r}\omega_0 = \dfrac{r_0}{r_0 - a\theta}\omega_0,\quad \underline{\omega = \dfrac{\omega_0}{1 - \frac{a}{r_0}\theta}}$

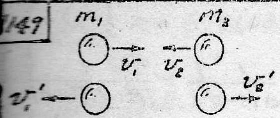
$\Sigma M_O = \dot{H}_O;\ -Ta = \dfrac{d}{dt}(m v r) = m v \dot{r}$ since v = const

But $\dot{r} = \dfrac{d}{dt}(r_0 - a\theta) = -a\dot{\theta} = -a\omega$

So $-Ta = m v(-a\omega)$, $T = m v \omega$ or $\underline{T = m r_0 \omega_0 \omega}$

since $v = v_0 = r_0 \omega_0$

3/149



With center of mass of system at rest,

$m_1 v_1 = m_2 v_2$

$m_1 v_1' = m_2 v_2'$

Thus $v_1' = v_1 \frac{v_2'}{v_2}$, $v_2' = v_2 \frac{v_1'}{v_1}$

so $e = \frac{v_1' + v_2'}{v_1 + v_2}$ or $v_1' = \left(1 + \frac{v_2}{v_1}\right) = e(v_1 + v_2)$ which gives $\begin{cases} v_1' = ev_1 \\ v_2' = ev_2 \end{cases}$

Now $\Delta E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_1 v_1'^2 - \frac{1}{2} m_2 v_2'^2$

$= \frac{1}{2}\{m_1 v_1^2 (1-e^2) + m_2 v_2^2 (1-e^2)\} = \frac{1-e^2}{2}(m_1 v_1^2 + m_2 v_2^2)$

But $m_1 v_1 + [m_2 v_1] = m_2 v_2 + [m_2 v_1]$, $v_1 = \frac{m_2}{m_1 + m_2}(v_1 + v_2)$

$m_2 v_2 + [m_1 v_2] = m_1 v_1 + [m_1 v_2]$, $v_2 = \frac{m_1}{m_1 + m_2}(v_1 + v_2)$

So $\Delta E = \frac{1-e^2}{2}\left\{\frac{m_1 m_2^2}{(m_1+m_2)^2} + \frac{m_2 m_1^2}{(m_1+m_2)^2}\right\}(v_1 + v_2)^2$

$\Delta E = \frac{1-e^2}{2} \cdot \frac{m_1 m_2}{m_1 + m_2}(v_1 + v_2)^2$

3/150 Maximum advantage of the earth's rotation is achieved for launch at the equator with trajectory headed east, in direction of velocity of earth's surface.

3/151 For circular orbit $a = R + H$, so from Eq. 82

$v^2 = 2gR^2\left(\frac{1}{R+H} - \frac{1}{2(R+H)}\right)$, $\underline{v = R\sqrt{\frac{g}{R+H}}}$

3/152 Angular momentum is conserved, so

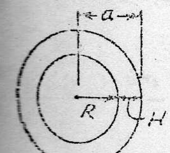
$v_a (R + H_a) = v_p (R + H_p)$ where a = apogee

p = perigee

$H_a = \frac{v_p}{v_a}(R + H_p) - R = 1.5(6370 + 389) - 6370$

$= \underline{3768 \text{ km}}$

3/153 From Eq. 82 with $r = a = R + H$,



$v^2 = 2gR^2\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{gR^2}{R+H}$

$= \frac{1.62}{1000}(3600)^2 \frac{(1738)^2}{1738 + 80} = 34.88 (10^6)$

$\underline{v = 5910 \text{ km/h}}$

3/154 From Eq. 79 with $r = a$, $\tau = 2\pi \frac{r^{3/2}}{R\sqrt{g}}$

For the moon (Appen. C), $R = 1738$ km

$g = 1.62$ m/s^2

$r = 1738 + 80$ km

so $\tau = 2\pi \frac{(1818)^{3/2}}{1738\sqrt{1.62(10^{-3})}} = 6962$ s

or $\underline{\tau = 1 \text{ h } 56 \text{ min } 2 \text{ s}}$

3/155 From Eq. 82 with $a = \infty$, $r = R + H$,

escape velocity is $v^2 = 2gR^2\left(\frac{1}{R+H} - \frac{1}{\infty}\right)$

$\underline{v = R\sqrt{\frac{2g}{R+H}}}$

3/156 From Eq. 78, $a = \frac{r_{min}}{1-e} = \frac{6370 + 145}{1 - 0.2} = 8144$ km

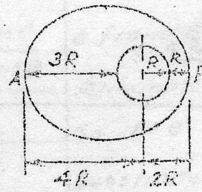
From Eq. 82, $v^2 = 2(9.824)(10^{-3})(3600)^2(6370)^2\left(\frac{1}{6690} - \frac{1}{16\,288}\right)$

where $r = 6370 + 320 = 6690$ km

Solve & get $v^2 = 910(10^6)$, $\underline{v = 30\,170 \text{ km/h}}$

3/157 Min. velocity occurs at Apogee A ...

at which $r = 4R$. Also $2a = 6R$ so from



Eq. 82, $v^2 = 2gR^2\left(\frac{1}{4R} - \frac{1}{6R}\right) = \frac{gR}{6}$

$v^2 = \frac{9.824\,(3600)^2\,6370}{10^3 \cdot 6} = 135.17\,(10^6)$

$\underline{v = 11\,630 \text{ km/h}}$

3/158 From Eq. 78 $a = \frac{r_{min}}{1-e} = \frac{640 + 6200}{1 - 0.25} = 9120$ km

& Appendix C

From Eq. 83 $v_{max} = v_p = 6200\sqrt{\frac{3.44(10^{-3})}{9120}}\sqrt{\frac{1+0.25}{1-0.25}}$

$= 6200(0.962)(10^{-3})(1.291)$

$= \underline{7.70 \text{ km/s}}$

3/159 From the equation for the elliptical orbit, Eq. 78, when $\theta = \pi/2$, $r = a(1 - e^2)$

But $r_{max} = a(1+e)$ & $r_{min} = a(1-e)$, so

$\underline{r = a(1-e)(1+e) = \frac{r_{max}\, r_{min}}{a}}$

3/160 From Eq. 82 at injection point

$\frac{1}{2a} = \frac{1}{R+H} - \frac{v_0^2}{2gR^2}$ so Eq. 82 gives for

any position $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{R+H} + \frac{v_0^2}{2gR^2}\right)$

or $v^2 = v_0^2 + 2gR^2\left(\frac{1}{r} - \frac{1}{R+H}\right)$

3/161 For given orbit injection conditions

Eq. 82 gives $a = \frac{(R+H)gR^2}{2gR^2 - v_0^2(R+H)}$

Substitute in Eq. 79 & get $\tau = \frac{2\pi g R^2}{\sqrt{\left(\frac{2gR^2}{R+H} - v_0^2\right)^3}}$

3/162 $r_{min} = 6370 + 240 = 6610$ km

$r_{max} = 6370 + 400 = 6770$ km

From Eq. 78, $\frac{r_{min}}{r_{max}} = \frac{1-e}{1+e}$

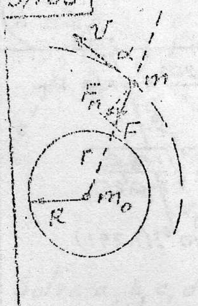
so $(1+e)(6610) = (1-e)(6770)$, $e = 0.01196$

From Eq. 79 with $a = \frac{1}{2}(r_{max} + r_{min}) = 6690$ km

$\tau = 2\pi \frac{(6690)^{3/2}}{6370\sqrt{9.821(10^{-3})}} = 5446$ s or

$\tau = 1$ h 30 min 46 s

3/163 $F = K\frac{m m_0}{r^2} = \frac{gR^2 m}{r^2}$



$\Sigma F_n = m a_n$; $\frac{gR^2 m}{r^2}\sin\alpha = m\frac{v^2}{\rho}$

so $\rho = \frac{r^2 v^2}{gR^2 \sin\alpha}$

3/164 Eq. 79, with $R = 6370$ km, $g = 9.821$ m/s^2, $\tau = 90(60)$ s,

$90(60) = 2\pi \frac{a^{3/2}}{6370}\frac{1}{\sqrt{9.824(10^{-3})}}$

$a^3 = 294(10^9)$ km^3, $a = 6652$ km

Eq. 82, $v^2 = 2(9.821)(10^{-3})(3600)^2(6370)^2\left(\frac{1}{6370+150} - \frac{1}{2(6652)}\right)$

$= 808(10^6)$ (km/h)^2

$v = 28\ 420$ km/h

3/165 $r_{min} = a(1-e) = R+H = 6370+500 = 6870$ km

$e = 0.7$

From Eq. 78, for $\theta = \pi/2$, $\frac{1}{r} = \frac{1}{a(1-e^2)} = \frac{1}{(R+H)(1+e)}$

so from Eq. 82, $v^2 = 2gR\left(\frac{1}{(R+H)(1+e)} - \frac{1-e}{2(R+H)}\right)$

or $v^2 = \frac{gR^2}{R+H}\frac{1+e^2}{1+e} = \frac{9.824(10^{-3})(3600)^2(6370)^2}{6370+500}\frac{1+0.7^2}{1+0.7}$

$= 659(10^6)$ (km/h)^2

$v = 25\ 670$ km/h

3/166 From Eq. 78, $a = \frac{6370+300}{1-0.2} = 8338$ km

From Eq. 79 the period is

$\tau = 2\pi \frac{(8338)^{3/2}}{(6370)\sqrt{9.824(10^{-3})}} = 7577$ s

Longitude change $\Delta\beta = \omega_{earth}\tau = (0.7292)(10^{-4})(7577)$

$= 0.5525$ rad

or $\Delta\beta = 0.5525\frac{180}{\pi} = 31.7°$ west

3/167 $\Delta E = E_{orbit} - E_{launch}$

From Eq. 81 $E_{orbit} = -\frac{gR^2 m}{2(R+H)}$

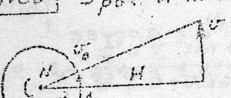
& $E_{launch} = T + V = \frac{1}{2}m(R\omega)^2 - \frac{mgR^2}{R}$

so $\Delta E = mgR\left(1 - \frac{R}{2(R+H)} - \frac{R\omega^2}{2g}\right)$

$= 450(9.824)(6370)(10^3)\left(1 - \frac{6370}{2(6370+250)} - \frac{6370(0.7292)^2(10^{?})}{2(9.821)(10^{?})}\right)$

$= 28.2(10^9)(0.5172) = 14.56$ GJ

3/168 Spot A must be on the equator (latitude zero) and v must be in the easterly direction.



$\frac{v}{R+H} = \omega = 0.729(10^{-4})$ rad/s

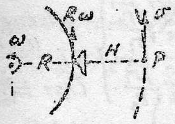
But $v = R\sqrt{\frac{g}{R+H}}$ for circular orbit. Combine & ?

$\frac{gR^2}{R+H} = (R+H)^2\omega^2$, $R+H = \sqrt[3]{\frac{gR^2}{\omega^2}} = \sqrt[3]{\frac{9.824(10^{-3})(6370)^2}{(0.729)^2(10^{-8})}}$

$= \sqrt[3]{75.0(10^{12})} = 42\ 170$ km

$H = 42\ 170 - 6370 = 35\ 800$ km, $v = 11\ 07?$

3/169 $a = \frac{1}{2}(2[6370] + 150 + 1500) = 7195$ km



From Eq. 82, at perigee P,

$v^2 = \frac{2(9.824)(3600)^2(6370)^2}{10^3}\left(\frac{1}{6370+150} - \frac{1}{2(7195)}\right)$

$v = 29\ 440$ km/h $= 8.18$ km/s

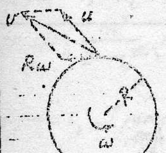
$R\omega = 6370(0.729)(10^{-4}) = 0.464$ km/s

Absolute angular velocity of dish $p_a = \frac{v - R\omega}{H}$

Relative " " " " $p = p_a - \omega$

$p = \frac{v - R\omega}{H} - \omega = \frac{8.18 - 0.464}{150} - 0.729(10^{-4}) = \underline{0.0515 \text{ rad/s}}$

3/170 From Eq. 82 for $a = \infty$, $r = R$, $v = \sqrt{2gR}$



$v = \sqrt{2(9.824)(6370)(10^3)} = 11\ 190$ m/s

or $v = 11.19$ km/s

Relative projectile velocity u is a minimum when fired horizontally at equator in E-direction.

Absolute velocity v is independent of its direction or point of origin.

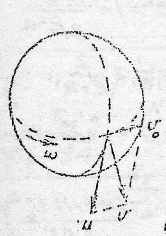
3/171 $\tan\beta = \frac{\dot r}{r\dot\theta}$; From Eq. 78, $r = \frac{a(1-e^2)}{1+e\cos\theta}$

so $\dot r = \frac{a(1-e^2)e\dot\theta\sin\theta}{(1+e\cos\theta)^2}$;

Thus $\tan\beta = \frac{a(1-e^2)e\dot\theta\sin\theta}{(1+e\cos\theta)^2} \Big/ \frac{a(1-e^2)\dot\theta}{1+e\cos\theta} = \underline{\frac{e\sin\theta}{1+e\cos\theta}}$

3/172 Absolute orbital velocity $v = R\sqrt{\frac{g}{R+H}}$

$= 6370\sqrt{\frac{(9.824)(10^3)}{6370+800}} = 7460$ m/s



or $v = 26\ 800$ km/h

Tangential velocity of tower is

$v_0 = (R+H)\omega = (6370+800)(0.729)(10^{-4})$

$= 0.5227$ km/s or 1882 km/h

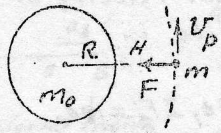
(a) $u = \sqrt{v^2 - v_0^2} = \sqrt{(26\ 800)^2 - (1882)^2}$

$= \underline{26\ 770 \text{ km/h}}$

(b) $\xleftarrow{u} \xrightarrow{v_0}$, $u = v_0 + v = 1882 + 26\ 800 = \underline{28\ 680 \text{ km/h}}$

3/173 $\Sigma F_n = ma_n$; $F = m\frac{v^2}{\rho}$ where $F = K\frac{mm_0}{(R+H)^2}$

$= g\frac{R^2}{(R+H)^2}m$



Thus $\frac{gR^2}{(R+H)^2} = \frac{v^2}{\rho}$

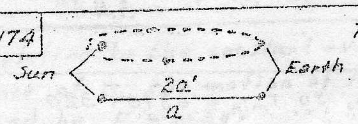
But at perigee $v^2 = \frac{gR^2}{a}\frac{1+e}{1-e}$ from Eq. 83

Also $R+H = r_{min.} = a(1-e)$, so $v^2 = \frac{gR^2}{R+H}(1+e)$

Combine & get $\frac{gR^2}{(R+H)^2} = \frac{gR^2}{R+H}\frac{1+e}{\rho}$

Hence $\underline{\rho = (R+H)(1+e)}$

3/174



The a for actual orbit becomes the $2a'$ for the degenerate ellipse.

Thus time $t = \frac{1}{2}\tau'$, τ' = period for degenerate ellipse

τ = " " actual earth orbit

From Eq. 79, $\tau'/\tau = (a'/a)^{3/2} = (1/2)^{3/2}$

So $t = \frac{1}{2}\tau' = \frac{\tau}{4\sqrt2} = \frac{365}{4\sqrt2} = \underline{64.6 \text{ days}}$

3/175 From Eq. 82, $a = \frac{gR^2(R+H)}{2gR^2 - v_p^2(R+H)}$

& from Eq. 78, $r_{min} = a(1-e)$ or $a = \frac{R+H}{1-e}$

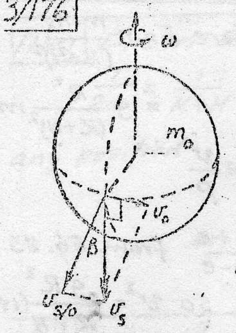
Eliminate a & get $1 - e = 2 - v_p^2(R+H)/gR^2$

So $1 + e = v_p^2(R+H)/gR^2$

Substitute in Eq 78 $r_{max} = a(1+e) = \frac{v_p^2(R+H)^2}{2gR^2 - v_p^2(R+H)}$

Substitute in Eq. 83 & simplify to get $v_a = \frac{2gR^2}{v_p(R+H)} - v_p$

3/176



v_s = velocity of satellite

v_o = velocity of point on equator

$v_{s/o}$ = apparent velocity of satellite

$$v_o = R\omega$$

For satellite, $\Sigma F_n = ma_n$

$$K\frac{mm_e}{(R+H)^2} = m\frac{v_s^2}{R+H}, \text{ so } v_s^2 = \frac{gR^2}{R+H}$$

where $Km_o = gR^2$; $\therefore v_s = R\sqrt{\dfrac{g}{R+H}}$

Hence $\beta = \tan^{-1}\dfrac{R\omega}{R\sqrt{\frac{g}{R+H}}} = \tan^{-1}\omega\sqrt{\dfrac{R+H}{g}}$

$$\beta = \tan^{-1}\left(0.729(10^{-4})\sqrt{\frac{6370+300}{9.824(10^{-3})}}\right) = \tan^{-1} 0.0601 = \underline{3.44°}$$

3/177 $R = 6370$ km, so $r_{min} = 6370+320 = 6690$ km

For circular orbit $e = 0$

& Eq. 83 gives $v_{cir} = R\sqrt{g/a} = 6370\sqrt{\dfrac{9.824(10^{-3})}{6690}}$

$= 7.72$ km/s

For elliptical orbit, $v_p = 7.72 + 0.300 = 8.02$ km/s

From Eq. 82,

$$\overline{8.02}^2 = 2(9.824)(10^{-3})(6370)^2\left(\frac{1}{6690} - \frac{1}{2a}\right)$$

Solve & get $2a = r_{min} + r_{max} = 14\,530$ km

so $r_{max} = R + H = 2a - r_{min}$

Hence $H = 14\,530 - 6690 - 6370 = \underline{1470 \text{ km}}$

3/178 From Eq. 83 the apogee velocity at A is

$v_a = R\sqrt{\dfrac{g}{a}}\sqrt{\dfrac{r_{min}}{r_{max}}}$ where R & g are given for Mars in Appendix C

$v_a = 3388\sqrt{\dfrac{3.73(10^{-3})}{13\,000/2}}\sqrt{\dfrac{5000}{8000}} = 2.03$ km/s relative to planet

$\Delta v_r = 5 - 2.03 = 2.97$ km/s

$$\int_0^t F_r\,dt = 300(2.97)(10^3) \text{ where } \int F_r\,dt = 500t$$

so $t = 1783$ s or $\underline{t = 29 \text{ min } 43 \text{ s}}$

3/179 For 1, $v_1 = R\sqrt{\dfrac{g}{r_1}}$; for 2, $v_2 = R\sqrt{\dfrac{g}{r_2}}$

For transfer ellipse at A, $v_1' = R\sqrt{\dfrac{g}{a}}\sqrt{\dfrac{r_2}{r_1}}$

" " " " B, $v_2' = R\sqrt{\dfrac{g}{a}}\sqrt{\dfrac{r_1}{r_2}}$

where $a = \dfrac{r_1+r_2}{2}$ (see Eq. 83)

So at A, $\Delta v_A = v_1' - v_1 = R\sqrt{\dfrac{g}{a}}\sqrt{\dfrac{r_2}{r_1}} - R\sqrt{\dfrac{g}{r_1}} = \underline{R\sqrt{\dfrac{g}{r_1}}\left(\sqrt{\dfrac{2r_2}{r_1+r_2}} - 1\right)}$

& at B, $\Delta v_B = v_2 - v_2' = R\sqrt{\dfrac{g}{r_2}} - R\sqrt{\dfrac{g}{a}}\sqrt{\dfrac{r_1}{r_2}} = \underline{R\sqrt{\dfrac{g}{r_2}}\left(1 - \sqrt{\dfrac{2r_1}{r_1+r_2}}\right)}$

$$\Delta v_A = 6370\sqrt{\frac{9.824(10^3)}{6870}}\left(\sqrt{\frac{2(7370)}{6870+7370}} - 1\right) = \underline{132.6 \text{ m/s}}$$

$$\Delta v_B = 6370\sqrt{\frac{9.824(10^3)}{7370}}\left(1 - \sqrt{\frac{2(6870)}{6870+7370}}\right) = \underline{130.2 \text{ m/s}}$$

3/180 For P, time to A, from Eq. 79, is

$$\tau/2 = \pi\frac{a^{3/2}}{R\sqrt{g}} = \pi\frac{[(7195)(10^3)]^{3/2}}{6370(10^3)\sqrt{9.824}} = 3037 \text{ s}$$

where $a = \frac{1}{2}(2[6370] + 1200 + 450) = 7195$ km

Period for S would be $(3037)(2)\left(\dfrac{7570}{7195}\right)^{3/2} = 6556$ s

Thus $\dfrac{180° - \theta}{360°} = \dfrac{3037}{6556}$, $\underline{\theta = 13.2°}$

$$v_s = \frac{2\pi(7570)(10^3)}{6556} = 7255 \text{ m/s}$$

$(v_P)_A = v_{apogee} = R\sqrt{\dfrac{g}{a}}\sqrt{\dfrac{r_{min}}{r_{max}}} = 6370(10^3)\sqrt{\dfrac{9.824}{7195(10^3)}}\sqrt{\dfrac{6820}{7570}}$

$= 7064$ m/s

$\int F\,dt = m\Delta v$; $900t = 800(7255 - 7064)$

$\underline{t = 170 \text{ s}}$

3/181 From Eq. 77 with $C = 1/d$ & $a = \dfrac{ed}{1-e^2}$, $Km_o = gR^2$

there results. $e = \dfrac{h^2}{gR^2}\dfrac{e}{a(1-e^2)}$ or $e^2 = 1 - \dfrac{h^2}{gR^2a}$

From Eq. 82 for starting conditions $\dfrac{1}{a} = \dfrac{2}{R+H} - \dfrac{v_o^2}{gR^2}$

Also $h = r^2\dot\theta = r(r\dot\theta) = (R+H)v_o\cos\beta$ = constant

Thus

$$\underline{e = +\sqrt{1 - \frac{(R+H)v_o^2\cos^2\beta}{gR^2}\left(2 - \frac{(R+H)v_o^2}{gR^2}\right)}}$$

Prob. 3/151 or 3/58, $v_c = R\sqrt{\dfrac{g}{R+H}}$

Repeat derivation of Prob. 3/183 & substitute a from Eq. 82

$r_{\substack{max\\min}} = a(1 \pm e)$ to get

$$= \frac{R+H}{2 - \dfrac{(R+H)v_0^2}{gR^2}}\left\{1 \pm \sqrt{1 - \frac{(R+H)v_0^2\cos^2\beta}{gR^2}\left(2 - \frac{(R+H)v_0^2}{gR^2}\right)}\right\}$$

Substitute $\eta = v_0/v_c$ & get

(a) For $\beta = 0$, $\quad r = \dfrac{R+H}{2-\eta^2}\left\{1 \pm \sqrt{1-\eta^2(2-\eta^2)}\right\}$

$$= \frac{R+H}{2-\eta^2}\left(1 \pm [1-\eta^2]\right)$$

$r_{max} = R+H$ if $\eta < 1$ | $r_{min} = R+H$ if $\eta > 1$

$r_{max} = (R+H)\dfrac{\eta^2}{2-\eta^2}$ if $\eta > 1$ | $r_{min} = (R+H)\dfrac{\eta^2}{2-\eta^2}$ if $\eta < 1$

(b) For $\eta = 1$, $r_{\substack{max\\min}} = (R+H)(1 \pm \sin\beta)$

3/183 Let τ_f = period for fixed sun

τ_M = " " moving "

From Eq. 84 $\quad \dfrac{\tau_f}{\tau_M} = \dfrac{\sqrt{m_0+m}}{\sqrt{m_0}} = \sqrt{1+\dfrac{m}{m_0}} = \sqrt{1+\dfrac{1}{333\,000}}$

$= \left(1 + 3.003003(10^{-6})\right)^{1/2}$

$= 1 + \frac{1}{2}(3.003003)(10^{-6}) - \frac{1}{8}\,9.018(10^{-12}) + \cdots$

$= 1.000\,001\,502\ldots$

Error is $1.502(10^{-6})(365.26)(23.9344)(60)^2$

$= 47.3$ s/y

Thus uncorrected period is $\underline{47 \text{ s longer}}$ than corrected period

---

3/184 Let τ_f = period for fixed earth

τ_M = " " moving "

From Eq. 84 $\quad \dfrac{\tau_f}{\tau_M} = \dfrac{\sqrt{m_0+m}}{\sqrt{m_0}} = \sqrt{1+\dfrac{m}{m_0}} = \sqrt{1+0.0123}$

$= (1+0.0123)^{1/2} = 1 + \dfrac{0.0123}{2} - \dfrac{0.0123^2}{8} + \cdots$

$= 1 + 0.00615 - 0.0000189 + \cdots$

$= 1.00613$

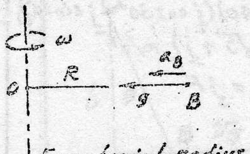
Error is $0.00613(27.32)(23.9344) = 4.0090$ h

or $\underline{4 \text{ h } 0 \text{ min } 33 \text{ s}}$ too great

---

3/186 O = center of earth, B = point on equator

From Eq. 89, $\underline{g}' = \underline{g} - \underline{a}_B$



$a_B = R\omega^2$

$= 6378(10^3)(0.7292)^2(10^{-4})^2$

$= 0.0339$ m/s²

Equatorial radius of earth (Appen C) is $R = 12\,755/2$

$= 6378$ km

$\omega = 0.7292(10^{-4})$ rad/s

$g' = 9.815 - 0.0339$

$= \underline{9.781 \text{ m/s}^2}$

3/187 Relative to carrier, $U_r = \Delta T_r$

$(22+P)75 = \frac{1}{2}3\left([240/3.6]^2 - 0\right)$

$22 + P = 88.9$ kN, $\quad \underline{P = 66.9 \text{ kN}}$

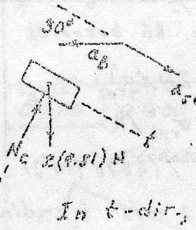
3/188 Relative to the constant-velocity reference of B, the motion of A is governed by $F_t = m\Delta v$

$F = 5\frac{1}{5}(16/3.6) = \underline{4.44 \text{ kN}}$

$G_r = mv_r = 5(16/3.6) = \underline{22.2 \text{ kN·s}}$ or $\underline{22.2(10^3) \text{ kg·m/s}}$

3/189 Let B = pt. on plate coincident with slider. $a_B = 8$ m/s²

From Eq. 85, $\Sigma\underline{F} = m(\underline{a}_B + \underline{a}_{rel})$



In n-dir.

$2(9.81)\cos 30° - N_c = 2(8\sin 30°)$

$\underline{N_c = 8.99 \text{ N}}$

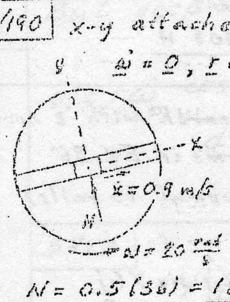
In t-dir., $2(9.81)\sin 30° = 2(a_{rel} - 8\cos 30°)$

$\underline{a_{rel} = 11.83 \text{ m/s}^2}$

3/190 x-y attached to the disk. $\underline{\omega} = 20\underline{k}$,

$\dot{\underline{\omega}} = \underline{0}$, $\underline{r} = 0$, $\underline{a}_B = \underline{a}_0 = \underline{0}$, $\underline{v}_{rel} = 0.9\,\underline{i}$ m/s

$\underline{a}_{rel} = \ddot{x}\underline{i}$, so Eq. 86 becomes



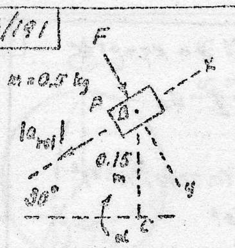
$\Sigma\underline{F} = m(2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel})$

$N\underline{j} = 0.5\left(2(20)(0.9)\underline{j} + \ddot{x}\underline{i}\right)$

so $\ddot{x} = 0$ if spring force $= kx$, &

$N = 0.5(36) = \underline{18 \text{ N}}$

3/191



With $\boldsymbol{\omega} = \mathbf{0}$, $\dot{\boldsymbol{\omega}} = \alpha\mathbf{k}$, $\mathbf{r} = \mathbf{0}$, $\mathbf{v}_{rel} = \mathbf{0}$

Eq. 86 gives $\Sigma\mathbf{F} = m(\mathbf{a}_B + \mathbf{a}_{rel})$

where $\mathbf{a}_B = 0.15(40)(\mathbf{i}\cos 30° + \mathbf{j}\sin 30°)$
$= 3.0(\mathbf{i}\sqrt{3} + \mathbf{j})$ m/s²

$\mathbf{a}_{rel} = \mathbf{i}\ddot{x}$, $\Sigma\mathbf{F} = \mathbf{j}F$

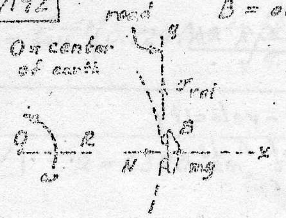
Thus

$$\mathbf{j}F = 0.5(3.0[\mathbf{i}\sqrt{3} + \mathbf{j}] + \mathbf{i}\ddot{x})$$

from which $F = 0.5(3) = \underline{1.5\ \text{N}}$

& $a_{rel} = \ddot{x} = -3.0\sqrt{3} = -5.20$ m/s; $\underline{|a_{rel}| = 5.20\ \text{m/s}^2,}$ $\underline{\text{direction shown}}$

3/192



B = origin of coordinates attached to earth

Eq. 86 with $\mathbf{r} = \mathbf{0}$, $\dot{\boldsymbol{\omega}} = \mathbf{0}$, $\mathbf{a}_{rel} = \mathbf{0}$

is $\Sigma\mathbf{F} = m(\mathbf{a}_B + 2\boldsymbol{\omega}\times\mathbf{v}_{rel})$

$a_B = R\omega^2$

$= \dfrac{12\,756}{2}(0.7292)^2(10^{-4})^2$

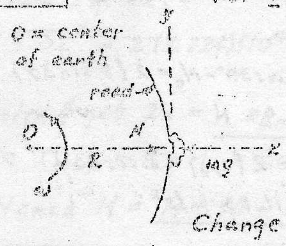
$= 33.9\,(10^{-6})$ km/s² $= 0.0339$ m/s²

$2\boldsymbol{\omega}\times\mathbf{v}_{rel} = 2(0.7292)(10^{-4})(150/3.6)(-\mathbf{i}) = -0.00608\,\mathbf{i}$ m/s²

so in x-dir, $N - 1500(9.815) = 1500(-0.0339 - 0.0061)$

$\underline{N = 14\,660\ \text{N}}$

3/193



For $\mathbf{r} = \mathbf{0}$, $\dot{\boldsymbol{\omega}} = \mathbf{0}$, Eq. 86 becomes

$\Sigma\mathbf{F} = m(\mathbf{a}_B + 2\boldsymbol{\omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel})$

where $a_{rel} = v_{rel}^2/R$

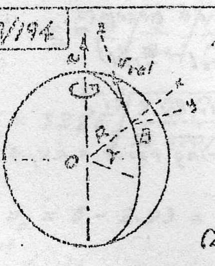
$= (150/3.6)^2/(6378[10^3])$

$= 0.000272$ m/s² (−x-dir.)

Change from straight road (Prob. 3/192) to curved road produces increment

$\Delta N = 1500(-0.000272) = \underline{-0.408\ \text{N}}$

3/194



Attach axes to earth with z north and y east. Terms in Eq. 86 in y-dir. (normal to rails) are

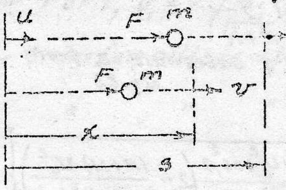
$2\boldsymbol{\omega}\times\mathbf{v}_{rel} = 2\omega v_{rel}\sin\gamma(-\mathbf{j})$

$(\Sigma\mathbf{F} = m\mathbf{a})_y$ gives

$T = 2m\omega v_{rel}\sin\gamma$ (west dir.)

so $T = 2(1500)(0.7292)(10^{-4})(2400/3.6)\sin 30° = \underline{72.9\ \text{N}}$

3/195 During launch plane goes distance s for moving carrier and distance x for stationary carrier



For moving carrier, relative to land $d\dot{U} = d$

$F\,ds = m\ddot{s}\,ds = m\dot{s}\,d\dot{s}$

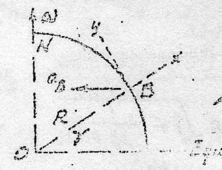
$$\int_0^s F\,ds = \int_0^x F\,ds + \int_x^s F\,ds = \int_u^{u+v} m\dot{s}\,d\dot{s}$$

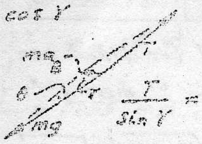
$$Fx + F(s-x) = \tfrac{1}{2}mv^2 + muv$$

For stationary carrier $Fx = \frac{1}{2}mv^2$

Subtract & get $F(s-x) = muv$, so difference between two cases is due to initial kinetic energy of plane $\frac{1}{2}mu^2$ and work done muv during added displacement for moving carrier.

3/196 $a_B = R\omega^2\cos\gamma$





$\mathbf{T} + m\mathbf{g} = m\mathbf{a}_B$

From geometry of figure,

$\dfrac{T}{\sin\gamma} = \dfrac{ma_B}{\sin\theta}$, $\sin\theta = \dfrac{mR\omega^2\sin}{2T}$

$T\cos\theta + mR\omega^2\cos^2\gamma = mg$

Combine & get

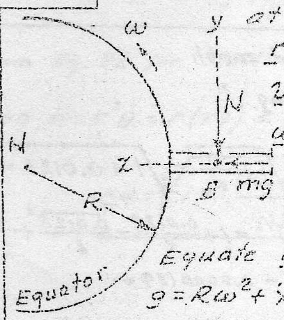
$$\underline{\tan\theta = \frac{R\omega^2}{2g}\,\frac{\sin 2\gamma}{1 - \frac{R\omega^2}{g}\cos^2\gamma}}$$

$a_{rel} = v_{rel} = \dot{\omega} = 0$

$R\omega^2 = 0.0339$ m/s²

so $\tan\theta = \dfrac{0.0339}{2(9.821)}\,\dfrac{0.866}{1 - \frac{0.0339}{9.821}\,\frac{3}{4}} = 0.001499$, $\underline{\theta = 5}$

3/197 Attach x-y to tube at position of ball at instant considered



$\mathbf{r} = \mathbf{i}x = 0$, $\dot{\boldsymbol{\omega}} = 0$, $\boldsymbol{\omega}\times\mathbf{r} = 0$, $\mathbf{a}_B = \mathbf{i}R$

$\mathbf{v}_{rel} = \mathbf{i}\dot{x}$, $\mathbf{a}_{rel} = \mathbf{i}\ddot{x}$, $\Sigma\mathbf{F} = \mathbf{i}mg - N$

$\boldsymbol{\omega} = -\omega\mathbf{k}$, so Eq. 86 becomes

$\mathbf{i}mg - N\mathbf{j} = m(\mathbf{i}R\omega^2 - 2\omega\mathbf{k}\times\mathbf{i}\dot{x}$

$= m(\mathbf{i}R\omega^2 - 2\omega\dot{x}\mathbf{j} + \mathbf{i}\ddot{x})$

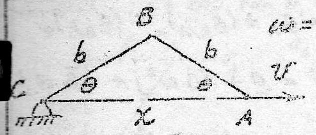
Equate $\mathbf{i}$ & $\mathbf{j}$ components & get

$g = R\omega^2 + \ddot{x}$, $\ddot{x} = g - R\omega^2$ so $\dot{x} = (g - R\omega^2)$

$N = m(2\omega\dot{x}) = \underline{2m\omega(g - R\omega^2)t}$

$g \gg R\omega^2$ so contact is on $\underline{\text{east side}}$ of tube as shown

5/8 $x = 2b\cos\theta$, $\dot x = -2b\dot\theta\sin\theta$, $v = \dot x$

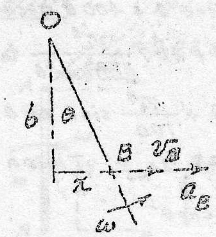


$\omega = \omega_{AB} = \dot\theta$; so $\omega = \dfrac{-v}{2b\sin\theta}$ CW

For $a = \ddot x$ const, $\dot x^2 = 2ax$

so $\omega = \dfrac{\sqrt{2ax}}{\sqrt{4b^2 - x^2}}$ (counterclockwise)

5/13 $x = b\tan\theta$, $v_B = \dot x = b\dot\theta\sec^2\theta$

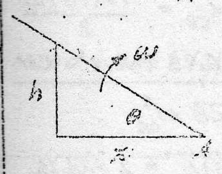


$v_B = b\omega\sec^2\theta$

$a_B = \dot v_B = 2b\omega\dot\theta\sec^2\theta\tan\theta$

$a_B = 2b\omega^2\sec^2\theta\tan\theta$

5/9 $\dot x = -v_A = -r\omega_o$, $h = x\tan\theta$

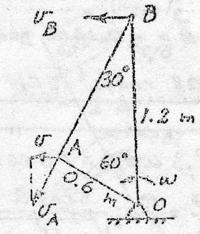


$0 = \dot x\tan\theta + x\dot\theta\sec^2\theta$

$\omega = \dot\theta = -\dfrac{\dot x}{x}\sin\theta\cos\theta = -\dfrac{\dot x}{x}\dfrac{hx}{x^2+h^2}$

$= \dfrac{rh\omega_o}{x^2+h^2}$

5/14 Given, $v = 0.15$ m/s,



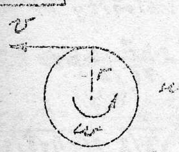
so $v_A = 0.15/\sin 30° = 0.30$ m/s

$\omega = v_A/\overline{AO} = v_B/\overline{BO}$, $v_B = \dfrac{1.2}{0.6}\,0.30$

$= 0.60$ m/s

$(a_B)_n = \dfrac{v_B^2}{\overline{BO}} = \dfrac{(0.60)^2}{1.2} = 0.30$ m/s²

5/10 $v = r\omega$, $\dot v = 0 = r\dot\omega + \dot r\omega$

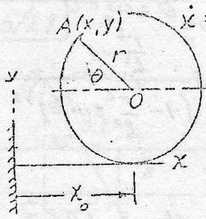


$\alpha = \dot\omega = -\dfrac{\dot r}{r}\omega$

where $\dot r = \dfrac{-t}{2\pi/\omega} = -\dfrac{tv}{2\pi r}$

so $\alpha = \dfrac{tv^2}{2\pi r^3}$

5/15 $x = x_o - r\cos\theta$, $y = r(1+\sin\theta)$



$\dot x = \dot x_o + r\dot\theta\sin\theta$, $\dot y = r\dot\theta\cos\theta$; $v_o = r\dot\theta = \dot x_o$

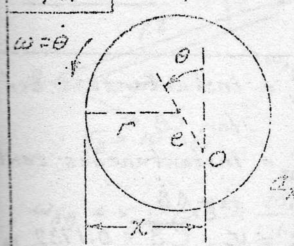
$v = \sqrt{\dot x^2 + \dot y^2} = v_o\sqrt{(1+\sin\theta)^2 + \cos^2\theta}$

$= v_o\sqrt{2(1+\sin\theta)}$

$\ddot x = 0 + r\dot\theta^2\cos\theta$, $\ddot y = -r\dot\theta^2\sin\theta$

$a = \sqrt{\ddot x^2 + \ddot y^2} = r\dot\theta^2 = v_o^2/r$ toward O

5/11 Displacement of fork & rod is



$x = r + e\sin\theta$

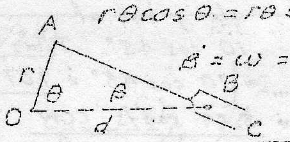
$v_x = \dot x = 0 + e\dot\theta\cos\theta$

or $v_x = e\omega\cos\theta$

$a_x = \dot v_x = -e\omega^2\sin\theta$

5/16 $\dot\theta = \omega_o$ constant ; $r\sin\theta = (d - r\cos\theta)\tan\beta$

$r\dot\theta\cos\theta = r\dot\theta\sin\theta\tan\beta + (d - r\cos\theta)\dot\beta\sec^2\beta$

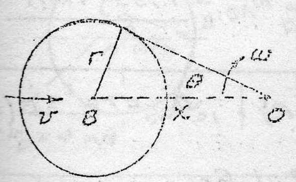


$\dot\beta = \omega = r\dot\theta\,\dfrac{\cos\theta - \sin\theta\tan\beta}{(d - r\cos\theta)\sec^2\beta}$

But $\sec\beta = \dfrac{\sqrt{d^2 - 2rd\cos\theta + r^2}}{d - r\cos\theta}$, so

$\omega = r\omega_o\,\dfrac{\cos\theta - \sin\theta\dfrac{r\sin\theta}{d - r\cos\theta}}{\dfrac{d^2 - 2rd\cos\theta + r^2}{d - r\cos\theta}} = \dfrac{r\omega_o(d\cos\theta - r)}{d^2 - 2rd\cos\theta + r^2}$

5/12 $v_B = v = -\dot x$ $\quad r = x\sin\theta$

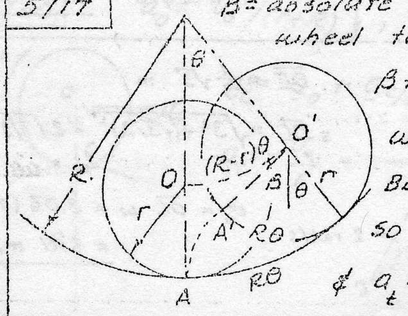


$0 = \dot x\sin\theta + x\dot\theta\cos\theta$

$\omega = \dot\theta = -\dfrac{\dot x}{x}\tan\theta$

$\omega = \dfrac{rv}{x\sqrt{x^2 - r^2}}$

5/17 β = absolute angle through which wheel turns



$\beta + \theta = \dfrac{R\theta}{r}$, $\beta = \left(\dfrac{R}{r} - 1\right)\theta$

$\omega = \dot\beta = \dfrac{R-r}{r}\dot\theta$

But $v = v_o = (R-r)\dot\theta$

so $v = r\omega$

& $a_t = \dot v = r\dot\omega = r\alpha$

5/18

$x = 200\tan\theta,\quad v_A = \dot{x} = 200\,\dot\theta\sec^2\theta$

$a_A = \ddot{x} = 200\,\ddot\theta\sec^2\theta + 400\,\dot\theta^2\sec^2\theta\tan\theta$

$= 200\,\ddot\theta\sec^2\theta + 400\dfrac{v_A^2}{200^2\sec^4\theta}\tan\theta$

$= 200\,\ddot\theta\sec^2\theta + \dfrac{v_A^2}{100}\sin\theta\cos\theta$

But for $\ddot{x} = \text{const } a_A$, $v_A^2 = 2a_A x$. Thus

$a_A = 200\,\ddot\theta\sec^2\theta + \dfrac{a_A x}{50}\sin\theta\cos\theta$

Substitute $a_A = 100$ mm/s², $\sec\theta = \frac{5}{4}$, $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$ for $x = 150$ mm

$100 = 200\,\ddot\theta\left(\frac{5}{4}\right)^2 + \dfrac{(100)(150)}{50}\dfrac{3}{5}\dfrac{4}{5},\quad \alpha = \ddot\theta = -0.1408 \text{ rad/s}^2$

or $\underline{\alpha = 0.1408 \text{ rad/s}^2 \text{ CCW}}$

5/19

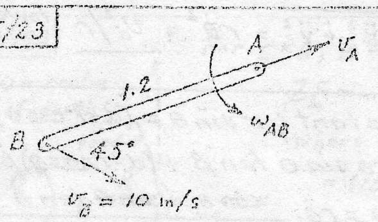
$l\sin\beta = r\sin\theta,\quad l\dot\beta\cos\beta = r\dot\theta\cos\theta$

So $\omega = \dot\beta = \dfrac{r}{l}\dot\theta\dfrac{\cos\theta}{\cos\beta} = \dfrac{r}{l}\omega_0\dfrac{\cos\theta}{\sqrt{1-\left(\frac{r}{l}\sin\theta\right)^2}}$

$l\ddot\beta\cos\beta - l\dot\beta^2\sin\beta = -r\dot\theta^2\sin\theta,\quad \ddot\theta = \dot\omega_0 = 0$

$\alpha = \ddot\beta = \dfrac{l\dot\beta^2\sin\beta - r\dot\theta^2\sin\theta}{l\cos\beta} = \dfrac{r\omega_0^2}{l}\sin\theta\dfrac{\frac{r^2}{l^2}-1}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{3/2}}$

5/23

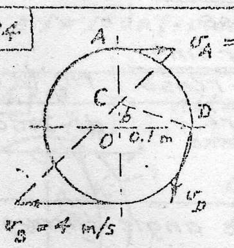


Length of AB is invariant, so A must have component of velocity along bar of $10\cos 45°$. Thus

$(v_A)_{min} = 10\cos 45° = \underline{7.07 \text{ m/s}}$

$\omega_{AB} = v_{B/A}/\overline{AB} = \dfrac{10/\sqrt{2}}{1.2} = \underline{5.89 \text{ rad/s CCW}}$

5/24

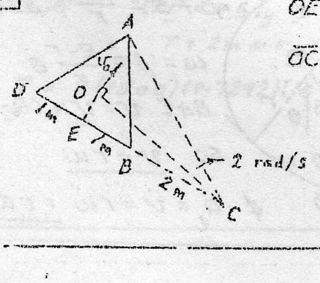


$\dfrac{2}{0.1-b} = \dfrac{4}{0.1+b},\quad b = \dfrac{0.1}{3}$ m

$v_D = \overline{CD}\dfrac{v_A}{\overline{CA}} = \dfrac{\sqrt{(0.1/3)^2+(0.1)^2}}{0.1-(0.1/3)}\,2$

$= \sqrt{10} = \underline{3.16 \text{ m/s}}$

5/25



$\overline{OE} = \frac{1}{3}\sqrt{3}$ m

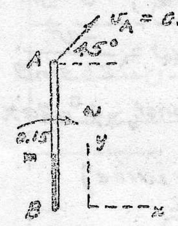
$\overline{OC} = \sqrt{3^2 + (\sqrt{3}/3)^2} = 2\sqrt{7/3}$

$= 3.06$ m

$v_O = \overline{OC}\,\omega = 3.06\,(2)$

$= \underline{6.11 \text{ m/s}}$

5/26



$\omega = v_{A/B}/\overline{AB} = \dfrac{0.08/\sqrt{2}}{0.15} = 0.377$ rad/s

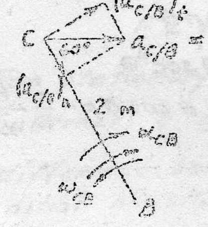
$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_n + (\mathbf{a}_{B/A})_t$

$a_B\mathbf{j} = \mathbf{0} + \overline{AB}\,\omega^2\mathbf{j} + \overline{AB}\,\alpha\,\mathbf{i}$

$a_B = 0.15\,(0.377)^2 = 0.0213$ m

$= \underline{21.3 \text{ mm}}$

5/27



$(a_{C/B})_t = \overline{CB}\,\alpha_{CB}$

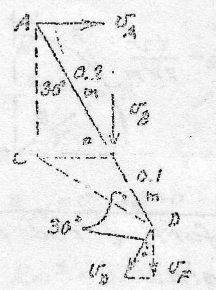
$\alpha_{GA} = \alpha_{CB} = \dfrac{16(\sqrt{3}/2)}{2}$

$= \underline{6.93 \text{ rad/s}^2 \text{ CW}}$

$(a_{C/B})_n = \overline{CB}\,\omega_{CB}^2$

$\omega_{GA} = \omega_{CB} = \sqrt{16(0.5)/2} = \underline{2 \text{ rad/s}}$ CW or CCW

5/28



C = instantaneous velocity of AB D

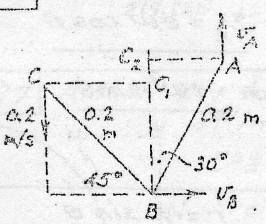
$v_F = 2$ m/s, $v_D = \dfrac{2}{\sqrt{3}/2}$ m/s

$\omega_{AD} = \omega_{CD} = \dfrac{v_D}{\overline{CD}} = \dfrac{2}{\sqrt{3}/2}\,\dfrac{1}{2(0.1)\sqrt{3}/2} = 13.33$ rad/s

$v_A = \overline{AC}\,\omega_{AC} = \overline{AC}\,\omega_{CD}$

$= 0.2\frac{\sqrt{3}}{2}\,13.33 = \underline{2.31 \text{ m/s}}$

5/29



C_1 = instantaneous center for CB

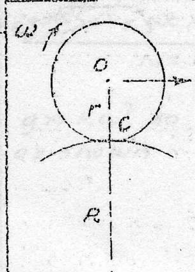
C_2 = instantaneous center for AB

$v_B = v_C$, $\overline{C_2B} = 0.1732$ m

$\omega_{AB} = v_B/\overline{C_2B} = \dfrac{0.2}{0.1732}$

$\omega_{AB} = \underline{1.155 \text{ rad/s CCW}}$

5/30



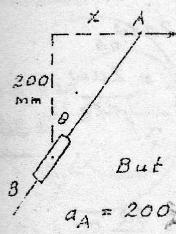
$a_0 = v_o^2/(r+R)$; $a_{C/O} = r\omega^2 = r\left(\dfrac{v_o}{r}\right)^2$

$\mathbf{a}_C = \mathbf{a}_O + \mathbf{a}_{C/O}$

$a_C = \dfrac{v_o^2}{r} - \dfrac{v_o^2}{r+R} = \underline{\dfrac{R v_o^2}{r(r+R)}}$

5/18

$x = 200 \tan\theta,\quad v_A = \dot{x} = 200\,\dot{\theta}\sec^2\theta$



$$a_A = \ddot{x} = 200\,\ddot{\theta}\sec^2\theta + 400\,\dot{\theta}^2\sec^2\theta\tan\theta$$

$$= 200\,\ddot{\theta}\sec^2\theta + 400\frac{v_A^2}{200^2\sec^2\theta}\tan\theta$$

$$= 200\,\ddot{\theta}\sec^2\theta + \frac{v_A^2}{100}\sin\theta\cos\theta$$

But for $\ddot{x} = \text{const } a_A$, $v_A^2 = 2a_A x$. Thus.

$$a_A = 200\,\ddot{\theta}\sec^2\theta + \frac{a_A x}{50}\sin\theta\cos\theta$$

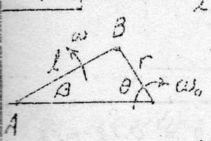
Substitute $a_A = 100$ mm/s², $\sec\theta = \frac{5}{4}$, $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$ for $x = 150$ mm

$$100 = 200\,\ddot{\theta}\left(\tfrac{5}{4}\right)^2 + \frac{(100)(150)}{50}\,\frac{3}{5}\,\frac{4}{5},\quad \alpha = \ddot{\theta} = -0.1408 \text{ rad/s}^2$$

or $\underline{\alpha = 0.1408 \text{ rad/s}^2 \text{ CCW}}$

***5/19**

$l\sin\beta = r\sin\theta,\quad l\dot{\beta}\cos\beta = r\dot{\theta}\cos\theta$

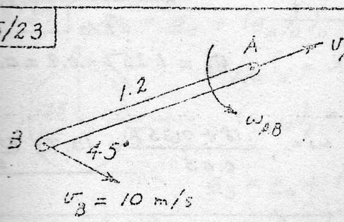


$$\text{So } \omega = \dot{\beta} = \frac{r}{l}\dot{\theta}\frac{\cos\theta}{\cos\beta} = \frac{r}{l}\omega_o\frac{\cos\theta}{\sqrt{1-\left(\frac{r}{l}\sin\theta\right)^2}}$$

$$l\ddot{\beta}\cos\beta - l\dot{\beta}^2\sin\beta = -r\dot{\theta}^2\sin\theta,\quad \ddot{\theta} = \dot{\omega}_o = 0$$

$$\alpha = \ddot{\beta} = \frac{l\dot{\beta}^2\sin\beta - r\dot{\theta}^2\sin\theta}{l\cos\beta} = \underline{\frac{r\omega_o^2}{l}\sin\theta\,\frac{\frac{r^2}{l^2}-1}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{3/2}}}$$

5/23

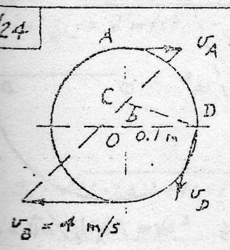


Length of AB is invariant, so A must have component of velocity along bar of 10 cos 45°. Thus

$$(v_A)_{min} = 10\cos 45° = \underline{7.07 \text{ m/s}}$$

$$\omega_{AB} = v_{B/A}/\overline{AB} = \frac{10/\sqrt{2}}{1.2} = \underline{5.89 \text{ rad/s CCW}}$$

5/24

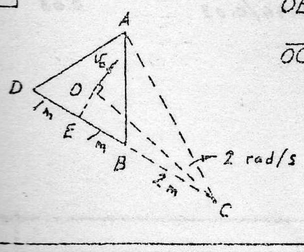


$$\frac{2}{0.1-b} = \frac{4}{0.1+b},\quad b = \frac{0.1}{3}\text{ m}$$

$$v_D = \overline{CD}\,\frac{v_A}{\overline{CA}} = \frac{\sqrt{(0.1/3)^2+(0.1)^2}}{0.1-(0.1/3)}\,2$$

$$= \sqrt{10} = \underline{3.16 \text{ m/s}}$$

5/25



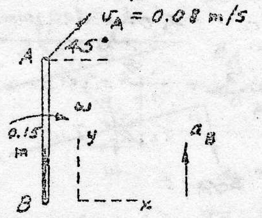
$$\overline{OE} = \tfrac{1}{3}\sqrt{3}\text{ m}$$

$$\overline{OC} = \sqrt{3^2+(\sqrt{3}/3)^2} = 2\sqrt{7/3} = 3.06\text{ m}$$

$$v_o = \overline{OC}\,\omega = 3.06\,(2) = \underline{6.11 \text{ m/s}}$$

5/26

$$\omega = v_{A/B}/\overline{AB} = \frac{0.08/\sqrt{2}}{0.15} = 0.377 \text{ rad/s}$$



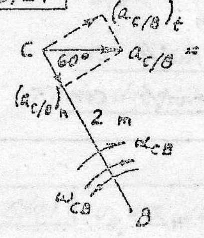
$$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$$

$$a_B\,\underline{j} = \underline{0} + \overline{AB}\,\omega^2\,\underline{j} + \overline{AB}\,\alpha\,\underline{i}$$

$$a_B = 0.15\,(0.377)^2 = 0.0213 \text{ m/s}^2$$

$$= \underline{21.3 \text{ mm/s}^2}$$

5/27



$$(a_{C/B})_t = \overline{CB}\,\alpha_{CB}$$

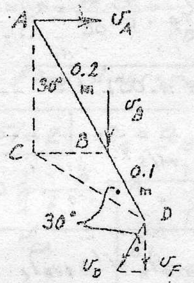
$$\alpha_{GA} = \alpha_{CB} = \frac{16(\sqrt{3}/2)}{2} = \underline{6.93 \text{ rad/s}^2 \text{ CW}}$$

$$(a_{C/B})_n = \overline{CB}\,\omega_{CB}^2$$

$$\omega_{GA} = \omega_{CB} = \sqrt{16(0.5)/2} = \underline{2 \text{ rad/s CW or CCW}}$$

5/28

C = instantaneous velocity of ABD



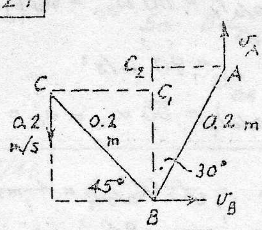
$$v_F = 2 \text{ m/s},\quad v_D = \frac{2}{\sqrt{3}/2}\text{ m/s}$$

$$\omega_{AD} = \omega_{CD} = \frac{v_D}{\overline{CD}} = \frac{2}{\sqrt{3}/2}\,\frac{1}{2(0.1)\sqrt{3}/2} = \underline{13.33 \text{ rad/s CW}}$$

$$v_A = \overline{AC}\,\omega_{AC} = \overline{AC}\,\omega_{CD}$$

$$= 0.2\frac{\sqrt{3}}{2}\,13.33 = \underline{2.31 \text{ m/s}}$$

5/29



C_1 = instantaneous center for CB

C_2 = instantaneous center for AB

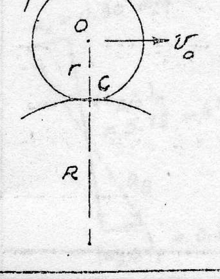
$$v_B = v_C,\quad \overline{C_2B} = 0.1732 \text{ m}$$

$$\omega_{AB} = v_B/\overline{C_2B} = \frac{0.2}{0.1732}$$

$$\omega_{AB} = \underline{1.155 \text{ rad/s CCW}}$$

5/30

$$a_o = v_o^2/(r+R)\downarrow;\quad a_{C/O} = r\omega^2 = r\left(\frac{v_o}{r}\right)^2 = \frac{v_o^2}{r}\uparrow$$



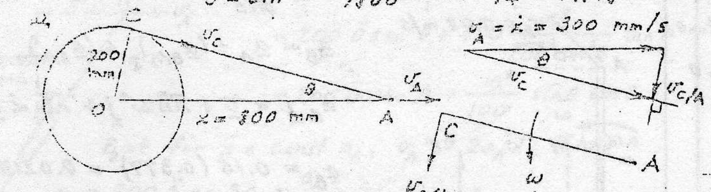
$$\underline{a}_C = \underline{a}_O + \underline{a}_{C/O}$$

$$a_C = \frac{v_o^2}{r} - \frac{v_o^2}{r+R} = \underline{\frac{R v_o^2}{r(r+R)}}\ \text{up}$$

5/31 $\underline{v}_C = \underline{v}_A + \underline{v}_{C/A}$, $v_A = 300$ mm/s

$\theta = \sin^{-1} 200/800 = \sin^{-1} 1/4 = 14.48°$

$v_A = \dot{x} = 300$ mm/s



$v_C = 300 \cos 14.48°$

$= 300(0.9682) = 290$ mm/s

$v_{C/A} = 300 \sin 14.49° = 300(1/4) = 75$ mm/s

$\overline{CA} = 800 \cos 14.49° = 775$ mm

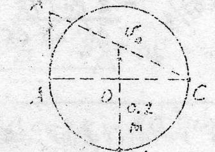
$\omega = v_{C/A}/\overline{CA} = 75/775 = \underline{0.0968 \text{ rad/s CCW}}$

$\omega_o = v_C/\overline{CO} = 290/200 = \underline{1.452 \text{ rad/s CW}}$

5/32 $\underline{a}_B = \underline{a}_O + (\underline{a}_{B/O})_n + (\underline{a}_{B/O})_t$

$v_A = 1.2$ m/s

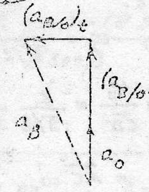
$a_O = a_{Load} = 3 \text{ m/s}^2$



$(a_{B/O})_n = \overline{BO}\,\omega_{BO}^2 = \overline{BO}\left(\frac{v_A}{\overline{AC}}\right)^2 = 0.2\left(\frac{1.2}{0.4}\right)^2$

$= 4.05 \text{ m/s}^2$

$(a_{B/O})_t = \overline{BO}\,\alpha_{BO} = \overline{BO}\,\frac{a_o}{\overline{OC}} = a_o = 3 \text{ m/s}^2$

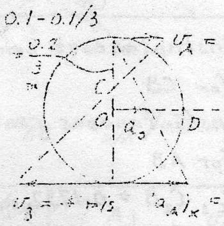


$a_B = \sqrt{(3 + 4.05)^2 + 3^2}$

$= \underline{7.66 \text{ m/s}^2}$

5/33 $a_O = 4 \text{ m/s}^2$; $\underline{a}_D = \underline{a}_O + (\underline{a}_{D/O})_n + (\underline{a}_{D/O})_t$

$0.1 - 0.1/3$



$(a_{D/O})_n = \overline{DO}\,\omega_{DO}^2 = 0.1\left(\frac{2}{0.2/3}\right)^2$

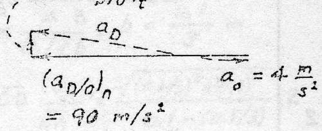
$= 90 \text{ m/s}^2$

$(a_{D/O})_t = \overline{DO}\,\alpha_{DO} = a_o = 4 \text{ m/s}^2$

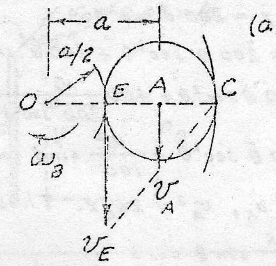
$v_B = 4$ m/s $(a_A)_x = 8 \text{ m/s}^2$

$a_D = \sqrt{(90-4)^2 + 4^2}$

$= \underline{86.1 \text{ m/s}^2}$



5/34



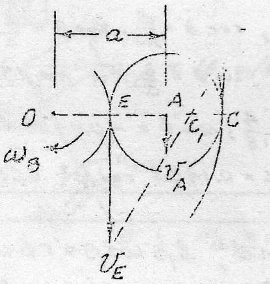
(a) $v_A = \omega_{OA}\, a$

$v_E = 2v_A = 2a\omega_{OA}$

$\omega_B = v_E/\frac{a}{2} = \frac{2a\omega_{OA}}{a/2} = 4(90)$

$= 360$ [rad/s] C[W]

$\overline{OC}\,\omega = \frac{3a}{2}\,90 = 1[35a]$



$v_A = \overline{OA}\,\omega_{CA} = 90a$

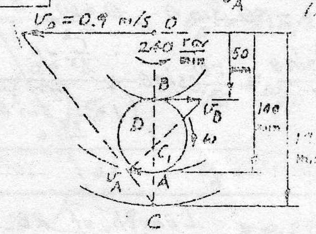
$\frac{v_A + v_O}{a} = \frac{v_E - v_A}{a}$, $v_E = v_D + [\ldots]$

$= 300a$

$\omega_{DE} = \omega_B = \frac{v_E}{a/2} = \frac{300a}{a/2} = 60[0]$

CW

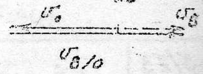
5/35 $v_A = \frac{125-100}{125}\,0.9 = 0.18$ m/s



$\underline{v}_B = \underline{v}_O + \underline{v}_{B/O}$

$v_{B/O} = \overline{BO}\,\omega_{OB}$

$= 0.05\,\frac{240(2\pi)}{60} = 1.25[7]$

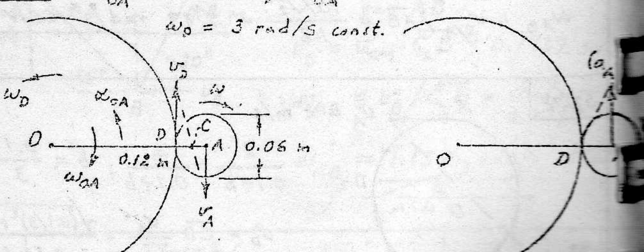


$v_B = 1.257 - 0.9 = 0.357$

$\frac{v_B}{\overline{BC_1}} = \frac{v_A + v_B}{\overline{AB}} = \omega$; $\omega = \frac{0.18 + 0.357}{0.05} = \underline{10.73 \text{ rad/s}}$ C[W]

5/36 $\omega_{OA} = 4$ rad/s, $\alpha_{OA} = 12 \text{ rad/s}^2$

$\omega_O = 3$ rad/s const.



$v_D = \overline{OD}\,\omega_D = 0.12(3) = 0.36$ m/s

$v_A = \overline{OA}\,\omega_{OA} = 0.15(4) = 0.60$ m/s

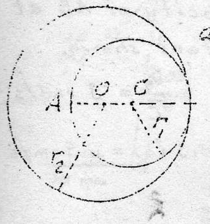
$\omega = (v_A + v_D)/0.03 = 0.96/0.03$

$= \underline{32 \text{ rad/s}}$ CW

$\alpha = (a_A)_t/\overline{AD} = \frac{\overline{AO}\ldots}{\overline{AD}}$

$= \frac{0.15(12)}{0.03} = 60$ ra[d/s²] CCW

5/37 $v_C = (r_2 - r_1)\frac{2\pi}{\tau}$, $a_C = \frac{v_C^2}{r_2 - r_1} = (r_2 - r_1)\left(\frac{2\pi}{\tau}\right)^2$ C to O



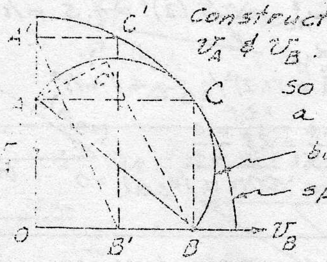
$a_{A/C} = \overline{AC}\,\omega_{AC}^2 = r_1\left(\frac{v_C}{r_1}\right)^2 = (r_2 - r_1)^2\left(\frac{2\pi}{\tau}\right)^2\frac{1}{r_1}$ A to C

$\mathbf{a}_A = \mathbf{a}_C + \mathbf{a}_{A/C}$

$a_A = (r_2 - r_1)\left(\frac{2\pi}{\tau}\right)^2\left(1 - \frac{r_2 - r_1}{r_1}\right)$

$= (r_2 - r_1)\left(2 - \frac{r_2}{r_1}\right)\left(\frac{2\pi}{\tau}\right)^2$

5/38



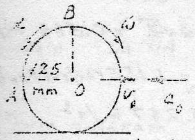
Construct C from ⊥'s to v_A & v_B. CO = AB = const. So space centrode is a circular arc.

body centrode

space centrode

For position A'B', locate C' & transfer to position AB of bar. Point C' falls on circular arc with AB as its diameter. This is body centrode which is seen to roll on space centrode.

5/39 $\mathbf{a}_A = \mathbf{a}_0 + (\mathbf{a}_{A/O})_n + (\mathbf{a}_{A/O})_t$



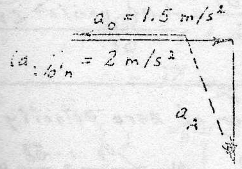
$(a_{A/O})_n = \overline{AO}\,\omega_{AO}^2 = 0.125\left(\frac{0.5}{0.125}\right)^2 = 2$ m/s²

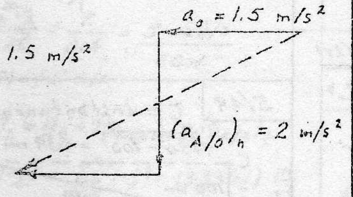
$(a_{A/O})_t = \overline{AO}\,\alpha_{AO} = 0.125\left(\frac{1.5}{0.125}\right) = 1.5$ m/s²

$\mathbf{a}_B = \mathbf{a}_0 + (\mathbf{a}_{B/O})_n + (\mathbf{a}_{B/O})_t$

$v_0 = 0.5$ m/s $\qquad (a_{B/O})_n = \overline{BO}\,\omega_{BO}^2 = 2$ m/s²

$a_0 = 1.5$ m/s² $\qquad (a_{B/O})_t = \overline{BO}\,\alpha_{BO} = 1.5$ m/s²



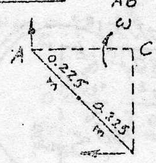


$a_A = \sqrt{(2-1.5)^2 + (1.5)^2}$

$= 1.581$ m/s²

$a_B = \sqrt{(1.5+1.5)^2 + 2^2} = 3.61$ m/s²

5/40 $\omega_{AB} = \omega_{BC} = \omega = \frac{v_B}{\overline{CB}} = \frac{1.8}{0.225\sqrt{2}} = 5.66$ rad/s



$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_n + (\mathbf{a}_{A/B})_t$

$a_B = 0$, $(a_{A/B})_n = \overline{AB}\,\omega_{AB}^2 = 0.45\,\overline{5.66}^2$

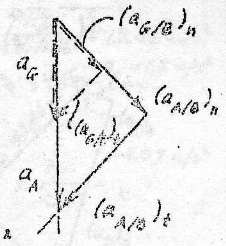
$= 14.4$ m/s²

$\mathbf{a}_G = \mathbf{a}_B + (\mathbf{a}_{G/B})_n + (\mathbf{a}_{G/B})_t$

$(a_{G/B})_n = \overline{GB}\,\omega^2 = 7.2$ m/s²

$(a_{G/B})_t = \overline{GB}\,\alpha_{AB} = \frac{\overline{GB}}{\overline{AB}}(a_{A/B})_t$

$= \frac{1}{2}(a_{A/B})_t$

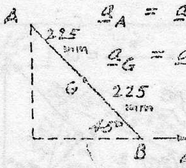


From diag., $a_G = \frac{1}{2}a_A = 7.2\sqrt{2}$

$= 10.18$ m/s²

5/41 $v_B = 0$ so $v_A = 0$ & $\omega_{AB} = 0$

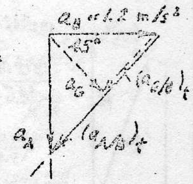
$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t$, $(a_{A/B})_n = \overline{AB}\,\omega_{AB}^2 = 0$



$\mathbf{a}_G = \mathbf{a}_B + (\mathbf{a}_{G/B})_t$

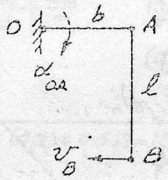
$(a_{G/B})_t = \overline{GB}\,\alpha = \frac{1}{2}(a_{A/B})_t$

$a_B = 1.2$ m/s²



From diagram, $a_G = 1.2/\sqrt{2} = 0.849$ m/s²

5/42 $\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_n + (\mathbf{a}_{A/B})_t$; $\omega_{AB} = v_B/l$

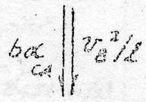


$v_A = 0$

$(a_A)_n = 0$; $(a_A)_t = b\alpha_{OA}$, $a_B = 0$

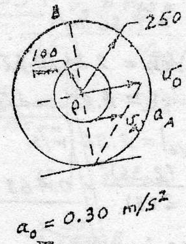
$(a_{A/B})_n = l\left(\frac{v_B}{l}\right)^2 = v_B^2/l$

So $b\alpha_{CA} = v_B^2/l$



$\alpha_{CA} = \frac{v_B^2}{bl}$ CW

5/43 $v_0^2 = 2a_0 s$, $a_0 = \frac{(1.2)^2}{2(2.4)} = 0.30$ m/s²

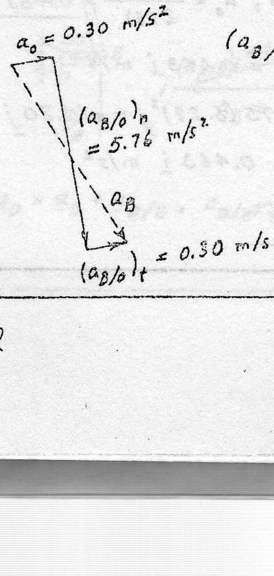


$a_A = \frac{250-100}{250}(0.30) = 0.18$ m/s²

$\mathbf{a}_B = \mathbf{a}_0 + (\mathbf{a}_{B/O})_n + (\mathbf{a}_{B/O})_t$

$(a_{B/O})_n = \overline{BO}\,\omega^2 = 0.25\left(\frac{1.2}{0.25}\right)^2 = 5.76$ m/s²

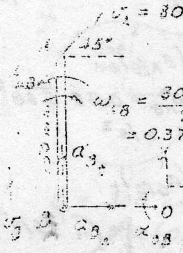
$(a_{B/O})_t = \overline{BO}\,\alpha = 0.25\left(\frac{0.30}{0.25}\right) = 0.30$ m/s²



$a_B = \sqrt{(0.30 + 0.30)^2 + (5.76)^2}$

$= 5.79$ m/s²

5/44



$v_A = 80 \frac{mm}{s}$

$\omega_{AB} = \frac{80/\sqrt{2}}{150} = 0.377 \frac{rad}{s}$

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$\underline{a}_A = \underline{0}$

$(\underline{a}_B)_n = \frac{v_B^2}{\overline{BO}}\underline{i} = \frac{(80/\sqrt{2})^2}{80}\underline{i}$

$(\underline{a}_B)_n = 40\,\underline{i}$ mm/s²

$(\underline{a}_{B/A})_t = \overline{AB}\,\alpha_{AB}\,\underline{i} = 150\,\alpha_{AB}\,\underline{i}$

$(\underline{a}_B)_t = \overline{OB}\,\alpha_{OB}\,\underline{j} = 80\,\alpha_{OB}\,\underline{j}$

$(\underline{a}_{B/A})_n = \overline{AB}\,\omega_{AB}^2\,\underline{j} = 150\,(0.377)^2\underline{j} = 21.3\,\underline{j}$ mm/s²

Substitute & get

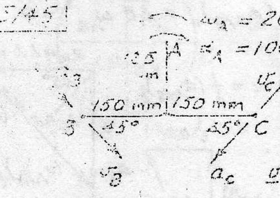
$40\,\underline{i} + 80\alpha_{OB}\,\underline{j} = 21.3\,\underline{j} + 150\,\alpha_{AB}\,\underline{i}$ giving

$\alpha_{AB} = \frac{40}{150} = 0.267$ rad/s² CCW

$\alpha_{OB} = \frac{21.3}{80} = 0.267$ rad/s² CW

$a_B = \sqrt{40^2 + 21.3^2} = \underline{45.5 \text{ mm/s}^2}$

5/45



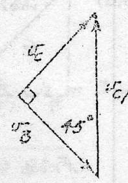
$\omega_A = 20$ rad/s $= \omega_{BC}$

$\alpha_A = 100$ rad/s² $= \alpha_{BC}$

$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$

$v = v_C = v_{C/B}/\sqrt{2}$

$v = \frac{\overline{BC}\,\omega_{BC}}{\sqrt{2}} = \frac{0.3(20)}{\sqrt{2}} = \underline{4.24 \text{ m/s}}$



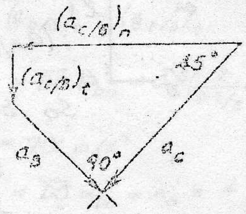
$a = a_C = a_B + (\underline{a}_{C/B})_n + (a_{C/B})_t$

$(a_{C/B})_n = \overline{CB}\,\omega_{CB}^2 = 0.3(20)^2 = 120$ m/s²

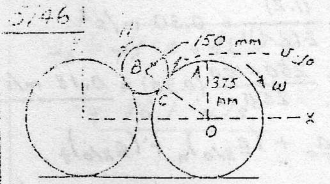
$(a_{C/B})_t = \overline{CB}\,\alpha_{CB} = 0.3(100) = 30$ m/s²

$a = a_C = 120/\sqrt{2} + 30/\sqrt{2}$

$a = \frac{150}{\sqrt{2}} = \underline{106.1 \text{ m/s}^2}$



5/46



$v_{C/O} = v_{C/B} = \overline{BC}\,\omega_{BC} = 0.15\,\frac{150(2\pi)}{60} = 2.36$ m/s

$\omega_{CO} = v_{C/O}/\overline{CO} = 2.36/0.375 = 6.28$ rad/s

$v_O = r\omega = |v_{C/O}| = 2.36$ m/s

For const. accel, $v_o^2 = 2a_o s$, $a_o = \frac{(2.36)^2}{2(6)} = \underline{0.463 \text{ m/s}^2}$

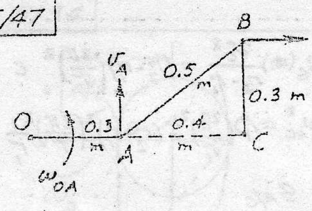
$\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$, $\underline{a}_O = 0.463\,\underline{i}$ m/s²

$(a_{A/O})_n = \overline{AO}\,\omega^2(-\underline{j}) = -0.375\,(6.28)^2\underline{j} = -14.80\,\underline{j}$ m/s²

$(a_{A/O})_t = -\overline{AO}\,\alpha_{AO}\,\underline{i} = a_o\,\underline{i} = 0.463\,\underline{i}$ m/s²

$\underline{\underline{a}_A = 0.925\,\underline{i} - 14.80\,\underline{j} \text{ m/s}^2}$

5/47



C is instantaneous center of zero velocity of AB at instant shown.

$\omega_{AB} = \frac{v_B}{\overline{BC}} = 3$ rad/s

$v_B = 0.3(3) = 0.9$ m/s

so $v_A = 0.4$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$;

$(a_B)_n = v_B^2/\overline{BC} = (0.9)^2/0.3 = 2.7$ m/s²

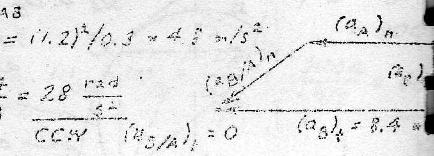
$(a_B)_t = \overline{BC}\,\alpha_{BC}$

$(a_{B/A})_n = \overline{AB}\,\omega_{AB}^2 = 0.5\,(3)^2 = 4.5$ m/s²

$(a_{B/A})_t = \overline{AB}\,\alpha_{AB}$

$(a_A)_n = v_A^2/\overline{OA} = (1.2)^2/0.3 = 4.8$ m/s²

$\alpha_{BC} = \frac{(a_B)_t}{\overline{BC}} = \frac{8.4}{0.3} = \underline{28 \frac{rad}{s^2} \text{ CCW}}$

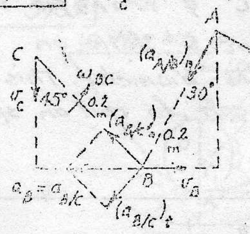


$(a_{B/A})_t = 0$

$(a_B)_t = 8.4$

5/48

$v_C = 0.2$ m/s constant



$\omega_{BC} = \frac{0.2}{0.2/\sqrt{2}} = 1.414$ rad/s CCW

$\underline{a}_B = \underline{a}_C + (\underline{a}_{B/C})_n + (a_{B/C})_t$

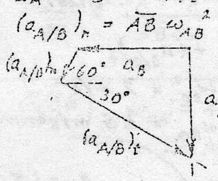
$a_C = 0$

$(a_{B/C})_n = \overline{BC}\,\omega_{BC}^2 = 0.2\,(1.414)^2 = 0.4$ m/s²

$a_B = a_{B/C} = 0.4\sqrt{2} = \underline{0.566}$ m/s²

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$, $\omega_{AB} = 1.155$ rad/s (Prob. 5

$(a_{A/B})_n = \overline{AB}\,\omega_{AB}^2 = 0.2\,(1.155)^2 = 0.267$ m/s²



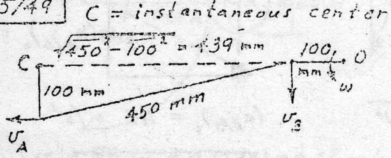
$(a_{A/B})_t\,\frac{\sqrt{3}}{2} = 0.566 + 0.5\,(0.267)$

$(a_{A/B})_t = 0.807$ m/s²

$\alpha_{AB} = \frac{0.807}{0.2} = \underline{4.04 \text{ rad/s}^2 \text{ CW}}$

5/49

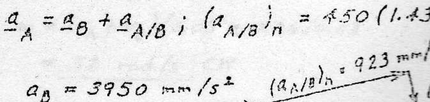
C = instantaneous center of zero velocity of



$\omega_{AB} = \frac{v_B}{\overline{CB}} = \frac{\overline{OB}}{\overline{CB}}\,\omega = \frac{100}{439}\,\frac{60(2\pi)}{60} = 1.43$

$a_B = (a_B)_n = 100\,(2\pi)^2 = 3950$

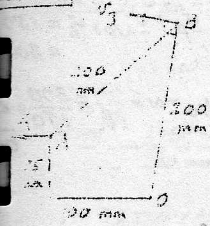
$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$; $(a_{A/B})_n = 450\,(1.432)^2 = 923$ mm/s²



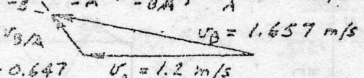
$(a_{A/B})_t = \underline{210 \text{ mm/s}^2}$

$\alpha_{AB} = \frac{(a_{A/B})_t}{\overline{AB}} = \frac{210}{450} = \underline{0.467 \text{ rad/s}^2 \text{ CCW}}$

5/50



$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$, $v_A = 1.2$ m/s



$\omega = \frac{0.647}{0.2} = \underline{3.23 \text{ rad/s CCW}}$

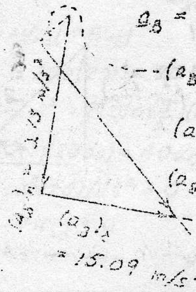
$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$(a_{B/A})_n = \frac{(0.647)^2}{0.2} = 2.09$ m/s²

$(a_B)_n = \frac{(1.657)^2}{0.2} = 13.73$ m/s²

$(a_{B/A})_t = 20.3$ m/s²

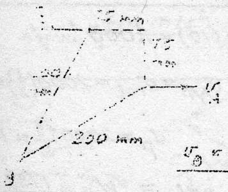
$\alpha = \frac{20.3}{0.2} = \underline{101.5 \frac{\text{rad}}{s^2} \text{ CW}}$

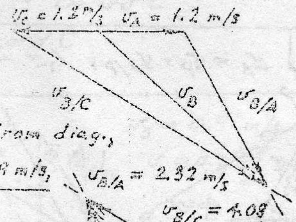


5/51

$\underline{v}_B = \underline{v}_{B/C} = \underline{v}_A + \underline{v}_{B/A}$

$v_C = 1.2$ m/s, $v_A = 1.2$ m/s





From diag.,

$v_B = 3.99$ m/s, $v_{B/A} = 2.32$ m/s

$v_{B/C} = 4.08$ m/s

With $a_C = 0$, $a_A = 0$,

$(\underline{a}_{B/C})_n + (\underline{a}_{B/C})_t = (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

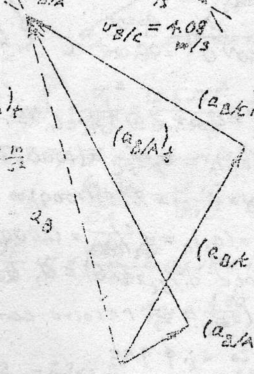
$(a_{B/C})_n = v_{B/C}^2/\overline{BC} = \frac{(4.08)^2}{0.2} = 83.2 \frac{m}{s^2}$

$(a_{B/A})_n = v_{B/A}^2/\overline{BA} = \frac{(2.32)^2}{0.2}$

$= 26.9$ m/s²

From diagram

$\underline{a_B = 120.6 \text{ m/s}^2}$



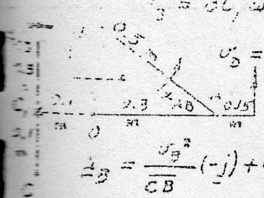
5/52

C_1 = instantaneous center for AB

$v_A = \frac{300}{150} 1 = 2$ m/s, $\omega_{AB} = \frac{v_A}{\overline{AC_1}} = \frac{2}{0.4} = \underline{5 \text{ rad/s CCW}}$

$v_B = \overline{BC_1}\,\omega_{BC_1} = 0.3(5) = 1.5$ m/s

$v_D = 3$ m/s



$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$\underline{a}_A = \frac{v_A^2}{\overline{OA}}(-\underline{i}) = \frac{2^2}{0.3}(-\underline{i}) = 13.33(-\underline{i}) \frac{m}{s^2}$

$\underline{a}_B = \frac{v_B^2}{\overline{CB}}(-\underline{j}) + \overline{CB}\,\alpha_{CB}(-\underline{i}) = \frac{(1.5)^2}{0.4}(-\underline{j}) + 0.4\,\alpha_{CB}(-\underline{i})$

$= -5.62\,\underline{j} - 0.4\,\alpha_{CB}\,\underline{i}$

$(\underline{a}_{A/B})_n = \overline{AB}\,\omega_{AB}^2\left(-\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right) = 0.5(5)^2\left(-\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right)$

$= 2.5(-4\underline{i} + 3\underline{j})$ m/s²

$(\underline{a}_{A/B})_t = \overline{AB}\,\alpha_{AB}\left(\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}\right) = 0.5\,\alpha_{AB}\left(\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}\right)$ m/s²

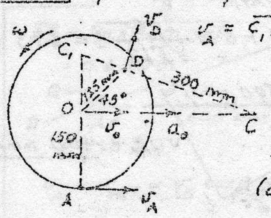
Substitute, equate $\underline{i}$- and $\underline{j}$-components, & get

α_{CB} ... $.33\,\alpha_{CB} = -11.11$, $1.33\,\alpha_{AB} + 6.25 = 0$

$\alpha_{AB} = -4.69$ rad/s² or $\underline{\alpha_{AB} = 4.69 \text{ rad/s}^2 \text{ CW}}$

5/53

C_1 = velocity center; $\overline{C_1O} = 115.6$ mm, $\overline{C_1D} = 92.5$ mm



$v_A = \overline{C_1A}\,\omega = \frac{\overline{C_1A}}{\overline{C_1O}} v_o = \frac{265.6}{115.6} 1.2 = \underline{2.76 \text{ m/s}}$

$v_D = \overline{C_1D}\,\omega = \frac{92.5}{115.6} 1.2 = 0.960$ m/s

$\underline{a}_D = \underline{a}_o + \underline{a}_{D/O}$; $a_o = 1.8$ m/s²

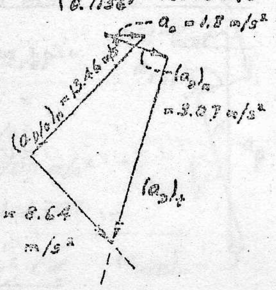
$(a_D)_n = v_D^2/\overline{CD} = (0.960)^2/0.3 = 3.07$ m/s²

$(a_{D/O})_n = \overline{OD}\,\omega^2 = 0.125\left(\frac{1.2}{0.1156}\right)^2 = 13.46$ m/s²

$\dot{v}_A = (a_A)_{Horiz.} = (\underline{a}_o + \underline{a}_{A/O})_{Horiz.}$

$= 1.8 + \overline{AO}\,\alpha_{AO}$

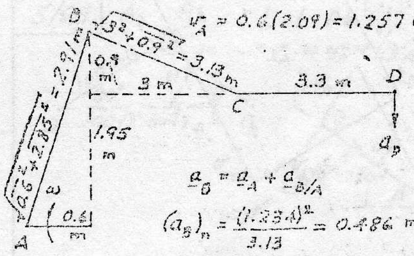
$= 1.8 - 0.15\,\frac{8.64}{0.125} = \underline{-8.57 \frac{m}{s^2}}$



5/54

$\omega = \frac{2\pi}{3} = 2.09$ rad/s

$v_A = 0.6(2.09) = 1.257$ m/s

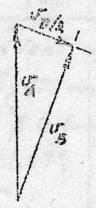


From diag.

$v_B = 1.234 \frac{m}{s}$

$v_{B/A} = 0.362 \frac{m}{s}$

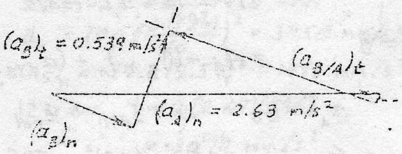
$\omega_{BA} = \frac{0.362}{2.91} = 0.1244 \frac{\text{rad}}{s}$



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$(a_B)_n = \frac{(1.234)^2}{3.13} = 0.486$ m/s²; $(a_A)_n = 0.6(2.09)^2 = 2.63$ m/s²

$(a_{B/A})_n = 2.91(0.1244)^2 = 0.0451$ m/s²

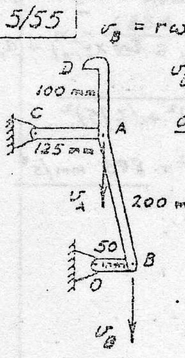


$a_D = r\alpha = 3.3\,\frac{(a_B)_t}{3.13} = 3.3\,\frac{0.539}{3.13} = \underline{0.568 \text{ m/s}^2}$

5/55

$v_B = r\omega = 50\,\frac{(120)(2\pi)}{60} = 200\pi = 628$ mm/s

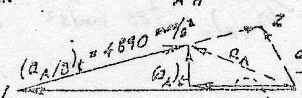
$\underline{v}_B$ parallel to $\underline{v}_A$ so $v_A = v_B$, $\omega_{AB} = 0$



$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$; $a_B = (a_B)_n = v_B^2/r = \frac{(200\pi)^2}{50}$

$= 7900$ mm/s²

$(a_A)_n = (200\pi)^2/125 = 3160$ mm/s²

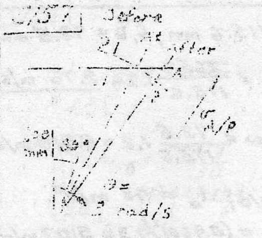


$a_D = 1997$ mm/s²

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$, $a_{D/B} = (a_{D/B})_t = \frac{\overline{BD}}{\overline{BA}}(a_{A/B})_t = \frac{300}{200} 4890$

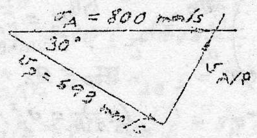
$= 7340$ mm/s² (1-2)

5/57



$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_P = r\dot\theta = \frac{200}{\sqrt{3}/2}\,3 = 693$ mm/s



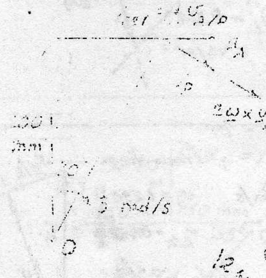
$v_{A/P} = \underline{400 \text{ mm/s}}$

5/58

P = point on OC coincident with point A.

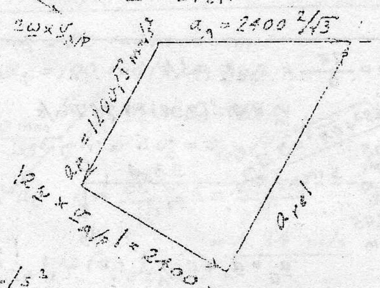
$v_{A/P} = 400$ mm/s (Prob. 5/57)

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$



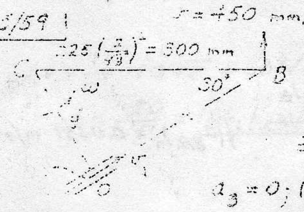
$a_P = (a_P)_n = 200\,\frac{2}{\sqrt{3}}\,3^2 = 1200\sqrt{3}$ mm/s²

$|2\underline{\omega}\times\underline{v}_{rel}| = 2(3)(400) = 2400$ mm/s²



$\underline{a_A = 2770 \text{ mm/s}^2}$

5/59



$\omega = \frac{450}{300} = 1.5$ rad/s

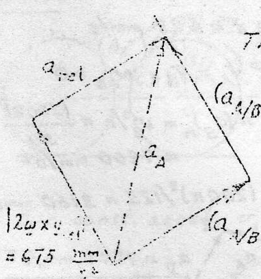
$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$\underline{a}_A = \underline{a}_o + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_B = 0;\ (\underline{a}_{A/B})_n = 225\,\frac{2}{\sqrt{3}}(1.5)^2 = 585$ mm/s²

$2\underline{\omega}\times\underline{v}_{rel} = 2(1.5)v_A\underline{j} = 3\,\overline{CA}\,\omega\underline{j} = 3(150)(1.5)\underline{j}$

$= 675\,\underline{j}$ mm/s²



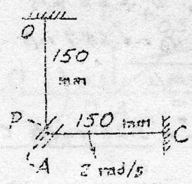
Thus $(\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t = 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

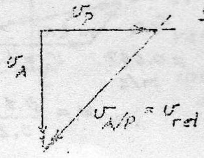
$a_A = \sqrt{(585)^2 + (675)^2}$

$= \frac{675}{2}\sqrt{7} = \underline{893 \text{ mm/s}^2}$

$(a_{A/B})_n = 585$ mm/s²

5/60 Let P be a point on CBO coincident wi[th A]





$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P};\ v_A = 150(2)$
$= 30[0]$

$\omega_{PO} = \omega = \frac{v_P}{PC} = \frac{300}{150} = 2$ [rad/s]

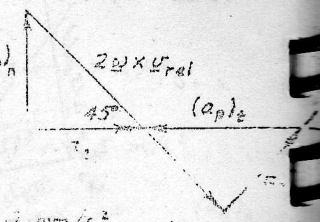
$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_A = (a_A)_n = 150(2)^2 = 600$ mm/s²

$|2\underline{\omega}\times\underline{v}_{rel}| = 2(2)(300\sqrt{2})$

$= 1200\sqrt{2}$ mm/s²

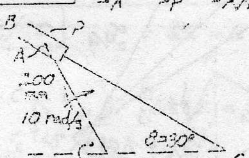
$(a_P)_n = 150(2)^2 = 60[0]$

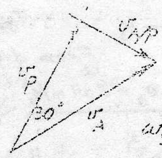


From diagram, $(a_P)_t$ = [1200] mm/s²

$\alpha_{PO} = \alpha = 1200/150 = \underline{8 \text{ rad/s}^2 \text{ CW}}$

5/61 $\underline{v}_A = \underline{v}_P + \underline{v}_{A/P},\ v_A = 200(10) = 2000$ mm/s





$v_{A/P} = v_{rel} = 2000(\frac{1}{2}) = 100[0]$

$v_P = 2000\frac{\sqrt{3}}{2} = 1732$ [mm/s]

$\omega_{BC} = v_P/\overline{PC} = \frac{1732}{2(200)\sqrt{3}/[2]}$

$= \underline{5 \text{ rad/s CW}}$

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$(a_P)_n = v_P^2/\overline{PC} = (1000\sqrt{3})^2/(2[200]\sqrt{3}/2) = 5000\sqrt{3}$ mm/s²

$|2\underline{\omega}\times\underline{v}_{rel}| = 2(5)(1000) = 10\,000$ mm/s²

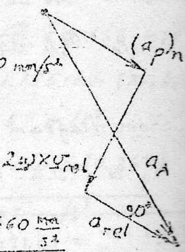
$a_A = (a_A)_n = v_A^2/\overline{OA} = (2000)^2/200 = 20\,000$ mm/s²

Since $d^2/dt^2\,(2\theta) = 0$, $\ddot\theta = \alpha_{BC} = 0$

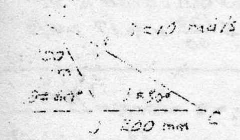
& $(a_P)_t = 0$; resolve components

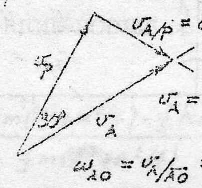
⊥ to a_A & get

$a_{rel}(0.5) = 10\,000\frac{\sqrt{3}}{2} - 5000\frac{\sqrt{3}}{2},\ \underline{a_{rel} = 8660 \text{ mm/s}^2}$



5/62 $\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$, $v_P = 2(200\sqrt{3}/2)10 = 3460$ mm/s





$v_{A/P} = v_{rel} = \frac{2000\sqrt{3}}{\sqrt{3}} = 2000$ mm/s

$v_A = 2(2000) = 4000$ mm/s

$\omega_{AO} = v_A/\overline{AO} = \frac{4000}{200} = \underline{20 \text{ rad/s CW}}$

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

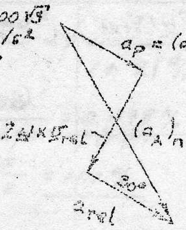
$(a_A)_n = \overline{AO}\,\omega_{AO}^2 = 200(20)^2 = 80\,000$ mm/s²

[illegible] $= 2(200\frac{\sqrt{3}}{2})(10)^2 = 20\,000\sqrt{3}$ mm/s²

[illegible] $= 2(10)(2000) = 40\,000$ mm/s²

[illegible] $(2\theta)/dt^2 = 0$

[illegible] $\alpha_{AO} = 0$



[illegible]

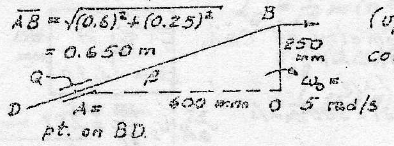
[illegible] $0\,000\sqrt{3}$ mm/s² toward C

$= \underline{[illegible] \text{ m/s}^2}$ toward C

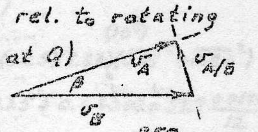
5/64 $v_B = 0.25(5) = 1.25$ m/s $\quad \underline{v}_A = \underline{v}_{rel} = \underline{v}_B + \underline{v}_{A/B}$

$\overline{AB} = \sqrt{(0.6)^2 + (0.25)^2}$
$= 0.650$ m

(v_{rel}: rel. to rotating collar at Q)



pt. on BD



$\sin\beta = \frac{250}{650} = 0.3846$

$\cos\beta = \frac{600}{650} = 0.9231$

$v_A = v_{rel} = v_B\cos\beta = 1.25(0.9231) = 1.154$ m/s

$\omega_{AB} = \frac{v_{A/B}}{\overline{AB}} = \frac{1.25(0.3846)}{0.650} = 0.740$ rad/s

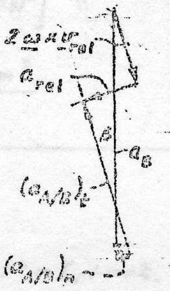
$\underline{a}_B + \underline{a}_{A/B} = \underline{a}_A = \underline{a}_Q + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_B = (a_B)_n = 0.25(5)^2 = 6.25$ m/s²

$(a_{A/B})_n = 0.65(0.740)^2 = 0.356$ m/s²

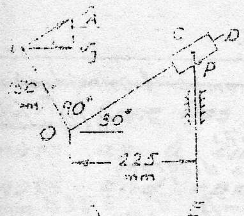
$|2\underline{\omega}\times\underline{v}_{rel}| = 2(0.740)(1.154) = 1.707$ m/s²

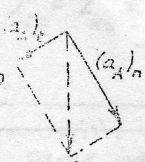
From diag. $(a_{A/B})_t = 6.25(0.9231) - 1.707 = 4.06$ m/s²



$\alpha_{AB} = \frac{(a_{A/B})_t}{\overline{AB}} = \frac{4.06}{0.65} = \underline{6.25 \text{ rad/s}^2 \text{ CW}}$

5/63





$v_B = 0.9$ m/s constant

$v_A = 0.9/\frac{\sqrt{3}}{2} = 1.039$ m/s

$a_{A_{Horiz.}} = 0$ so a_A vertical

$(a_A)_n = \frac{v_A^2}{\overline{AO}} = \frac{(0.6\sqrt{3})^2}{0.15} = 7.20$ m/s²

$(a_A)_t = 7.2/\sqrt{3} = 4.16$ m/s²

$\alpha_{OD} = \alpha_{AO} = \frac{(a_A)_t}{\overline{OA}} = \frac{4.16}{0.15} = 27.7$ rad/s² CCW

$v_P = \frac{\overline{OC}}{\overline{OA}} v_A = \frac{225\frac{2}{\sqrt{3}}}{150} 0.6\sqrt{3} = 1.80$ m/s

$v_P = v_{rel}$

$v_{rel} = \frac{1.80}{\sqrt{3}}$ m/s

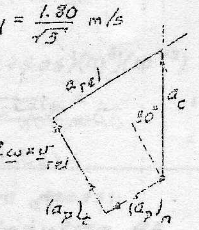
[illegible] $2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

[illegible] $= (1.80)^2/0.225\frac{2}{\sqrt{3}} = 7.2\sqrt{3}$ m/s²

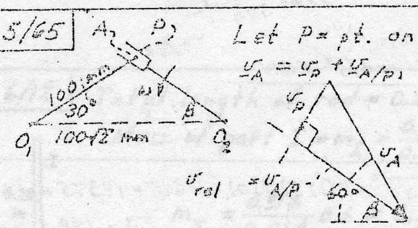
[illegible] $= \overline{OP}\alpha_{OD} = 0.225\frac{2}{\sqrt{3}}\left(\frac{48}{\sqrt{3}}\right) = 7.2$ m/s²

[illegible] $= 14.4$ m/s²

[illegible] $(7.2 + 14.4)\frac{2}{\sqrt{3}} = \underline{24.9 \frac{\text{m}}{\text{s}^2} \text{ up}}$



5/65



Let P = pt. on disk C coincident with A

$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$ $\quad v_A = 100\frac{60(2\pi)}{60} = 628$ mm/s

From O_1O_2P β is found to be $\beta = 42.37°$; $\overline{PO_2} = 74.2$ mm

$v_P = 628\sin(60°\text{-}\beta) = 190.3$ mm/s

$v_{rel} = v_{A/P} = 628\cos(60°\text{-}\beta) = 599$ mm/s

$\omega = 190.3/74.2 = \underline{2.57 \text{ rad/s CCW}}$

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_A = 100\left(\frac{60(2\pi)}{60}\right)^2 = 3950$ mm/s²

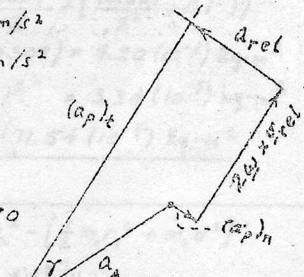
$(a_P)_n = 74.2(2.57)^2 = 488$ mm/s²

$|2\underline{\omega}\times v_{rel}| = 2(2.57)(599) = 3070$ mm/s²

From diagram

$(a_P)_t = 3950\cos 17.63° + 3070 = 6834$ mm/s²

$\alpha = \frac{6834}{74.2} = \underline{92.1 \text{ rad/s}^2}$ CCW



$\gamma = 90° - \beta - 30° = 17.63°$

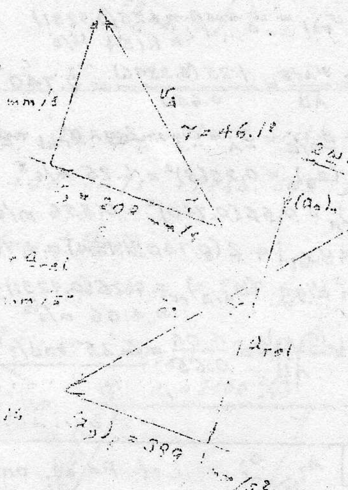
5/66 Point P on BC coincident with A

from geometry of COA, $\overline{CA} = 361$ mm

$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$ $\qquad \beta = 13.90°$

$v_A = 100(3) = 300$ mm/s

$v_P = v_A \frac{\overline{BC}}{\overline{BC}} = 288$ mm/s



$v_{rel} = v_{A/P} = 216$ mm/s

$\gamma = 46.1°$

$v_P = 288$ mm/s

$2\omega \times v_{rel}$ $\quad (a_P)_n$

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$= 1700$ mm/s²

$= 120.0$ mm/s²

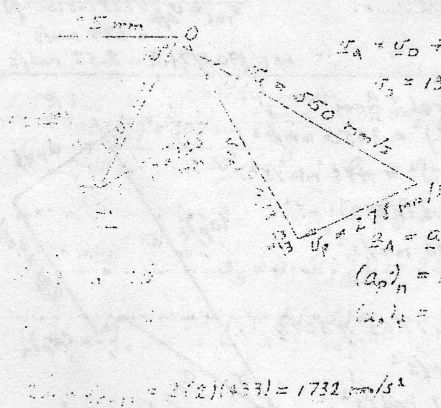
a_{rel}

$(a_P)_t = 399$ mm/s²

$\alpha_{BC} = \frac{399}{361} = 1.107$ rad/s² CCW

$\overline{CP}^2 = \overline{125}^2 + \overline{150}^2 - 2(125)(150)\cos 60° = 19375$ mm²,

$\overline{CP} = 139.2$ mm



$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$, $\underline{v}_{rel} = \underline{v}_{A/P}$

$v_A = 550$ mm/s

$v_P = 139.2(2) = 278$ mm/s

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

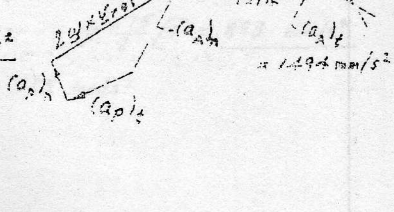
$(a_P)_n = 139.2(2)^2 = 557$ mm/s²

$(a_P)_t = 139.2(6) = 835$ mm/s²

$2\omega v_{rel} = 2(2)(433) = 1732$ mm/s²

$(a_{rel})_n = \frac{(433)^2}{125} = 1500$ mm/s²

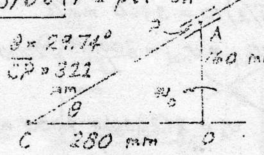
$(a_A)_n = 550^2/150 = 2020$ mm/s²



$\alpha_{AB} = \frac{1494}{150} = 9.96$ rad/s² CCW

$(a_A)_t = 1494$ mm/s²

5/68 P = pt. on screw & is coincident with A



$\theta = 29.74°$

$\overline{CP} = 322$ mm

160 mm

280 mm

$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$, $\underline{v}_{A/P} = \underline{v}_{rel}$

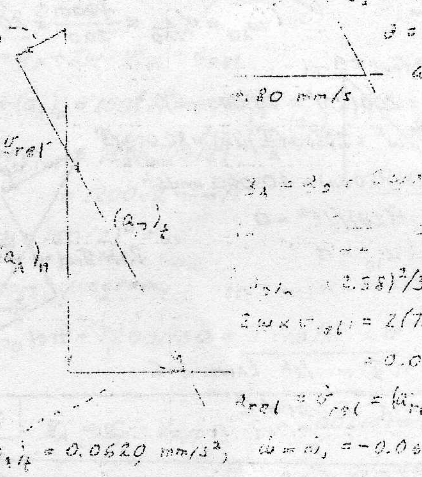
$v_{A/P} = 25\frac{100}{60} = 4.17$ mm/s

$v_P = 2.35$ mm/s

$\dot\theta = \frac{2.35}{322} = 7.30(10^{-3})$

$\omega = \dot\theta$

4.80 mm/s



$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_A = 0.1439$ mm/s²

$(a_P)_n = (2.35)^2/322 = 0.01758$ mm/s²

$2\omega \times v_{rel} = 2(7.30)(10^{-3})(4.17)$

$= 0.0615$ mm/s²

$a_{rel} = \dot v_{rel} = (a_{rel})_t = 0$

$(a_P)_t = 0.0620$ mm/s², $\quad \dot\omega = \alpha = -0.0620/160 = -3.88(10^{-4})$ rad/s

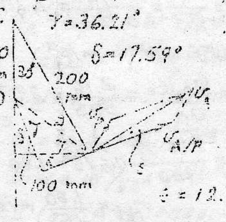
5/69 Let P be on slotted member & coincident with A

$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$; $v_A = 200(6) = 1200$ mm/s

$\beta = 23.79°$

$\gamma = 36.21°$

$\delta = 17.59°$



From sol. ~~to~~ diagram,

$v_P = 437$ mm/s, $v_{A/P} = 820$ mm/s

$\omega_{OP} = \omega = \frac{437}{123.9} = 3.53$ rad/s CCW

$\epsilon = 12.41°$

$\overline{OP} = \sqrt{\overline{100}^2 + (200\cos 30° - 100)^2}$

$= 123.9$ mm

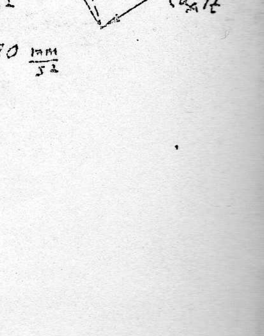
$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$(a_A)_n = 200(6)^2 = 7200$ mm/s²

$(a_A)_t = 200(-30) = -6000$ mm/s²

$(a_P)_n = 123.9(3.53)^2 = 1539$ mm/s²

$|2\underline{\omega}\times\underline{v}_{rel}| = 2(3.53)(820) = 5780$ mm/s²



$(a_P)_t = 2160$ mm/s²

$\alpha_{OP} = \alpha = \frac{2160}{123.9}$

$= 17.42$ rad/s² CW

CHAPTER SIX

PLANE KINETICS OF RIGID BODIES

6/4 error $e = \dfrac{\frac{1}{3}m(\ell^2+a^2/4)-\frac{1}{3}m\ell^2}{\frac{1}{3}m(\ell^2+a^2/4)} = \dfrac{1}{1+\left(\frac{2\ell}{a}\right)^2}$

% error $e = \dfrac{100\%}{1+(2\ell/a)^2} = \dfrac{100\%}{401} = 0.249\%$

6/5 error $= \dfrac{(\frac{1}{2}mr^2+md^2)-md^2}{\frac{1}{2}mr^2+md^2} = \dfrac{1}{1+2(d/r)^2}$

$d = 10r$ % error $e = \dfrac{100}{1+200} = \underline{0.498\%}$

$d = 2r$ % error $e = \dfrac{100}{1+8} = \underline{11.1\%}$

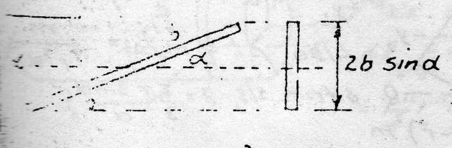
6/6 Complete sphere of mass $2m$, $I_{zz} = \frac{2}{5}(2m)r^2$

For hemisphere $I_{zz} = I_{xx} = \underline{\frac{2}{5}mr^2}$

6/7 Cone has same radial distribution of mass as a circular disk of the same mass and radius. Thus $\underline{I_{zz} = \frac{1}{2}mr^2}$

6/8 ρt = mass per unit area
$= 20.5$ kg/m^2

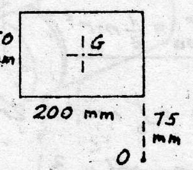
$I_{zz} = \rho t J_z = \rho t(I_x+I_y) = 20.5(1.40+3.05)(10^8)(10^{-12})$
$= \underline{9.12(10^{-3})\ \text{kg}\cdot\text{m}^2}$



Same mass distribution as for rod of length $2b\sin\alpha$ normal to x-axis.

Thus $I_{xx} = \frac{1}{12}m\ell^2 = \frac{1}{12}m(4b^2\sin^2\alpha)$
$= \underline{\frac{1}{3}mb^2\sin^2\alpha}$

6/10 From Sample Prob. 6/3,



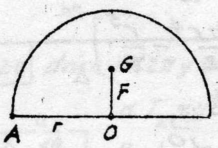
$I_G = \frac{1}{12}m(a^2+b^2)$
$= \frac{1}{12}(0.15)(0.20)(0.25)(7.83)(10^3)(\overline{0.15}^2+\overline{0.20}^2)$
$= \frac{1}{12}(58.7)(0.0225+0.0400) = 58.7\frac{0.0625}{12}$

$\overline{OG} = \sqrt{(150)^2+(100)^2} = 180.3$ mm $= 0.1803$ m

$I_O = I_G + md^2$; $I_O = 58.7\left[\frac{0.0625}{12} + (0.1803)^2\right]$

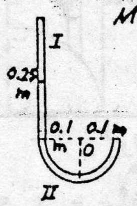
so $I_{OO} = 58.7(0.0377) = \underline{2.21\ \text{kg}\cdot\text{m}^2}$

6/11 $r = 0.2$ m, $\bar{r} = \frac{4r}{3\pi}$; $m = 45$ kg



$I_A = I_G + m(\bar{r}^2+r^2)$
$= I_O - m\bar{r}^2 + m\bar{r}^2 + mr^2 = I_O + mr^2.$
$= \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$
$= \frac{3}{2}(45)(0.2)^2 = \underline{2.70\ \text{kg}\cdot\text{m}^2}$

6/12 Total length of rod $= 0.25 + 0.1\pi = 0.564$ m



Mass of part I $= m_I = \frac{0.250}{0.564}0.6 = 0.266$ kg

$m_{II} = \frac{0.314}{0.564}0.6 = 0.334$ kg

(I): $I_O = \bar{I} + md^2 = m\left(\frac{\ell^2}{12}+d^2\right)$
$= 0.266\left(\frac{(0.25)^2}{12} + \left[\frac{(0.25)^2}{2^2}+\overline{0.1}^2\right]\right)$
$= 0.266(0.0308) = 8.20(10^{-3})$ kg·m^2

(II): $I_O = mr^2 = 0.334(0.1)^2 = 3.34(10^{-3})$ kg·m^2

Total $\underline{I_{OO} = 11.54(10^{-3})\ \text{kg}\cdot\text{m}^2}$

6/13 $I = I_{2_0} - I_{1_0} = \frac{1}{2}m_2r_2^2 - \left[\frac{1}{2}m_1r_1^2 + m_1d^2\right]$
$= \rho\pi t\left[\frac{r_2^4}{2} - \frac{r_1^4}{2} - r_1^2d^2\right]$

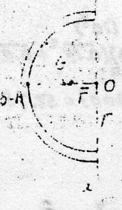
$m = \rho\pi t\left[r_2^2 - r_1^2\right]$

$k_0^2 = \frac{I}{m} = \frac{1}{2}(r_2^2+r_1^2) - \frac{r_1^2d^2}{r_2^2-r_1^2}$

$= \frac{1}{2}(\overline{0.3}^2+\overline{0.15}^2) - \frac{\overline{0.15}^2\,\overline{0.1}^2}{\overline{0.3}^2-\overline{0.15}^2} = 0.0529\ \text{m}^2$; $\underline{k_0 = 0.230\ \text{m}}$

6/14 For complete ring of mass 2m, $I_o = (2m)r^2$ & $I_{aa} = \frac{1}{2}(2m)r^2$

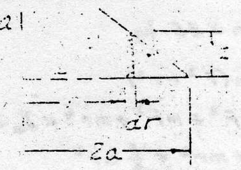
So for half ring $\underline{I_{aa} = \frac{1}{2}mr^2}$



$$I_{oo} = \bar{I} + (r-\bar{r})^2 m$$
$$= I_o - m\bar{r}^2 + (r-\bar{r})^2 m = I_o + m(r^2 - 2r\bar{r})$$
$$= 2mr^2 - 2mr\frac{2r}{\pi} = \underline{2mr^2\left(1-\frac{2}{\pi}\right)}$$

6/15 $\frac{r}{2a}+\frac{z}{2a}=1$, $z = 2a - r$ For elemental shell.

$$dI_{zz} = r^2dm = r^2\rho(2\pi r)z\,dr = 2\pi\rho r^3(2a-r)dr$$

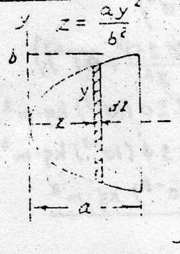


$$I_{zz} = 2\pi\rho\int_a^{2a} r^3(2a-r)\,dr = 2\pi\rho\left[\frac{ar^4}{2}-\frac{r^5}{5}\right]_a^{2a} = 2\pi\rho a^5\frac{13}{10}$$

$$m = \int dm = \int_a^{2a} 2\pi\rho r(2a-r)\,dr = 2\pi\rho\left[ar^2-\frac{r^3}{3}\right]_a^{2a} = 2\pi\rho a^3\frac{2}{3}$$

Thus $\underline{I_{zz} = \frac{39}{20}ma^2}$

6/16 $dI_{zz} = \frac{1}{2}(dm)y^2 = \frac{1}{2}(\pi y^2\rho\,dz)y^2$

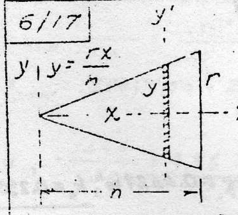


$z = \frac{ay^2}{b^2}$

$$I_{zz} = \frac{\rho\pi b^4}{2a^2}\int_0^a z^2dz = \frac{\rho\pi ab^4}{6}$$
$$V = \int dV = \pi\int_0^a y^2dz = \pi\frac{b^2}{a}\int_0^a z\,dz = \frac{\pi b^2a}{2}$$
$$m = \rho V = \frac{\rho\pi b^2a}{2}$$

So $\underline{I_{zz} = \frac{1}{3}mb^2}$

6/17 $dI_{xx} = \frac{1}{2}(dm)y^2 = \frac{1}{2}(\pi y^2\rho\,dx)y^2$



$y = \frac{rx}{h}$

$$I_{xx} = \frac{\pi\rho r^4}{2h^4}\int_0^h x^4dx = \frac{\pi r^2h\rho}{3}\,\frac{3r^2}{10}$$

$\underline{I_{xx} = \frac{3}{10}mr^2}$

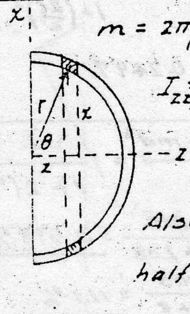
$$dI_{yy} = dI_{y'y'} + x^2dm = \frac{1}{4}dm\,y^2 + x^2dm = \frac{\rho\pi r^2}{h^2}\left[\frac{r^2x^4}{4h^2}+x^4\right]dx$$
$$I_{yy} = \frac{\pi\rho r^2}{h^2}\int_0^h\left(\frac{r^2x^4}{4h^2}+x^4\right)dx = \frac{\pi\rho r^2}{h^2}\left(\frac{r^2h^3}{20}+\frac{h^5}{5}\right)$$
$$= \frac{\pi r^2h\rho}{3}\left(\frac{3r^2}{20}+\frac{3h^2}{5}\right) = \underline{\frac{3}{5}m\left(\frac{r^2}{4}+h^2\right)}$$

6/18 Spherical wedge $m = \frac{\theta}{2\pi}m'$, m' = mass of complete sphere

(a) $I_{zz} = \frac{\theta}{2\pi}\,\frac{2}{5}m'a^2 = \underline{\frac{2}{5}ma^2}$

(b) Similarly for conical wedge $\underline{I_{zz} = \frac{3}{10}mr^2}$

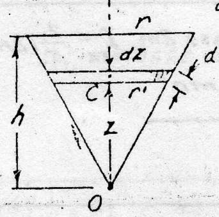
6/19 $dm = 2\pi\rho tr\sin\theta\, r\,d\theta$



$$m = 2\pi\rho tr^2\int_0^{\pi/2}\sin\theta\,d\theta = 2\pi\rho tr^2$$
$$I_{zz} = \int x^2dm = 2\pi\rho t r^4\int_0^{\pi/2}\sin^3\theta\,d\theta$$
$$= \frac{4}{3}\pi\rho tr^4 = \underline{\frac{2}{3}mr^2}$$

Also $\underline{I_{xx} = I_{zz}} = \frac{2}{3}mr^2$ since each is half that for whole shell of mass 2m

6/20 $dm = 2\pi\rho r'dl$ where ρ = mass/unit area



$$dI_{oo} = dI_{cc} + z^2dm$$
$$= \frac{1}{2}(dm)r'^2 + z^2dm$$
$$= \frac{1}{2}\left(2\pi\rho\frac{r}{h}z\frac{\sqrt{r^2+h^2}}{h}dz\right)\frac{r^2}{h^2}z^2 + \left(2\pi\rho\frac{r}{h}z\frac{\sqrt{r^2+h^2}}{h}dz\right)z^2$$

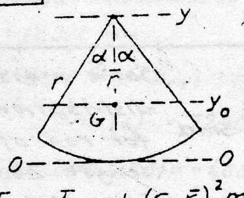
$r' = \frac{r}{h}z$, $\quad I_{oo} = 2\pi\rho\frac{r}{h^2}\sqrt{r^2+h^2}\left(\frac{r^2}{2h^2}+1\right)\int_0^h z^3dz$

$\frac{dl}{\sqrt{r^2+h^2}} = \frac{dz}{h}$, $\quad = \frac{\pi\rho r\sqrt{r^2+h^2}}{4}(r^2+2h^2)$

But $m = \rho\frac{2\pi r}{2\pi l}\pi l^2 = \pi\rho rl = \pi\rho r\sqrt{r^2+l^2}$

$\underline{I_{oo} = \frac{1}{4}m(r^2+2h^2)}$

6/21 For t small $I_{yy} = \rho t I_y$



From Prob. 8/23 (STATICS) $I_y = \frac{r^4}{4}\left(\alpha + \frac{\sin 2\alpha}{2}\right)$

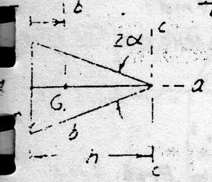
Also $\rho t = m/A = \frac{m}{r^2\alpha}$

& from 5/3, $\bar{r} = \frac{2}{3}\frac{r\sin\alpha}{\alpha}$

$$I_{oo} = I_{y_oy_o} + (r-\bar{r})^2m$$
$$= I_{yy} - m\bar{r}^2 + (r-\bar{r})^2m = I_{yy} + (r^2-2r\bar{r})m$$
$$= \frac{m}{r\alpha}\frac{r^4}{4}\left(\alpha+\frac{\sin2\alpha}{2}\right) + \left(r^2-2r\frac{2}{3}\frac{r\sin\alpha}{\alpha}\right)m$$
$$= \underline{mr^2\left[\frac{5}{4}+\frac{\sin\alpha}{\alpha}\left(\frac{\cos\alpha}{4}-\frac{4}{3}\right)\right]}$$

6/122 $I_{aa} = \frac{3}{10}mr^2$; $I_{cc} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$

$$I_{bb} = I_{cc} - m(h-\bar{h})^2$$



$$= \frac{3}{20}mr^2 + \frac{3}{5}mh^2 - \frac{9}{16}mh^2$$
$$= \frac{3}{20}mr^2 + \frac{3}{80}mh^2$$

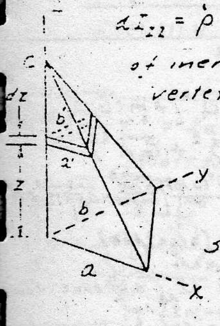
For $I_{aa} = I_{bb}$, $\frac{3}{10}mr^2 = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$

So $\left(\frac{r}{h}\right)^2 = \frac{20}{80}$, $\frac{r}{h} = \frac{1}{2}$ & $\tan\alpha = \frac{1}{2}$, $\underline{\alpha = 26°34'}$

6/123 $a' = a\frac{c-z}{c}$ Area of elemental triangle

$b' = b\frac{c-z}{c}$ $\quad A = \frac{1}{2}a'b' = \frac{ab}{2c^2}(c-z)^2$

$dI_{zz} = \rho\, dz\,(J_z)$ where J_z = polar moment of inertia of triangular area about vertex; $J_z = \frac{1}{12}a'b'^3 + \frac{1}{12}b'a'^3 = \frac{1}{12}a'b'(a'^2+b'^2)$



$$= \frac{1}{12}\frac{ab}{c^4}(a^2+b^2)(c-z)^4$$

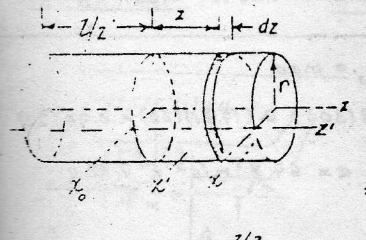
So $I_{zz} = \frac{\rho ab(a^2+b^2)}{12c^4}\int_0^c (c-z)^4 dz$

$$= \frac{\rho ab(a^2+b^2)}{12c^4}\left(-\frac{[c-z]^5}{5}\right)_0^c$$

$$= \frac{\rho abc(a^2+b^2)}{60}$$

But $m = \frac{1}{3}\frac{ab}{2}c\rho$, so $\underline{I_{zz} = \frac{1}{10}m(a^2+b^2)}$

6/124 $I_{z'z'} = I_{zz} + mr^2 = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$



$$dI_{x'x'} = \frac{1}{4}(dm)r^2 = \frac{\rho\pi}{4}r^4 dz$$
$$dI_{x_0x_0} = dI_{x'x'} + z^2 dm$$
$$= \frac{\rho\pi}{4}r^4 dz + \rho\pi r^2 z^2 dz$$

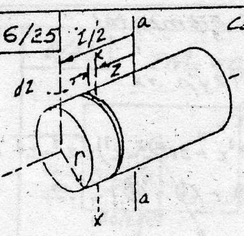
So $I_{x_0x_0} = \rho\pi r^2\int_{-l/2}^{l/2}\left(\frac{r^2}{4}+z^2\right)dz = \rho\pi r^2\left(\frac{r^2 l}{4} + \frac{l^3}{12}\right)$

$$\underline{I_{x_0x_0} = \frac{m}{12}(3r^2+l^2)}$$

$I_{xx} = I_{x_0x_0} + m\left(\frac{l}{2}\right)^2 = \frac{m}{12}(3r^2+l^2) + \frac{m}{4}l^2$

$$\underline{I_{xx} = \frac{m}{12}(3r^2+4l^2)}$$

6/125 Consider complete cylindrical shell with mass ρ per unit area:



$$dI_{xx} = \frac{1}{2}dm\, r^2$$
$$dI_{aa} = dI_{xx} + dm\, z^2$$
$$= \frac{1}{2}(2\pi r\, dz\,\rho)r^2 + (2\pi r\, dz\,\rho)z^2$$
$$= \pi\rho r\,(r^2 + 2z^2)dz$$

$I_{aa} = \pi\rho r\int_{-l/2}^{l/2}(r^2+2z^2)dz = \pi\rho r\left[r^2 z + \frac{2z^3}{3}\right]_{-l/2}^{l/2}$

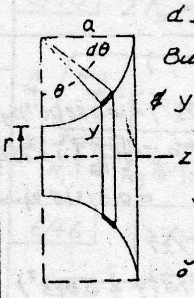
$$= 2\pi\rho r l\left(\frac{r^2}{2} + \frac{l^2}{12}\right)$$

For half-shell of mass $m = \pi\rho r l$

$$\underline{I_{aa} = \frac{m}{2}\left(r^2 + \frac{l^2}{6}\right)}$$

6/126 $dm = \rho(2\pi y\, a\, d\theta)$ where ρ = mass/unit area

$$dI_{zz} = y^2 dm = 2\pi\rho a y^3 d\theta$$



But $y = r + a(1-\cos\theta) = (r+a) - a\cos\theta$

& $y^3 = (r+a)^3 - 3a(r+a)^2\cos\theta + 3a^2(r+a)\cos^2\theta - a^3\cos^3\theta$

$$\int_0^{\pi/2}\cos\theta\, d\theta = \sin\theta\Big]_0^{\pi/2} = 1$$

$$\int_0^{\pi/2}\cos^2\theta\, d\theta = \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi/2} = \frac{\pi}{4}$$

$$\int_0^{\pi/2}\cos^3\theta\, d\theta = \left[\frac{\sin\theta}{3}(2+\cos^2\theta)\right]_0^{\pi/2} = \frac{2}{3}$$

So $I_{zz} = 2\pi\rho a\left[(r+a)^3\frac{\pi}{2} - 3a(r+a)^2(1) + 3a^2(r+a)\frac{\pi}{4} - a^3\frac{2}{3}\right]$

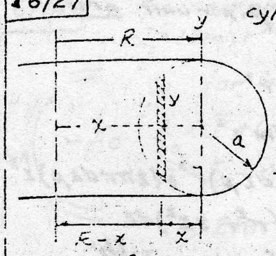
& for $r = a/3$, $I_{zz} = 2\pi\rho a^4\left(\frac{59}{27}\pi - 6\right) = 2\pi\rho a^4(0.865)$

$m = 2\pi\rho a\int_0^{\pi/2}[(r+a) - a\cos\theta]d\theta = 2\pi\rho a^2\left(\frac{2\pi}{3} - 1\right)$

$$= 2\pi\rho a^2(1.0944)$$

$k_z^2 = \frac{I_{zz}}{m} = \frac{0.865}{1.0944}a^2$, $\underline{k_z = 0.890a}$

6/27 Cross section of elemental cylindrical shell



$dm = 2\pi(R-x)(2y\rho\,dx)$

$dI = (R-x)^2 dm$

$= 4\pi\rho\,(R^3 - 3R^2x + 3Rx^2 - x^3)\sqrt{a^2-x^2}\,dx$

$I = 4\pi\rho\,[① - ② + ③ - ④]$

$① = R^3\int_{-a}^{a}\sqrt{a^2-x^2}\,dx = \dfrac{\pi a^2 R^3}{2}$

$② = 3R^2\int_{-a}^{a} x\sqrt{a^2-x^2}\,dx = 0$

$③ = 3R\int_{-a}^{a} x^2\sqrt{a^2-x^2}\,dx = \dfrac{3Ra^4\pi}{8}$

$④ = \int x^3\sqrt{a^2-x^2}\,dx = 0$

$m = \rho V$
$= \rho(2\pi R)(\pi a^2)$
$= 2\rho\pi^2 a^2 R$

Thus $I = 2\rho\pi^2 a^2 R\left(R^2 + \frac{3}{4}a^2\right) = \underline{m\left(R^2 + \frac{3}{4}a^2\right)}$

6/28 Hub; $I = \frac{1}{2}m(r_1^2 + r_2^2) = \frac{1}{2}\rho\pi t(r_2^4 - r_1^4)$

$= \frac{1}{2}(7830)\pi(0.075)(\overline{0.025}^4 - \overline{0.0125}^4) = 0.338(10^{-3})\,\text{kg}\cdot\text{m}^2$

Spokes; $I = 6\left[\frac{ml^2}{12} + md^2\right] = 6(7830)(250)(10^{-6})(0.125)\left[\frac{\overline{0.125}^2}{12} + \overline{0.0875}^2\right]$

$= 0.01315\ \text{kg}\cdot\text{m}^2$

Rim; $I = m(R^2 + \frac{3}{4}a^2)$ (see Prob. 6/27)

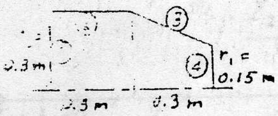
$= 2\pi(0.175)(\pi\,\overline{0.025}^2)(7830)(\overline{0.175}^2 + \frac{3}{4}\overline{0.025}^2)$

$= 0.526\ \text{kg}\cdot\text{m}^2$

Total $I = 0.000338 + 0.01315 + 0.526$

$= \underline{0.539\ \text{kg}\cdot\text{m}^2}$

6/29 ① $I = \frac{1}{2}mr^2 = \frac{1}{2}(17.5)\pi(0.3)^2(0.3)^2 = 0.223\ \text{kg}\cdot\text{m}^2$



② $I = mr^2 = 17.5(2\pi)(0.3)(0.3)(0.3)^2$

$= 0.891\ \text{kg}\cdot\text{m}^2$

③ $I = \frac{1}{2}m_2 r_2^2 - \frac{1}{2}m_1 r_1^2 = \frac{1}{2}\rho(\pi r_2 l_2)r_2^2 - \frac{1}{2}\rho(\pi r_1 l_1)r_1^2$

where ρ = mass/unit area, l = slant height,

& cone area $= \frac{2\pi r}{2\pi l}\pi l^2 = \pi r l$.

So $I = \frac{1}{2}\rho\pi(r_2^3 l_2 - r_1^3 l_1)$; $l_2 = 0.3\sqrt{5} = 0.671$ m, $l_1 = 0.335$ m

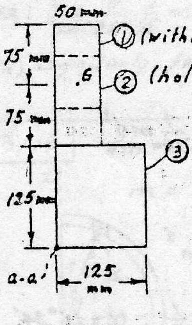
$I = \frac{1}{2}(17.5)\pi(\overline{0.3}^3\,0.671 - \overline{0.15}^3\,0.335) = 0.467\ \text{kg}\cdot\text{m}^2$

④ $I = \frac{1}{2}mr^2 = \frac{1}{2}17.5\pi(0.15)^2(0.15)^2 = 0.014\ \text{kg}\cdot\text{m}^2$

Total $I = 0.223 + 0.891 + 0.467 + 0.014$

$= \underline{1.594\ \text{kg}\cdot\text{m}^2}$

6/30



① (without hole)

② (hole)

①; $I_{aa} = \bar{I} + md^2$

$I_{aa} = 7830(0.05)(0.15)(0.125)\left[\frac{\overline{0.05}^2 + \overline{0.15}^2}{12} + \overline{0.025}^2 + \overline{0.2}^2\right]$

$= 0.314\ \text{kg}\cdot\text{m}^2$

② $I_{aa} = \bar{I} + md^2$; from Prob. 6/24

$\bar{I} = \frac{m}{12}(3r^2 + l^2)$

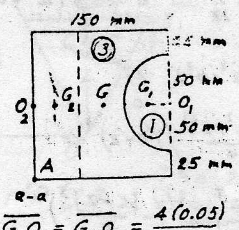
so $I_{aa} = 7830\,\pi\frac{(0.075)^2}{4}(0.05)\left[\frac{3(0.0375)^2 + (0.05)^2}{12} + (0.025)^2 + (0.2)^2\right] = 0.0712\ \text{kg}\cdot\text{m}^2$ (negative)

③ By symmetry $I_{aa} = \frac{1}{2}$ of that for complete cylinder so from Prob. 6/24 $I_{aa} = \frac{1}{2}\frac{(2m)}{12}(3r^2 + 4l^2)$

$= \frac{1}{2}7830\frac{0.125\pi(0.125)^2}{12}[3(0.125)^2 + 4(0.125)^2] = 0.219\ \text{kg}\cdot\text{m}^2$

Total $I_{aa} = 0.314 + 0.07[illegible] + 0.219 = \underline{0.461\ \text{kg}\cdot\text{m}^2}$

6/31 Groove ① $I_{aa} = \bar{I} + m(\overline{G_1A})^2$



$= I_{O_1O_1} - m(\overline{G_1O_1}^2) + m(\overline{G_1A}^2)$

$= m\left(\frac{1}{2}r^2 - \overline{G_1O_1}^2 + \overline{G_1A}^2\right)$

$= (11\,370)\frac{\pi(0.05)^2(0.15)}{2}\left[\frac{\overline{0.05}^2}{2} - \overline{0.0212}^2 + 0.0222\right]$

$= 0.1541\ \text{kg}\cdot\text{m}^2$ (negative)

$\overline{G_1O_1} = \overline{G_2O_2} = \frac{4(0.05)}{3\pi}$

$= 0.0212$ m

$\overline{G_1A}^2 = (0.15 - 0.0212)^2 + (0.075)^2$

$= 0.0222\ \text{m}^2$

Groove ② $I_{aa} = \frac{1}{2}$ that for complete cyl. by symmetry

From Prob. 6/24, $I_{aa} = \frac{1}{2}\frac{2m}{12}(3r^2 + 4l^2)$

$I_{aa} = \frac{(11\,370)\pi(0.05)^2(0.15)}{2}\frac{[3(0.05)^2 + 4(0.15)^2]}{12}$

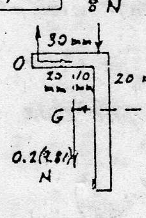
$= 0.0544\ \text{kg}\cdot\text{m}^2$ (negative)

③ $I_{aa} = \frac{1}{12}m(a^2 + a^2) + m\left(\frac{a^2}{4} + \frac{a^2}{4}\right) = \frac{2}{3}ma^2$

$= \frac{2}{3}(11\,370)(0.15)^3(0.15)^2 = 0.576\ \text{kg}\cdot\text{m}^2$

Total $I_{aa} = 0.576 - 0.0544 - 0.1541 = \underline{0.367\ \text{kg}\cdot\text{m}^2}$

6/36

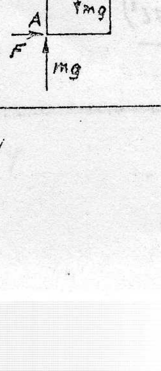


$\Sigma M_O = ma\,d$

$8(30) + 0.2(9.81)(20) = 0.2a(20)$

$a = 69.8\ \text{m/s}^2 = \underline{7.12\,g}$

6/37 $v = 1.2 + 0.9t^2$ m/s



$a = \dot{v} = 1.8t\ \text{m/s}^2$

$\Sigma M_A = ma\,d$; $mg\frac{250}{2} = m(1.8t)(200)$

$t = \frac{250g}{720} = \frac{250(9.81)}{720} = \underline{3.41\ \text{s}}$

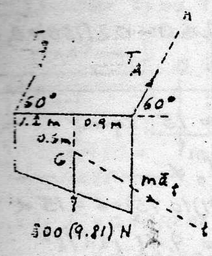
6/38

$\Sigma F_n = 0;\ T_A + T_B - 800(9.81)(0.866) = 0$

$\Sigma M_G = 0;\ (T_A + T_B)0.5(0.6) + 0.866(1.2T_B - 0.9T_A) = 0$

Solve for T_B & get

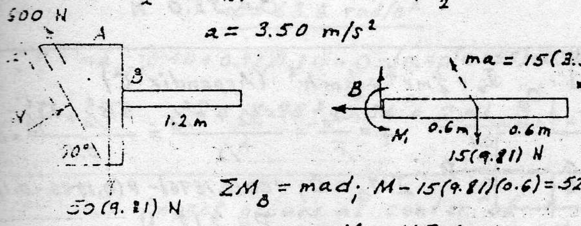
$\underline{T_B = 1792\ \text{N}}$



6/39

$\Sigma F_x = ma_x;\ 600 - 50(9.81)\frac{\sqrt{3}}{2} = 50\,a$

$a = 3.50\ \text{m/s}^2$

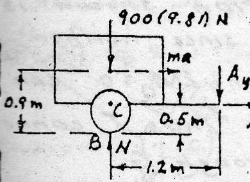


$\Sigma M_B = m\bar a d;\ M - 15(9.81)(0.6) = 52.6\frac{\sqrt{3}}{2}(0.6)$

$\underline{M = 115.6\ \text{N·m}}$

6/40

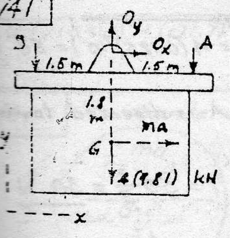
$v^2 = 2as,\quad a = \frac{v^2}{2s} = \frac{(60/3.6)^2}{2(30)} = 4.63\ \text{m/s}^2$



$\Sigma M_C = m\bar a d;\ 1.2A_y = 900(4.63)(0.9 - 0.5)$

$\underline{A_y = 1389\ \text{N}}$

6/41



$A = 400\pi(0.05)^2 = 3.14\ \text{kN}$

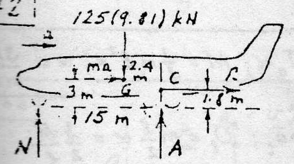
$B = 500\pi(0.05)^2 = 3.93\ \text{kN}$

$\Sigma M_O = m\bar a d;$

$3.93(1.5) - 3.14(1.5) = 4a(1.8)$

$\underline{a = 0.1636\ \text{m/s}^2}$

6/42



$v^2 = v_0^2 + 2as$

$a = \frac{1}{2(450)}(200^2 - 50^2)/(3.6)^2$

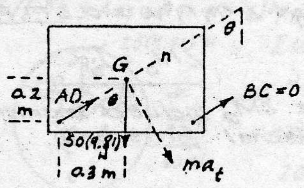
$= 3.22\ \text{m/s}^2$

$\Sigma M_C = m\bar a d;\ 15\,N - 2.4(125)(9.81) = 125(3.22)(3 - 1.8)$

$\underline{N = 228\ \text{kN}}$

6/43

$\Sigma M_G = 0;$ For $\theta = \tan^{-1}\frac{0.3}{0.2} = 56.31°$ & $BC = 0$

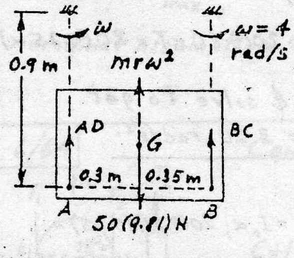


$\Sigma F_n = 0;\ AD - 50(9.81)\cos 56.31° = 0$

$\underline{AD = 272\ \text{N}}$

6/44

$mr\omega^2 = 50(0.9)(4)^2 = 720\ \text{N}$



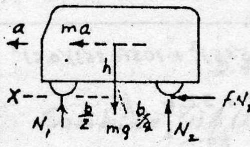
$\Sigma M_B = m\bar a d;$

$AD(0.3 + 0.35) - 50(9.81)(0.35) = 720(0.35)$

$\underline{AD = 652\ \text{N}}$

6/45

$\Sigma F_x = ma_x;\ fN_2 = ma$



$\Sigma M_{N_1} = m\bar a d;\ -mg\frac{b}{2} + N_2 b = mah$

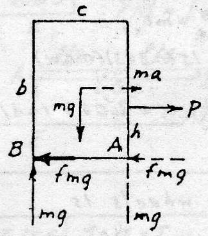
Solve & get

$-mg\frac{b}{2} + \frac{ma}{f}b = mah,\quad a = \frac{1}{2}\frac{bfg}{b - hf}$

$v^2 = 2as$ so $\underline{v = \sqrt{\frac{fbgs}{b - hf}}}$

6/46

$P > fmg;\ \Sigma F = ma;\ P - fmg = ma$



(a) Tipping about B

$\Sigma M_B = m\bar a d;\ Ph + mg\frac{c}{2} = ma\frac{b}{2} = \frac{b}{2}(P - fmg)$

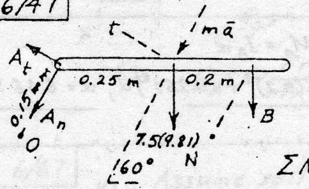
$\underline{h = h_{min} = \frac{1}{2}\left[b - \frac{mg}{P}(c + fb)\right]}$

(b) Tipping about A

$\Sigma M_A = m\bar a d;\ Ph - mg\frac{c}{2} = ma\frac{b}{2} = \frac{b}{2}(P - fmg)$

$\underline{h = h_{max} = \frac{1}{2}\left[b + \frac{mg}{P}(c - fb)\right]}$

6/47



$\bar a = \bar a_n = a_O + (a_{A/O})_n$

where $a_O = 0$

$\bar a = (a_{A/O})_n = 0.15\left(\frac{2.4}{0.175}\right)^2 = 28.2\ \frac{\text{m}}{\text{s}^2}$

$\Sigma M_A = m\bar a d;$

$7.5(9.81)(0.25) + B(0.45) = 7.5(28.2)0.25\sin 60°$

$\underline{B = 60.9\ \text{N}}$

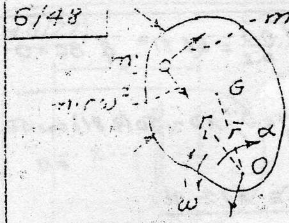
$\Sigma F_t = 0;\ A_t = (7.5(9.81) + 60.9)\cos 60° = 67.3\ \text{N}$

$\Sigma F_n = m\bar a_n;\ A_n + (7.5(9.81) + 60.9)\sin 60° = 7.5(28.2)$

$A_n = 95.1\ \text{N}$

$A = \sqrt{(67.3)^2 + (95.1)^2} = \underline{116.5\ \text{N}}$

6/48



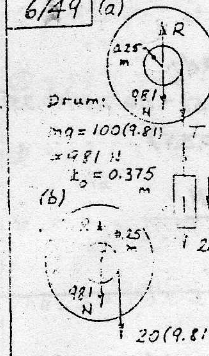
$-m_i r_i \alpha \qquad M_{O_i} = m_i r_i \alpha (r_i) + 0$

$\Sigma M_O = \Sigma m_i r_i^2 \alpha = \alpha \Sigma m_i r_i^2$

$\underline{\Sigma M_O = I_O \alpha}$.

where ΣM_O includes moments of external forces only.

6/49 (a)



$\Sigma M_O = I_O \alpha;\ T(0.25) = 100(0.375)^2 \alpha$

$\Sigma F = ma;\ 20(9.81) - T = 20(0.25\alpha)$

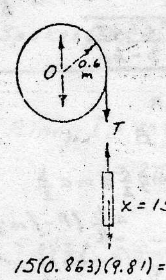
Combine & solve to get

$\underline{\alpha_a = 3.20 \text{ rad/s}^2}$.

$\Sigma M_O = I_O \alpha;\ 20(9.81)(0.25) = 100(0.375)^2 \alpha$

$\underline{\alpha_b = 3.49 \text{ rad/s}^2}$.

6/50



Drum & cable $I_O = 110(0.546)^2 + 105(0.863)(0.6)^2$

$= 32.8 + 32.6$

$= 65.4 \text{ kg}\cdot\text{m}^2$

$\Sigma M_O = I_O \alpha;\ 0.6T = 65.4\alpha$

15-m section: $\Sigma F_x = ma_x;$

$127.0 - T = 15(0.863)(0.6\alpha)$

Combine & get $\underline{\alpha = 1.087 \text{ rad/s}^2}$.

6/51 Accelerating force on rear wheels is

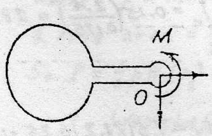
$F = ma = 2.8(0.6)(9.81) = 16.48$ kN

$\alpha_{Drum} = \dfrac{a_t}{r} = \dfrac{0.6(9.81)}{0.9} = 6.54 \text{ rad/s}^2$

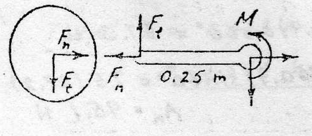
$\Sigma M_O = I_O \alpha;\quad I_O = \dfrac{16.48(0.9)}{6.54} = \underline{2.27 \text{ t}\cdot\text{m}^2}$

6/52 (a) Pin in place; $\Sigma M_O = I_O \alpha$

$30 = 43\left(\frac{1}{2}(0.2)^2 + (0.25)^2\right)\alpha,\ \alpha = 8.46 \frac{\text{rad}}{\text{s}^2}$



(b) Pin removed: $F_t = M/r = 30/0.25 = 120$ N

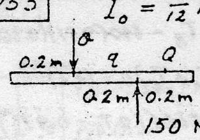


Rotor: $\Sigma F_t = ma_t$

$120 = 43(0.25\alpha)$

$\underline{\alpha = 11.16 \text{ rad/s}^2}$

6/53

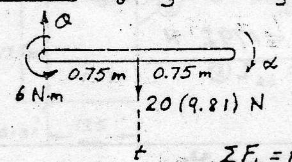


$I_O = \frac{1}{12}10(0.6)^2 + 10(0.1)^2,\ k_o^2 = 0.04 \text{ m}^2$

$q = k_o^2/\bar r = 0.04/0.1 = 0.4$ m

$\Sigma M_Q = 0;\ 0.4\,O - 0.2(150) = 0,\ O = $

6/54 $I_O = \frac{1}{3}ml^2 = \frac{1}{3}(20)(1.5)^2 = 15 \text{ kg}\cdot\text{m}^2$



$\Sigma M_O = I_O \alpha;$

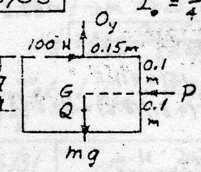
$20(9.81)(0.75) - 6 = 15\alpha$

$\alpha = 9.41 \text{ rad/s}^2$

$\Sigma F_t = m\bar a_t;\ 20(9.81) - O = 20(0.75)(9.$

$O = 55.0$ N

6/55 $I_O = \frac{1}{4}mr^2 + \frac{1}{3}mh^2$ (Appendix C)

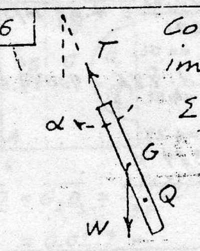


$q = \dfrac{k_o^2}{\bar r} = \dfrac{r^2/4 + h^2/3}{h/2} = \dfrac{\overline{0.15}^2/4 + \overline{0.2}^2/3}{0.1} = 0.1$

$\Sigma M_Q = 0;\ 100(0.1896) - P(0.1896 - 0.1)$

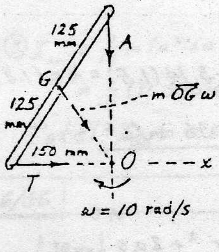
$\underline{P = 212 \text{ N}}$

6/56



Configuration shown is impossible since neither $\Sigma \overline{M} = \bar I \alpha$ nor $\Sigma M_Q = 0$ can satisfied. Thus T cannot pass through G.

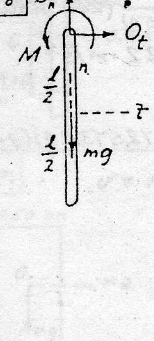
6/57 $\overline{GO} = 125$ mm, $m\overline{OG}\omega^2 = 4(0.125)(10^2) = 50$ N



$\Sigma M_O = 0$ so A is directed toward O

$\Sigma F_x = m\bar a_x;\ T = 50\frac{3}{5} = \underline{30 \text{ N}}$

6/58



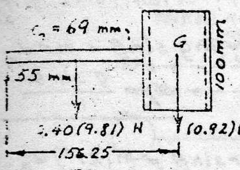
$\Sigma M_O = I_O \alpha;\ M = \frac{1}{3}ml^2\alpha,\ \alpha = \underline{\dfrac{3M}{ml^2}}$

$\Sigma F_t = m\bar a_t;\ O_t = m\frac{l}{2}\alpha = \underline{\frac{3}{2}\frac{M}{l}}$

$\Sigma F_n = m\bar a_n;\ O_n - mg = m\frac{l}{2}\omega^2$

$\underline{O_n = mg\left(1 + \dfrac{l\omega^2}{2g}\right)}$

6/59 For tube (see Appendix C)



$I_G = \frac{1}{2}m\left(r^2 + \frac{l^2}{6}\right)$

$= \frac{1}{2}(0.92)\left(\overline{31.25}^2 + \frac{\overline{100}^2}{6}\right)(10^{-6})$

$= 0.001216 \ \text{kg}\cdot\text{m}^2$

$I_o = I_G + m\bar{r}^2$

$= 0.001216 + 0.92(0.15625)^2$

$= 0.0237 \ \text{kg}\cdot\text{m}^2$

Link $I_o = 0.40(0.069)^2$

$= 0.001904 \ \text{kg}\cdot\text{m}^2$

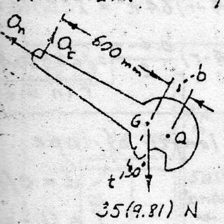
$\Sigma M_o = I_o\alpha$; $0.92(9.81)(0.15625) + 0.40(9.81)(0.055) = (0.0237 + 0.001904)\alpha$

$\underline{\alpha = 63.6 \ \text{rad/s}^2}$

$\Sigma F_t = m\bar{a}_t$; $(0.40 + 0.92)(9.81) - O = (0.40[0.055] + 0.92[0.15625])\alpha$

$O = 12.95 - 10.54 = \underline{2.41 \ \text{N}}$

6/60 If impact occurs at center of percussion, bearing reaction will have vertical component only & hence will be minimum. So $q = 600 + b = k_o^2/\bar{r} = \overline{650}^2/600$



$\underline{b = 104.2 \ \text{mm}}$

$\Sigma F_n = m\bar{a}_n = 0$;

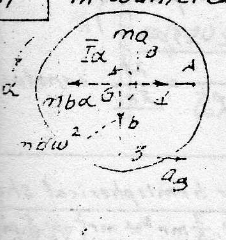
$O_n = 35(9.81)(0.5) = 171.7 \ \text{N}$

$\Sigma M_Q = 0$; $(600 + 104.2)O_t - 35(9.81)(0.866)(104.2) = 0$

$O_t = 44.0 \ \text{N}$

$O = \sqrt{(44.0)^2 + (171.7)^2} = \underline{177.2 \ \text{N}}$

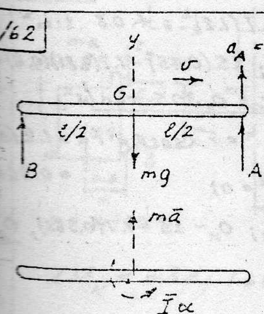
6/61 In counterclockwise sense,



$\underline{\Sigma M_A = \bar{I}\alpha + mbd\omega^2}$

$\underline{\Sigma M_B = \bar{I}\alpha + mb^2\alpha - ma_B b}$

6/62



$\bar{a} = a_B + a_{G/B} = 0 + \frac{a_A}{2}j = \frac{v^2}{2r}j$

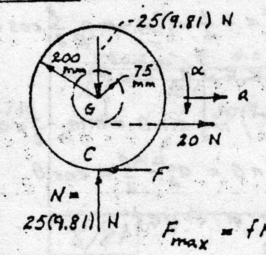
$\Sigma M_B = \bar{I}\alpha + m\bar{a}d$

$Al - mg\frac{l}{2} = \frac{1}{12}ml^2\alpha + m\frac{v^2}{2r}\frac{l}{2}$

$\alpha = \frac{a_A}{l} = \frac{v^2}{rl}$

$\underline{A = mg\left(\frac{1}{2} + \frac{1}{3}\frac{v^2}{gr}\right)}$

6/63 Assume no slipping: $a = r\alpha$



$\Sigma M_C = I_C\alpha$; $20(0.20 - 0.075) = 25(\overline{0.175}^2 + \overline{0.20}^2)\alpha$; $\alpha = 1.416 \ \text{rad/s}^2$

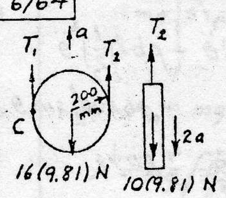
Check: $\Sigma F = m\bar{a}$;

$20 - F = 25(0.20)1.416$

$F = 20 - 7.08 = 12.92 \ \text{N}$

$F_{max} = fN = 0.10(25)(9.81) = 24.5 \ \text{N} > 12.92 \ \text{N}$, so original assumption OK, & $\underline{\alpha = 1.416 \ \text{rad/s}^2}$

6/64



Counterweight $\Sigma F = ma$;

$10(9.81) - T_2 = 10(2a)$ ---- (1)

Cylinder $\Sigma M_C = I_C\alpha$;

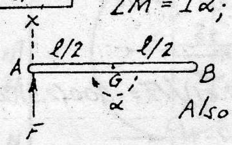
$I_C = \frac{1}{2}mr^2 + mr^2$

$= \frac{3}{2}16(0.2)^2 = 0.96 \ \text{kg}\cdot\text{m}^2$

$0.4T_2 - 16(9.81)(0.2) = 0.96\frac{a}{0.2}$ ----- (2)

Combine (1) & (2) & get $a = 0.613 \ \text{m/s}^2$

6/65 $\Sigma \bar{M} = \bar{I}\alpha$; $F\frac{l}{2} = \frac{1}{12}ml^2\alpha$, $F = \frac{1}{6}ml\alpha$

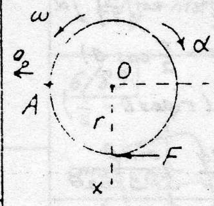


$\Sigma F_x = m\bar{a}_x$; $F = m\bar{a}$

Also $a_B = \bar{a} + a_{B/G}$; $a_B = \bar{a} - \frac{l}{2}\alpha = \frac{F}{m} - \frac{l}{2}\frac{6F}{ml}$

$\underline{a_B = -2\frac{F}{m}}$ (-x-dir.)

6/66 $\Sigma M_O = I_O\alpha$; $Fr = mk^2\alpha$, $\alpha = \frac{Fr}{mk^2}$



$\Sigma F_y = ma_y$; $F = ma_o$

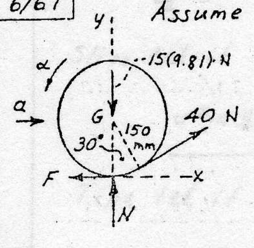
$a_A = a_o + a_{A/O}$

$(a_{A/O})_n = r\omega^2 j$, $(a_{A/O})_t = -r\alpha i = -\frac{Fr^2}{mk^2}i$

$a_o = -\frac{F}{m}j$

So $\underline{a_A = -\frac{Fr^2}{mk^2}i - \left(\frac{F}{m} - r\omega^2\right)j}$

6/67 Assume roll slips, $F = 0.2N$



$\Sigma \bar{M} = \bar{I}\alpha$; $(40 - 0.2N)(0.15) = \frac{1}{2}15(0.15)^2\alpha$

$\Sigma F_y = 0$; $N + 40(0.5) - 15(9.81) = 0$,

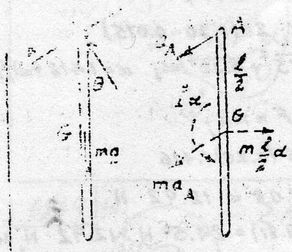
$N = 127.2 \ \text{N}$

Substitute N & get

$\underline{\alpha = 12.95 \ \text{rad/s}^2 \ \text{CCW}}$

$\Sigma F_x = m\bar{a}_x$; $40\frac{\sqrt{3}}{2} - 0.2(127.2) = 15\bar{a}$, $\underline{\bar{a} = 0.614 \ \text{m/s}^2}$

6/68



$$\Sigma M_A = \bar{I}\alpha + m\frac{l\alpha}{2}\frac{l}{2} - ma_A\frac{l}{2}\cos\theta$$

$$0 = \frac{1}{12}ml^2\alpha + \frac{1}{4}ml^2\alpha - ma_A\frac{l}{2}\cos\theta$$

$$2l\alpha = 3a_A\cos\theta$$

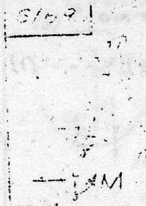
$$\Sigma F_x = m\bar{a}_x;\ mg\sin\theta = m(a_A - \frac{l}{2}\alpha\cos\theta)$$

$$g\sin\theta = a_A - \frac{l}{2}\alpha\cos\theta$$

Eliminate α & get

$$\underline{a_A = \frac{g\sin\theta}{1-\frac{3}{4}\cos^2\theta}}$$

6/69

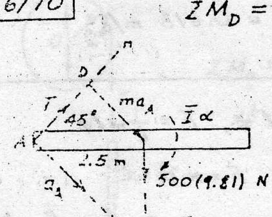


$$\Sigma M_A = \bar{I}\alpha + m\bar{a}d;$$

$$-M = \frac{1}{12}\rho b l(l^2)\ddot{\theta} + \rho b l\left(\frac{l}{2}\right)^2\ddot{\theta} - \rho b l a\frac{l}{2}\sin\theta$$

$$\underline{M = \rho b l^2\left(\frac{a}{2}\sin\theta - \frac{1}{3}l\ddot{\theta}\right)}$$

6/70



$$\Sigma M_D = \bar{I}\alpha + m\bar{a}d;$$

$$500(9.81)(1.25) = \frac{1}{12}500(5)^2\alpha + 500(2.5)\cdot(1.25)\alpha$$

$$\alpha = 2.35\ \text{rad/s}^2$$

$$\Sigma F_n = m\bar{a}_n$$

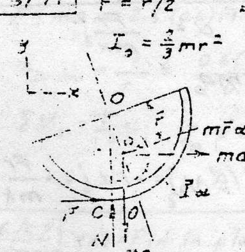
$$T - 500(9.81)/\sqrt{2} = -500(2.5)(2.35)/\sqrt{2}$$

$$\underline{T = 1387\ \text{N}}$$

6/71

$\bar{r} = r/2$

$I_o = \frac{2}{3}mr^2$

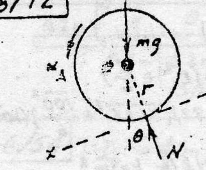


$$\Sigma M_C = \bar{I}\alpha + m\bar{a}d;$$

$$mg\frac{r}{2}\sin\theta = \left(\frac{2}{3}mr^2 - m\left[\frac{r}{2}\right]^2\right)\alpha + mr\alpha\left(r - \frac{r}{2}\cos\theta\right) - m\frac{r}{2}\alpha\left(r\cos\theta - \frac{r}{2}\right)$$

Solve & get $\underline{\alpha = \frac{3g}{2r}\,\frac{\sin\theta}{5-3\cos\theta}}$

6/72

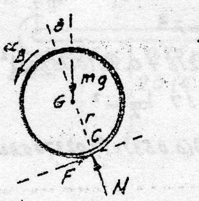


$\Sigma\bar{M} = \bar{I}\alpha$, but $\bar{I} = 0$ so $\Sigma\bar{M} = 0$.

Hence no friction force & $f_A = 0$

$$\Sigma F_x = m\bar{a}_x;\ mg\sin\theta = mr\alpha_A$$

$$\boxed{\alpha_A = \frac{g\sin\theta}{r}}$$



$$\Sigma M_C = I_C\alpha;\ mgr\sin\theta = m(2r^2)\alpha_B$$

$$\underline{\alpha_B = \frac{g\sin\theta}{2r}}$$

$$\Sigma\bar{M} = \bar{I}\alpha;\ Fr = mr^2\frac{g\sin\theta}{2r}$$

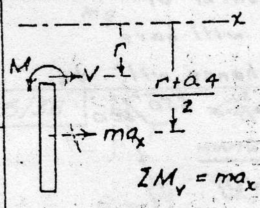
$$F = \frac{1}{2}mg\sin\theta$$

$$f_B = \frac{F}{N} = \frac{1}{2}mg\sin\theta/mg\cos\theta$$

$$\underline{f_B = \frac{1}{2}\tan\theta}$$

6/73

$x = x_o\sin pt$, $\dot{x} = x_o p\cos pt$, $a_x = \ddot{x} = -x_o p^2 \sin pt$



$$|a_x|_{max} = x_o p^2 = 10(40\times2\pi)^2\ \text{mm/s}^2$$

$$= 64\pi^2\ \text{m/s}^2$$

$$m|a_x|_{max} = 4(0.4-r)64\pi^2 = 2530(0.4-r)$$

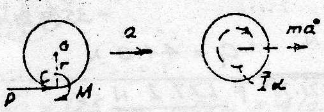
$$\Sigma M_V = ma_x d;\ M = 2530(0.4-r)\frac{0.4-r}{2} = \underline{1263(0.4-r)^2}$$

$\underline{r \text{ in meters}}$

6/74

Let ρ = mass per unit length of tape

$\alpha = a/r$



$\bar{I} = (L-x)\rho r^2$

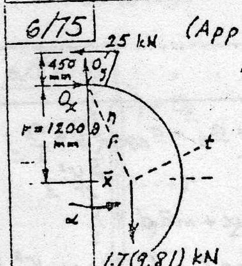
$$\Sigma M_C = m\bar{a}r + \bar{I}\alpha;\ M = (L-x)\rho ar + (L-x)\rho r^2\frac{a}{r}$$

$$= 2(L-x)\rho ar$$

so $\underline{a = M/2(L-x)\rho r}$

$$\Sigma F = m\bar{a};\ P = (L-x)\rho\frac{M}{2(L-x)\rho r} = \underline{\frac{M}{2r}}\ \text{constant}$$

6/75



(Appendix C) For hemispherical shell of mass m, $I_o = \frac{2}{3}mr^2 + mr^2 = \frac{5}{3}mr^2$

so $I_o = \frac{5}{3}1.7(1.2)^2 = 4.08\ \text{t}\cdot\text{m}^2$

$$\Sigma M_o = I_o\alpha;\ 25(0.45) - 1.7(9.81)(0.6) = 4.08\alpha$$

$$\underline{\alpha = 0.305\ \text{rad/s}^2}$$

$$\bar{a}_x = \bar{a}_t\cos\theta = \bar{r}\alpha\cos\theta = r\alpha = 1.2(0.305)$$

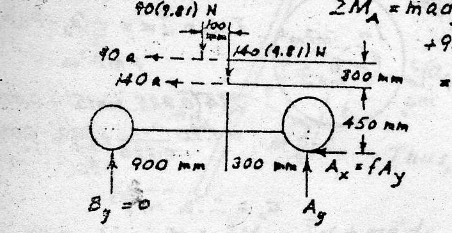
$$= 0.366\ \text{m/s}^2$$

$$(\bar{a}_n = \bar{r}\omega^2 = 0)$$

$\bar{x} = r/2 = 600$ mm

$$\Sigma F_x = m\bar{a}_x;\ O_x - 25 = 1.7(0.366),\ \underline{O_x = 25.6\ \text{kN}}$$

6/76



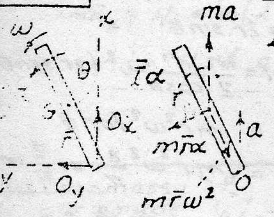
$\Sigma F_y = 0;\ A_y = (90+140)(9.81) = 2260$ N

$\Sigma M_A = m\bar{a}d;\ 140(9.81)(0.3) + 90(9.81)(0.4) = 140a(0.45) + 90a(0.75)$

$a = 5.86$ m/s^2

$\Sigma F_x = ma_x;\ 2260f = (90+140)5.86,\ f = \frac{5.86}{9.81} = \underline{0.598}$

6/77



$\Sigma M_O = 0 = \bar{I}\alpha + m\bar{r}^2\alpha - ma\bar{r}\sin\theta$

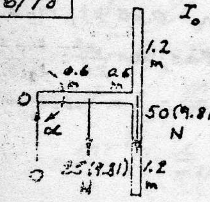
$\bar{I} = m(k_O^2 - \bar{r}^2)$ substitute & get

$\alpha = \frac{a\bar{r}}{k_O^2}\sin\theta$

$\int_0^\omega \dot{\theta}\,d\dot{\theta} = \frac{a\bar{r}}{k_O^2}\int_0^{\pi/2}\sin\theta\,d\theta$

$\frac{\omega^2}{2} = \frac{a\bar{r}}{k_O^2}(-0+1),\ \underline{\omega = \frac{1}{k_O}\sqrt{2a\bar{r}}}$

6/78



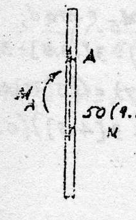
$I_o = \frac{1}{3}25(1.2)^2 + 50\left(\frac{2.4^2}{12} + 1.2^2\right) = 108$ kg·m^2

$\Sigma M_o = I_o\alpha;\ 25(9.81)0.6 + 50(9.81)1.2 = 108\alpha$

$\underline{\alpha = 6.81 \text{ rad/s}^2}$

$\Sigma F_t = m\bar{a}_t;\ (25+50)9.81 - O = [25(0.6) + 50(1.2)]6.81$

$O = 75(9.81) - 75(6.81) = \underline{225 \text{ N}}$



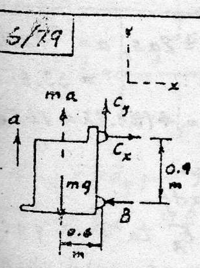
$\Sigma F_t = m\bar{a}_t;\ 50(9.81) - A = 50(6.81)(1.2),\ A = 81.8$ N

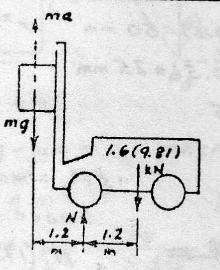
$\Sigma M_o = I_o\alpha;\ [50(9.81) - 81.8]1.2 + M_A = 50\left(\frac{2.4^2}{12} + 1.2^2\right)(6.81)$

$\underline{M_A = 163.5 \text{ N·m}}$

(or merely $\Sigma\bar{M} = \bar{I}\alpha;\ M_A = 50\frac{2.4^2}{12}6.81 = 163.5$ N·m

6/79





$m = 0.900$ t

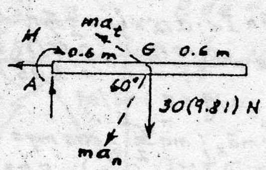
$\Sigma M_N = m\bar{a}d;\ 1.6(9.81)(1.2) - 0.9(9.81)(1.2) = 0.9a(1.2)$

$a = 7.63$ m/s^2

$\Sigma M_C = m\bar{a}d;\ 0.9B - 0.9(9.81)(0.6) = 0.9(7.63)(0.6)$

$\underline{B = 10.46 \text{ kN}}$

6/80



$ma_t = 30(0.6)(10) = 180$ N

$ma_n = 30(0.6)(6^2) = 648$ N

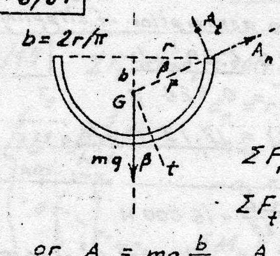
$\Sigma M_A = m\bar{a}d;$

$M + 30(9.81)(0.6) = 648(0.6)\sin 60° - 180(0.6)\cos 60°$

$\underline{M = 106.1 \text{ N·m}}$

6/81

$b = 2r/\pi$



$I_A = \frac{1}{2}(2mr^2 + 2mr^2) = 2mr^2$

$\Sigma M_A = I_A\alpha;\ mgr = 2mr^2\alpha$

$\alpha = \frac{g}{2r}$

$\Sigma F_n = m\bar{a}_n = 0;\ A_n = mg\sin\beta$

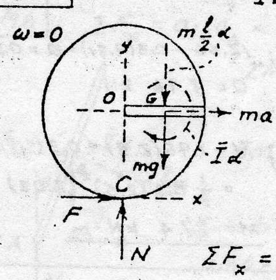
$\Sigma F_t = m\bar{a}_t;\ mg\cos\beta - A_t = m\bar{r}\alpha$

or $A_n = mg\frac{b}{\bar{r}},\ A_t = m\left(g\frac{r}{\bar{r}} - \bar{r}\frac{g}{2r}\right) = mg\left(\frac{r}{\bar{r}} - \frac{\bar{r}}{2r}\right)$

$A = mg\sqrt{\frac{b^2}{\bar{r}^2} + \frac{r^2}{\bar{r}^2} - 1 + \frac{\bar{r}^2}{4r^2}} = mg\frac{\bar{r}}{2r} = \frac{mg}{2}\sqrt{1 + \frac{4}{\pi^2}}$

or $A = \frac{\sqrt{4+\pi^2}}{2\pi}mg = \underline{0.593\ mg}$

6/82



$\bar{I}\alpha = \frac{1}{12}ml^2\frac{a}{l} = \frac{mla}{12}$

$\Sigma M_C = \bar{I}\alpha + m\bar{a}d;$

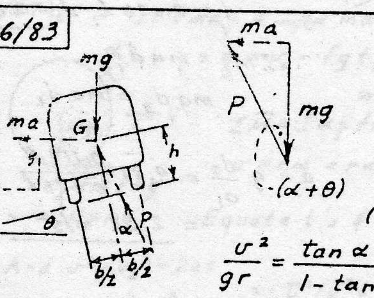
$mg\frac{l}{2} = \frac{mla}{12} + mal + m\frac{l}{2}\left(\frac{a}{l}\right)\frac{l}{2}$

Solve & get $a = \frac{3}{8}g,\ \alpha = \frac{3g}{8l}$

$\Sigma F_x = m\bar{a}_x;\ F = m\frac{3}{8}g = \underline{\frac{3}{8}mg}$

$\Sigma F_y = m\bar{a}_y;\ N - mg = -m\frac{l}{2}\frac{3g}{8l},\ \underline{N = \frac{13}{16}mg}$

6/83



$ma = mv^2/r$

(a) $\alpha = 0;\ ma = mg\tan\theta$

$\underline{\theta = \tan^{-1}\frac{v^2}{gr}}$

(b) $ma = mg\tan(\alpha+\theta)$

$\frac{v^2}{gr} = \frac{\tan\alpha + \tan\theta}{1 - \tan\alpha\tan\theta}$

Slips 1st if $f < \frac{b/2}{h}$ & $f = \tan\alpha$ so

$v^2 = gr\frac{f + \tan\theta}{1 - f\tan\theta}$

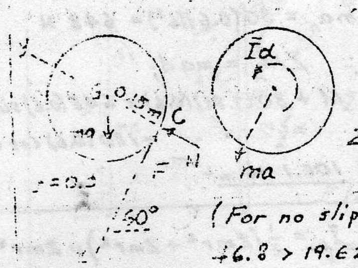
Tips 1st if $f > \frac{b/2}{h}$ & $\tan\alpha = \frac{b}{2h}$

$v^2 = gr\frac{b/2h + \tan\theta}{1 - \frac{b}{2h}\tan\theta}$

6/84 Assume cylinder slips so $F = 0.3N$

$\Sigma F_y = 0;\ N - 0.5\,mg = 0,\ N = 0.5\,mg,\ F = 0.3(0.5\,mg) = 0.15\,mg$

$\Sigma \bar{M} = \bar{I}\alpha;\ 0.15\,mgr = \frac{1}{2}mr^2\alpha$



$\alpha = \frac{0.15(2g)}{r} = \frac{0.15(19.62)}{0.15}$

$= 19.62\ \text{rad/s}^2$

$\Sigma F_x = m\bar{a}_x;\ mg\frac{\sqrt{3}}{2} - 0.15\,mg = ma$

$a = g\left(\frac{\sqrt{3}}{2} - 0.15\right) = 7.02\ \text{m/s}^2$

(For no slipping $\alpha = a/r = 7.02/0.15 = 46.8\ \frac{\text{rad}}{\text{s}^2}$

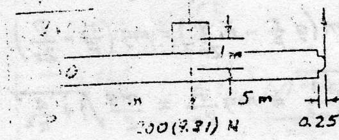
$46.8 > 19.62$ so assumption of slipping OK)

$v^2 = 2ax;\ v = \sqrt{2(7.02)(3)} = \underline{6.49\ \text{m/s}}$

$x = \frac{1}{2}at^2;\ t = \sqrt{2(3)/7.02} = 0.924\ \text{s}$

$\omega = \alpha t;\ \omega = 19.62(0.924) = \underline{18.13\ \text{rad/s}}$

6/85



$T = \frac{4000}{0.25} = 16\,000\ \text{N}$

$I_{O_{Beam}} = \frac{1}{3}1200(10)^2 = 40\,000\ \text{kg·m}^2$

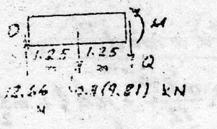
$I_{O_{Motor}} = 240(5^2 + 1^2) = \underline{\ \ 6\,240\ \ }$ "

Total $I_O = \quad 46\,240$ "

$\Sigma M_O = I_O\alpha;\ 16000(10.25) - 1440(9.81)(5) = 46\,240\,\alpha$

$\alpha = 2.02\ \text{rad/s}^2$

$\Sigma F_y = m\bar{a}_y;\ 16000 - 1440(9.81) + P = 1440(5)(2.02);\ P = 12\,660\ \text{N}$



$\Sigma F_y = m\bar{a}_y;\ 12.66 - 0.3(9.81) - Q = 0.3(1.25)2.02$

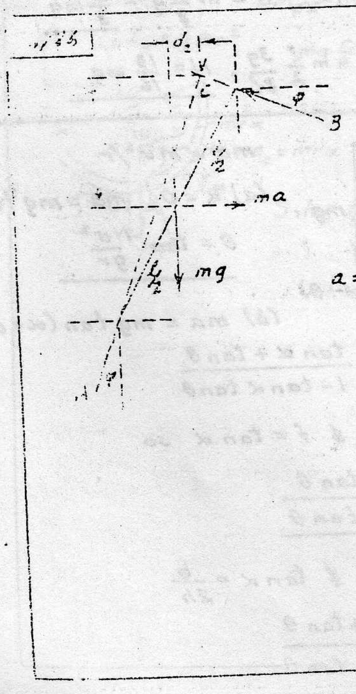
$Q = 8.96\ \text{kN}$

$\Sigma M_O = I_O\alpha;\ M - 8.96(2.5) - 0.3(9.81)(1.25)$

$= \frac{1}{3}0.3(2.5)^2(2.02)$

$\underline{M = 27.4\ \text{kN·m}}$

6/86



$\varphi = \tan^{-1} 0.4$

Moment arms measure

$d_1 = 0.486\,l,\ d_2 = 0.118\,l$

$\Sigma M_C = mad;$

$mgd_2 = mad_1$

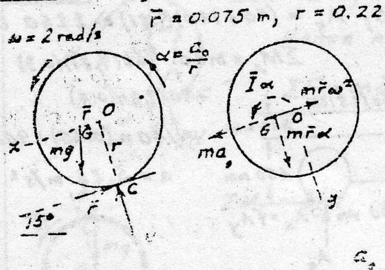
$a = g\frac{d_2}{d_1} = 9.81\frac{0.118\,l}{0.486\,l}$

$= \underline{2.38\ \text{m/s}^2}$

6/87 $mg = 30(9.81) = 294\ \text{N}$

$\bar{r} = 0.075\ \text{m},\ r = 0.225\ \text{m},\ \bar{k} = 0.2\ \text{m},\ \bar{I} = 30(0.2)^2$

$= 1.20\ \text{kg·m}^2$



$\Sigma M_C = \bar{I}\alpha + m(a_0 - \bar{r}\omega^2)r$

$+ m\bar{r}^2\alpha$

$294(0.225\sin 15° + 0.075\cos 15°)$

$= 1.20\frac{a_0}{0.225} + 30(a_0 - 0.075(2^2))$

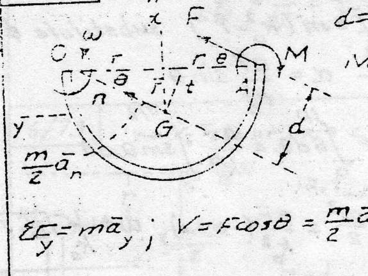
$+ 30(0.075)^2\frac{a_0}{0.225}$

$a_0 = 3.15\ \text{m/s}^2$

$\Sigma F_y = m\bar{a}_y;\ 294\cos 15° - N = 30(0.075)\frac{3.15}{0.225},\ \underline{N = 253\ \text{N}}$

6/88 $\bar{r} = \frac{2r}{\pi};\ \frac{m}{2}\bar{a}_n = \frac{m}{2}\overline{OG}\,\omega^2;\ \Sigma M_A = \frac{m}{2}\bar{a}_n d$

$d = 2r\sin\theta$



$M = \frac{m}{2}\frac{r}{\cos\theta}\omega^2(2r\sin\theta)$

$= mr^2\omega^2\tan\theta$

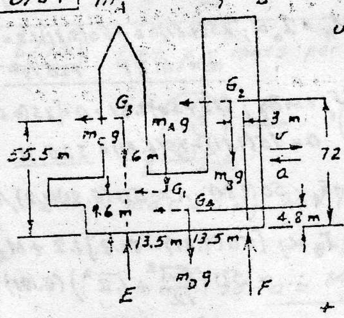
$= mr^2\omega^2\frac{2}{\pi} = \frac{2}{\pi}mr^2\omega^2$

$\Sigma F_y = m\bar{a}_y;\ V = F\cos\theta = \frac{m}{2}\bar{a}_n\cos\theta = \frac{m}{2}\overline{OG}\,\omega^2\frac{r}{\overline{OG}} = \frac{mr\omega^2}{2}$

6/89 $m_A = 3\ \text{kt},\ m_B = 3.3\ \text{kt},\ m_C = 0.23\ \text{kt},\ m_D = 3\ \text{kt}$

$v^2 = 2as;\ a = \frac{v^2}{2s} = \frac{(1.5/3.6)^2}{2(0.1)}$

$= 0.868\ \text{m/s}^2$



$\Sigma M_E = mad;$

$(-3[6] - 3[13.5] - 3.3[24])(9.81)$

$+ F(27) = (3.3[72] + 0.23[55.5]$

$+ 3[9.6] + 3[4.8])(0.868)$

$\underline{F = 59.5\ \text{MN}}$

6/90 $d = 100(0.5) = 50\ \text{mm}$

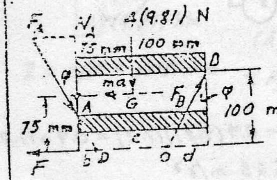
$b = 50(0.5) = 25\ \text{mm}$

$\varphi = \tan^{-1} 0.5$

$v^2 = 2as,\ a = 9^2/2(1.2)$

$= 33.8\ \text{m/s}^2$

$ma = 4(33.8) = 135.0\ \text{N}$



$\Sigma M_O = mad;$

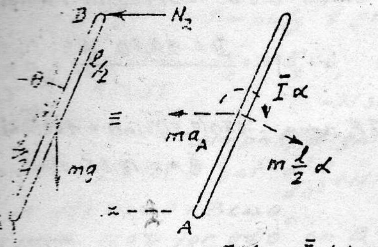
(move F_A to D & get)

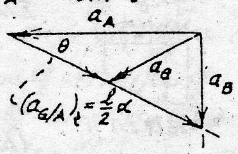
$N_A(175 - 50 - 25) + 4(9.81)(100)$

$= 135.0(75)$

$\underline{N_A = 81.6\ \text{N}}$

6/91 $\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_t$, $\underline{a}_G = \underline{a}_A + (\underline{a}_{G/A})_t$





Thus, $a_A = 2\frac{\ell}{2}\alpha\cos\theta$
$= \ell\alpha\cos\theta$

$\Sigma M_A = \bar{I}\alpha + m\bar{a}d;$

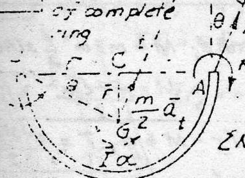
$$mg\frac{\ell}{2}\sin\theta - N_2\ell\cos\theta = \frac{1}{12}m\ell^2\alpha + m\frac{\ell}{2}\alpha\frac{\ell}{2} - m\ell\alpha\cos\theta\frac{\ell}{2}\cos\theta \quad -(1)$$

$\Sigma F_x = m\bar{a}_x;$ $N_2 = m\left(\ell\alpha\cos\theta - \frac{\ell}{2}\alpha\cos\theta\right)$ ------- (2)

Eliminate N_2 & get

$$\underline{\alpha = \frac{3g}{2\ell}\sin\theta}$$

6/92 m = mass of complete ring



$d = r\cos\theta - \bar{r}\sin\theta$

$= r\left[\frac{r}{\overline{OG}} - \frac{2^2}{\pi^2}\frac{r}{\overline{OG}}\right] = \frac{r^2}{\overline{OG}}\left(1 - \frac{4}{\pi^2}\right)$

For complete ring about O

$\Sigma M_O = I_O\alpha;$ $T = 2mr^2\alpha,$ $\alpha = \frac{T}{2mr^2}$

For split ring, $\Sigma M_A = \bar{I}\alpha - \frac{m}{2}\bar{a}_t d$

where $\bar{I} = I_C - \frac{m}{2}\bar{r}^2 = \frac{m}{2}\left(r^2 - \frac{4r^2}{\pi^2}\right) = \frac{mr^2}{2}\left(1 - \frac{4}{\pi^2}\right)$

So $M = \frac{mr^2}{2}\left(1 - \frac{4}{\pi^2}\right)\frac{T}{2mr^2} - \frac{m}{2}\left(\overline{OG}\frac{T}{2mr^2}\right)\frac{r^2}{\overline{OG}}\left(1 - \frac{4}{\pi^2}\right)$

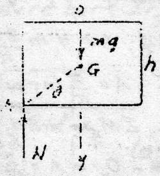
$= \frac{T}{4}\left(1 - \frac{4}{\pi^2}\right) - \frac{T}{4}\left(1 - \frac{4}{\pi^2}\right) = 0,$ $\underline{M = 0}$

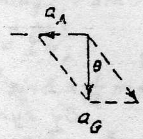
$\Sigma F = m\bar{a}_y;$ $V = F\sin\theta = \frac{m}{2}\bar{a}_t\sin\theta = \frac{m}{2}\left(\overline{OG}\frac{T}{2mr^2}\right)\frac{2r/\pi}{\overline{OG}}$

$$\underline{V = \frac{T}{2\pi r}}$$

6/93

$\bar{I} = m\bar{k}^2$
$= \frac{m}{12}(b^2 + h^2)$





$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A}$

$a_{G/A} = \overline{AG}\,\alpha$

$\Sigma F_y = m\bar{a}_y;$ $mg - N = m\,a_G$ where $a_G = \overline{AG}\,\alpha\cos\theta$
$= b\alpha/2$

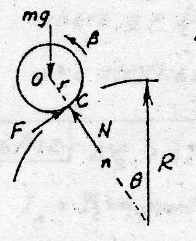
$\Sigma\bar{M} = \bar{I}\alpha;$ $N\frac{b}{2} = m\bar{k}^2\alpha$

Combine & get $\alpha = \frac{2bg}{b^2 + 4\bar{k}^2} = \frac{6bg}{4b^2 + h^2}$

$a_A = \overline{AG}\,\alpha\sin\theta = \overline{AG}\,\alpha\frac{h/2}{\overline{AG}} = \frac{3bhg}{4b^2 + h^2}$

or $\underline{a_A = \frac{3g}{4\frac{b}{h} + \frac{h}{b}}}$

6/94 $\Sigma M_C = I_C\alpha;$ $mgr\sin\theta = \frac{3}{2}mr^2\alpha,$ $\alpha = \frac{2g}{3r}\sin\theta$



$R\,d\theta = r(d\beta - d\theta),$ $\alpha = \ddot{\beta} = \left(\frac{R}{r} + 1\right)\ddot{\theta}$

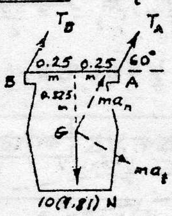
$\int\dot{\theta}\,d\dot{\theta} = \int\ddot{\theta}\,d\theta,$ $\dot{\theta}^2 = 2\int_0^{\theta}\frac{2g}{3(R+r)}\sin\theta\,d\theta$

$\dot{\theta}^2 = \frac{4g}{3(R+r)}(1 - \cos\theta)$

$\Sigma F_n = m\ddot{a}_n;$ $mg\cos\theta - N = m\frac{(R+r)4g}{3(R+r)}(1 - \cos\theta)$

$$\underline{N = \frac{mg}{3}(7\cos\theta - 4)}$$

6/95 $\Sigma F_t = ma_t;$ $10(9.81)(0.5) = 10\,a_t$, $a_t = 4.90$ m/s²



$a_n = r\dot{\theta}^2 = 2.1(2)^2 = 8.4$ m/s²

$ma_n = 10(8.4) = 84$ N

$ma_t = 10(4.90) = 49.0$ N

$\Sigma M_B = mad;$ $T_A(0.50)\frac{\sqrt{3}}{2} - 10(9.81)(0.25)$

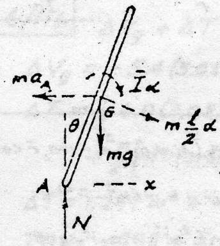
$= 84\left(0.25\frac{\sqrt{3}}{2} + 0.325\frac{1}{2}\right) + 49.0\left(-0.25\frac{1}{2} + 0.325\frac{\sqrt{3}}{2}\right)$

$\underline{T_A = 147.9\text{ N}}$

$\Sigma F_n = ma_n;$ $T_B + 147.9 - 10(9.81)\frac{\sqrt{3}}{2} = 84$

$\underline{T_B = 21.1\text{ N}}$

6/96 $\Sigma F_x = 0;$ $a_A = \frac{\ell}{2}\alpha\cos\theta;$ ℓ = length of bar



$\Sigma\bar{M} = \bar{I}\alpha;$ $N\frac{\ell}{2}\sin\theta = \frac{1}{12}m\ell^2\alpha$

$\alpha = 6N\sin\theta/m\ell$

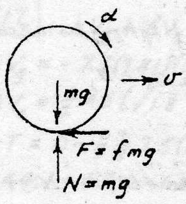
$\Sigma M_A = \bar{I}\alpha + m\bar{a}d;$ $mg\frac{\ell}{2}\sin\theta = \frac{1}{12}m\ell^2\alpha + m\frac{\ell}{2}\alpha\frac{\ell}{2} - ma_A\frac{\ell}{2}\cos\theta$

Substitute a_A

$\sin\theta = \frac{\ell\alpha}{6g} + \frac{\ell\alpha}{2g} - \frac{\ell\alpha\cos^2\theta}{2g} = \frac{\ell\alpha}{6g}(1 + 3\sin^2\theta)$

Substitute α & get $\underline{N = \frac{W}{1 + 3\sin^2\theta}}$

6/97 $\Sigma F = ma;$ $fmg = ma$, $a = fg$ ←



$v = v_0 - fgt,$ $t = \frac{v_0 - v}{fg}$ = time to acquire velocity v

$\Sigma\bar{M} = \bar{I}\alpha;$ $fmgr = m\bar{k}^2\alpha,$ $\alpha = \frac{fgr}{\bar{k}^2}$

$v = r\omega = r\alpha t,$ $t = \frac{v}{r}\frac{\bar{k}^2}{fgr}$

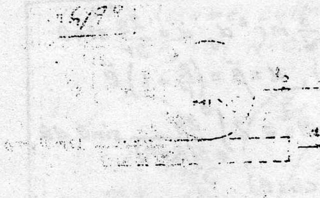
Equate t's & get $v = \frac{v_0 r^2}{r^2 + \bar{k}^2}$

And $v^2 = v_0^2 - 2as$

so $s = \frac{v_0^2 - v^2}{2a} = \underline{\frac{v_0^2}{2fg}\cdot\frac{\bar{k}^2(\bar{k}^2 + 2r^2)}{(\bar{k}^2 + r^2)^2}}$

6/99

$\alpha = (a_C/G)_t/r = \dfrac{a - a_0}{r}$

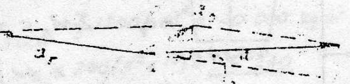


$\Sigma F = ma_0$; $\Sigma M = \bar I\alpha$: $Fr = \frac{1}{2}mr^2\dfrac{a-a_0}{r}$ — solve & get $a_0 = \dots a$

$a_{rel} = a - a_0 = \frac{2}{3}a$ to the left

$d = \frac{1}{2}a_{rel}t^2$, $d = \frac{1}{2}(\frac{2}{3}a)t^2$, $t^2 = \frac{3d}{a}$

$s = \frac{1}{2}at^2 = \frac{1}{2}a\frac{3d}{a} = \frac{3d}{2}$



accel. of truck

$a_0 = a + a$

$\Sigma M = \bar I\alpha$; $F_1 r = \frac{1}{2}mr^2\,\dfrac{a\cos\theta - a_1}{r}$

$\Sigma F_1 = ma_1$; $F_1 = ma_1$

Combine & get $3a_1 = a\cos\theta$

$a_1 = a\cos\theta$; $\alpha = \dfrac{a\cos\theta - \frac{1}{3}a\cos\theta}{r} = \underline{\dfrac{2a\cos\theta}{3r}}$

6/100

$\Sigma M_O = \bar I\alpha + m\bar r^2\alpha - ma_O\bar r\cos\theta$

$mg\bar r\sin\theta = m(k_O^2 - \bar r^2)\alpha + m\bar r^2\alpha - ma_O\bar r\cos\theta$

$\alpha = \dfrac{1}{k_O^2}\left(g\bar r\sin\theta + a_O\bar r\cos\theta\right)$

$\omega d\omega = \alpha d\theta$; $\int \omega d\omega = \dfrac{1}{k_O^2}\int_0^\theta (g\bar r\sin\theta + a\bar r\cos\theta)d\theta$

$\dots) + a\bar r\sin\theta]$ with $a_O = a$

$\omega = \dfrac{\sqrt{2}}{k_O}\sqrt{g\bar r(1-\cos\theta) + a\bar r\sin\theta}$

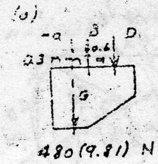
For $\bar r = 0.45$ m, $r = 0.8$ m, $k_O = 0.55$ m, $\theta = 45°$, $a = 10g$:

$v = \dfrac{0.80}{0.55}\sqrt{2}\sqrt{9.81(0.45)(1 - 1/\sqrt{2}) + 10(9.81)(0.45)1/\sqrt{2}} = \underline{11.73 \text{ m/s}}$

16/101

$ma = 480\dfrac{1.5^2}{0.6} = 1800$ N

(a)

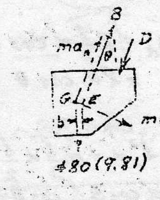


$\Sigma M_B = mad$; $0.6D - 0.3(480)(9.81) = 0.3(1800)$

$\underline{D = 3250 \text{ N}}$

(b) $\Sigma F_t = ma_t$; $480(9.81)\sin\theta = 480(0.6)(-\ddot\theta)$

$\ddot\theta = -16.35\sin\theta$



$\omega d\omega = \ddot\theta d\theta$;

$\int_{\omega_0}^{0}\omega d\omega = \int_0^\theta(-16.35\sin\theta)d\theta$

$\dfrac{\omega^2}{2}\Big|_{1.5/0.6}^{0} = 16.35\cos\theta\Big|_0^\theta$, $\cos\theta - 1 = -\dfrac{2.25}{0.72(16.35)}$

$\theta = 36.01°$

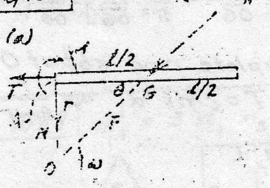
$\Sigma M_G = 0$ with $a_n = r\omega^2 = 0$

$D(0.6\cos 36.01°) = 480(9.81)b$ where $b = \dfrac{0.3 - 0.3\tan\theta}{\sec^2\theta} = 0.0555$

$D = \dfrac{480(9.81)(0.0555)}{0.6(0.8089)} = \underline{520 \text{ N}}$

16/102

(a)



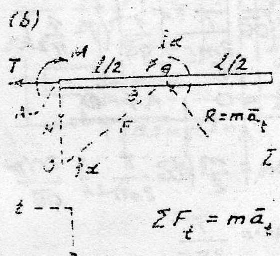
$\Sigma M_A = m\bar a d$; $M = m\bar r\omega^2\dfrac{l}{2}\sin\theta$

$\underline{M = \dfrac{mrl\omega^2}{2}}$

$\Sigma M_O = 0$; $Tr - M = 0$, $\underline{T = \dfrac{ml\omega^2}{2}}$

$\Sigma \bar M = 0$; $N\dfrac{l}{2} - M = 0$, $\underline{N = mr\omega^2}$

(b)



$\Sigma M_A = \bar I\alpha + m\bar a d$;

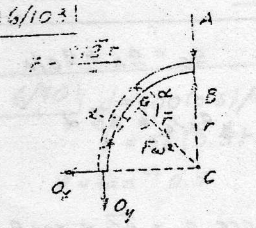
$-M = \frac{1}{12}ml^2\alpha + m\bar r\alpha\dfrac{l}{2}\cos\theta$

$\underline{M = -\dfrac{ml^2\alpha}{3}}$

$\Sigma F_n = m\bar a_n$; $-N = m\bar r\alpha\cos\theta$, $\underline{N = -m\dfrac{l}{2}\alpha}$

$\Sigma F_t = m\bar a_t$; $T = m\bar r\alpha\sin\theta$, $\underline{T = mr\alpha}$

16/103



(a) $\alpha = 0$: $\Sigma M_C = 0$; $O_y = 0$

$\Sigma M_O = m\bar a d$; $Ar = m\dfrac{2\sqrt{2}}{\pi}r\omega^2\dfrac{r}{\sqrt{2}}$

$\underline{A = \dfrac{2}{\pi}mr\omega^2}$

By symm., $\Sigma M_A = m\bar a d$, or $\Sigma F_n = m\bar a_n$

$\underline{O = -O_x = A = \dfrac{2}{\pi}mr\omega^2}$

(b) $\omega = 0$; $\Sigma M_C = I_C\alpha$: $O_y r = mr^2\alpha$, $O_y = mr\alpha$

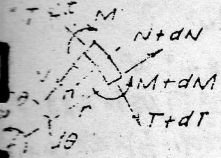
$\Sigma F_x = m\bar a_x$; $O_x = m\dfrac{2\sqrt{2}r}{\pi}\alpha\dfrac{1}{\sqrt{2}} = \dfrac{2mr\alpha}{\pi}$

$\Sigma F_y = m\bar a_y$; $O_y - B = m\dfrac{2\sqrt{2}r}{\pi}\alpha\dfrac{1}{\sqrt{2}}$, $B = mr\alpha - \dfrac{2mr\alpha}{\pi}$

$= mr\alpha(1 - \frac{2}{\pi})$

Thus $O = \sqrt{O_x^2 + O_y^2} = \underline{mr\alpha\sqrt{1 + \frac{4}{\pi^2}}}$

6/104 $\Sigma F_n = m\bar{a}_n$; $2T\sin\frac{d\theta}{2} + dT\sin\frac{d\theta}{2} + N\cos\frac{d\theta}{2} - (N+dN)\cos\frac{d\theta}{2} = \rho r d\theta\, r\omega^2$



Simplify & get $T - \rho r^2\omega^2 = \frac{dN}{d\theta}$ ... (1)

$\Sigma F_t = m\bar{a}_t = 0$; $T\cos\frac{d\theta}{2} - (T+dT)\cos\frac{d\theta}{2} - N\sin\frac{d\theta}{2} - (N+dN)\sin\frac{d\theta}{2} = 0$

Simplify & get $N = -\frac{dT}{d\theta}$ - - - - - - (2)

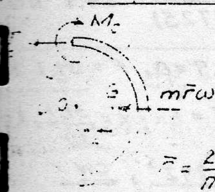
Combine (1) & (2) & get $\frac{d^2N}{d\theta^2} + N = 0.$

Sol. is $N = A\sin\theta + B\cos\theta$;

By symm. $N = 0$ for $\theta = 0$ so $B = 0$ & $N = A\sin\theta$

From (1) $T = \rho r^2\omega^2 + A\cos\theta$. $T = 0$ for $\theta = \pi$ so $A = \rho r^2\omega^2$

Thus $\underline{N = \rho r^2\omega^2\sin\theta}$ & $\underline{T = \rho r^2\omega^2(1+\cos\theta)}$

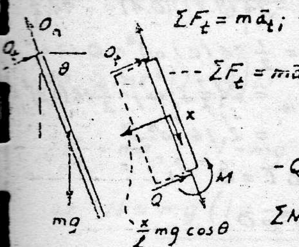


$\Sigma M_c = m\bar{a}d$; $M_c = m\bar{r}\omega^2 r = \rho\pi r\frac{2r}{\pi}\omega^2 r$

$\underline{M_c = 2\rho r^3\omega^2}$

6/105 $\Sigma M_O = I_O\alpha$; $mg\frac{l}{2}\cos\theta = \frac{1}{3}ml^2\alpha$, $\alpha = \frac{3}{2}\frac{g}{l}\cos\theta$

$\Sigma F_t = m\bar{a}_t$; $-O_t + mg\cos\theta = m\frac{l}{2}\frac{3g}{2l}\cos\theta$, $O_t = \frac{mg}{4}\cos\theta$



$\Sigma F_t = m\bar{a}_t$; $\frac{x}{l}mg\cos\theta - \frac{1}{4}mg\cos\theta - Q = \frac{x}{l}m\frac{x}{2}\frac{3}{2}\frac{g}{l}\cos\theta$

$-Q = mg\cos\theta\left[\frac{1}{4} - \frac{x}{l} + \frac{3}{4}\left(\frac{x}{l}\right)^2\right]$

$\Sigma M_O = I_O\alpha$; $\frac{x}{l}mg\cos\theta\frac{x}{2} - M + mg\cos\theta\left[\frac{1}{4} - \frac{x}{l} + \frac{3}{4}\left(\frac{x}{l}\right)^2\right]x = \frac{1}{3}\frac{x}{l}mx^2\frac{3}{2}\frac{g}{l}\cos\theta$

Solve for M & get $\underline{M = mg\frac{x}{4}\left(1 - \frac{x}{l}\right)^2\cos\theta}$

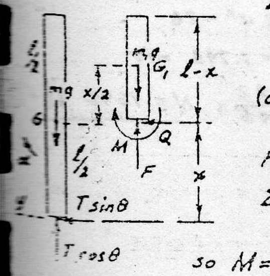
$\frac{dM}{dx} = \frac{mg}{4}\cos\theta\left(1 - \frac{4x}{l} + \frac{3x^2}{l^2}\right) = 0$ gives $\underline{x = l/3}$ (max.), $x = l$ (min.)

$\underline{M_{max} = \frac{mgl}{27}\cos\theta}$

6/106 Entire bar: $\Sigma F_y = m\bar{a}_y$; $T\sin\theta = m\bar{a}_y$, $\bar{a}_y = \frac{T\sin\theta}{m}$

$\Sigma\bar{M} = \bar{I}\alpha$;

$T\frac{l}{2}\sin\theta = \frac{1}{12}ml^2\alpha$, $\alpha = \frac{6T\sin\theta}{ml}$



$(a_{G_1})_y = \bar{a}_y - \frac{x}{2}\alpha = \frac{T\sin\theta}{m}\left(1 - \frac{3x}{l}\right)$ ← +

For section

$\Sigma M_{\substack{cut\\section}} = \bar{I}\alpha - m(a_{G_1})_y d$

so $M = \frac{1}{12}\frac{l-x}{l}m(l-x)^2\frac{6T\sin\theta}{ml} - \frac{l-x}{l}m\frac{T\sin\theta}{m}\left(1 - \frac{3x}{l}\right)\frac{l-x}{2}$

Simplify & get

$\underline{M = \left(\frac{l-x}{l}\right)^2 x\,T\sin\theta}$.

6/111 $\Delta V_g + \Delta T = 0$; $-6mg + \frac{1}{2}m(v^2 - 1.8^2) = 0$

where g = lunar gravity 1.62 m/s²

$v^2 = 2(6)(1.62) + (1.8)^2 = 22.7$, $\underline{v = 4.76 \text{ m/s}}$

6/112 $\Delta V_g + \Delta T = 0$; $-2mg\frac{l}{2}\sin\theta + \frac{1}{2}I_c\omega^2 + \frac{1}{2}I_A\omega^2 = 0$

$I_A = I_c = \frac{1}{3}ml^2$; thus $mgl\sin\theta = \frac{1}{3}ml^2\left(\frac{v}{l}\right)^2$

$\underline{v = \sqrt{3gl\sin\theta}}$

6/113 $\Delta V_g + \Delta T = 0$; $\Delta V_g = -mg(9 - 2.5) = -6.5\,mg$

$\Delta T = \frac{1}{2}mv^2$

$-6.5\,mg + \frac{1}{2}mv^2 = 0$; $v^2 = 13g$, $\underline{v = 11.29 \text{ m/s}}$

6/114 $\Delta V_g + \Delta T = 0$; Let m = mass of AB

so 2m = " " OA

$-2mg(0.9/2) - mg(0.9/2) + \frac{1}{2}\frac{1}{3}2m(0.9)^2\left(\frac{v}{0.9}\right)^2 + \frac{1}{2}mv^2 = 0$

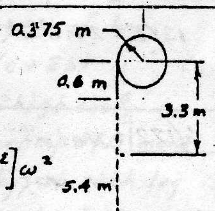
$v^2 = 15.89$, $\underline{v = 3.99 \text{ m/s}}$

6/115 $\Delta V_g + \Delta V_e + \Delta T = 0$

$-30(9.81)(1.2) + \frac{1}{2}3000(2.4 - 1.2\sqrt{2})^2 - \frac{1}{2}\frac{1}{3}30(2.4)^2\omega^2 = 0$

$\omega^2 = 13.47$, $\underline{\omega = 3.67 \text{ rad/s}}$

6/116 $\Delta V_g + \Delta T = 0$



$\Delta V_g = -5.4(3.08)(9.81)(3.3) = -538$ J

$\Delta T = \frac{1}{2}6.0(3.08)(0.375\,\omega)^2 + \frac{1}{2}\left[41(0.30)^2 + (3.08)(18-6)(0.375)^2\right]\omega^2$

$\Delta T = 1.299\,\omega^2 + 4.44\,\omega^2 = 5.74\,\omega^2$

Thus $-538 + 5.74\,\omega^2 = 0$, $\omega^2 = 93.8$,

$\underline{\omega = 9.68 \text{ rad/s}}$

6/117 $\Delta V_g + \Delta V_e + \Delta T = 0.$

$\Delta V_g = -7.5(9.81)(0.6) = 44.1$ J

$\Delta V_e = \frac{1}{2}45\left(\sqrt{1.8^2 + 1.2^2} - 0.6\right)^2 = 55.0$ J

$\Delta T = 0 - \frac{1}{2}\frac{1}{3}7.5(1.2)^2\omega^2 = -1.800\,\omega^2$

$-44.1 + 55.0 - 1.800\,\omega^2 = 0$, $\omega^2 = 6.03$, $\underline{\omega = 2.45 \text{ rad/s}}$

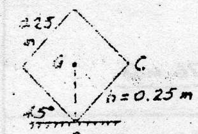
6/118 $\Delta V_g = -mg(2\pi l)\sin\theta$

$\Delta T = \frac{1}{2}I_c\omega^2 = \frac{1}{2}m\left(\frac{l^2}{12} + \left[\frac{3l}{2}\right]^2\right)\omega^2 = \frac{7}{6}ml^2\omega^2 = \frac{7}{6}mv^2$

$U = 0 = \Delta V_g + \Delta T$; $0 = -2\pi l\,mg\sin\theta + \frac{7}{6}mv^2$

$\underline{v = 2\sqrt{\frac{3\pi l g\sin\theta}{7}}}$

6/119 $\Delta V_g + \Delta T = 0; \; -mg\left(\frac{b}{\sqrt{2}} - \frac{b}{2}\right) + \frac{1}{2} I_o \omega^2 = 0$



$\omega = v_c/b, \; I_o = \bar{I} + m\bar{r}^2 = \frac{1}{6} m b^2 + m\left(\frac{b}{\sqrt{2}}\right)^2 = \frac{2}{3} m b^2$

so $\frac{mgb}{2}(\sqrt{2}-1) = \frac{1}{2}\frac{2}{3} m b^2 \frac{v_c^2}{b^2}, \; v_c^2 = \frac{3(\sqrt{2}-1)}{2} g b$

$v_c = \sqrt{\frac{3(\sqrt{2}-1)(9.81)(0.25)}{2}} = \sqrt{1.524} = \underline{1.234 \text{ m/s}}$

6/120 $\Delta V_g + \Delta V_e + \Delta T = 0$

$\Delta V_g = -15(9.81)(0.3) = -44.1 \text{ J}, \; \Delta V_e = \frac{1}{2} 20(10^3) h^2 = 10^4 h^2 \text{ J}$

$\Delta T = \frac{1}{2}\frac{1}{3} 15 (0.6)^2 (4)^2 = 14.40 \text{ J}$

So $-44.1 + 10^4 h^2 + 14.40 = 0, \; h^2 = 29.7(10^{-4}), \; \underline{h = 54.5 \text{ mm}}$

L has no effect on the energy change but does influence the bearing force at O.

6/121 $U = \Delta T + \Delta V_e + \Delta V_g$

$U = M\theta = 880\pi/2 = 1382 \text{ J}$

$\Delta V_e = \frac{1}{2} 450 (1.2)^2 = 324 \text{ J}$

$\Delta V_g = 100(9.81)(0.6) = 589 \text{ J}$

$\Delta T = \frac{1}{2} I_o \omega^2 = \frac{1}{2}\left[\frac{1}{12} 100 (1.2)^2 + 100(0.6)^2\right]\omega^2 = 24.0\,\omega^2$

$1382 = 24.0\,\omega^2 + 324 + 589, \; \omega^2 = 19.54, \; \underline{\omega = 4.42 \text{ rad/s}}$

6/122 Note: the wheel has no motion in initial & final positions, so $\Delta T_{wheel} = 0$.

$U = \Delta V_g + \Delta T, \quad U = Fb\sin\theta$

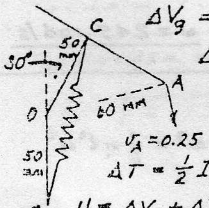
$\Delta V_g = -2 m_o g \frac{b}{2} \sin\theta$

$\Delta T = 2\left(\frac{1}{2} I_C \omega^2\right) = \frac{1}{3} m_o b^2 \omega^2$

Thus $Fb\sin\theta = -m_o g b \sin\theta + \frac{1}{3} m_o b^2 \omega^2$

$$\underline{\omega = \sqrt{\frac{3(F + m_o g)\sin\theta}{m_o b}}}$$

6/123 Let l_o = unstretched length of spring



$\Delta V_g = 0$ (horizontal plane); $U = 0$.

$\Delta V_e = \frac{1}{2} 1400 ([\overline{BC} - l_o]^2 - [0.10 - l_o]^2)$

$\overline{BC}^2 = [2(0.05)\cos 15°]^2 = 0.00933 \text{ m}^2$

$\overline{BC} = 0.0966 \text{ m}$

$\Delta T = \frac{1}{2} I_o \omega^2 = \frac{1}{2} 1.5 (0.055)^2 (0.25/0.060)^2 = 0.0394 \text{ J}$

$U = \Delta V_g + \Delta V_e + \Delta T$

$0 = 0 + 700([0.0966 - l_o]^2 - [0.10 - l_o]^2) + 0.0394$

$0 = 0 + 700(0.00933 - 0.1932 l_o - 0.0100 + 0.20 l_o) + 0.0394$

$4.77 l_o = 0.430, \; l_o = 0.0900 \text{ m} = \underline{90.0 \text{ mm}}$

6/124 $U = \Delta T + \Delta V_g + \Delta V_e; \quad U = M\theta = 12\frac{\pi}{4} = 9.42 \text{ J}$

$\Delta T = \frac{1}{2}\left(\frac{1}{12} 6 (0.50)^2\right)\omega^2 + \frac{1}{2}\left(\frac{1}{3} 3 (0.25)^2\right)\omega^2$

$= 0.0625\,\omega^2 + 0.0312\,\omega^2 = 0.0938\,\omega^2 \text{ J}$

$\Delta V_g = 3(9.81)\left(\left[0.25 + \frac{0.25}{2}\right] - \left[0.25 + \frac{0.25}{2}\right]/\sqrt{2}\right) = 3.23 \text{ J}$

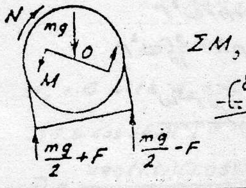
$\Delta V_e = \frac{1}{2} 140 (x_2^2 - x_1^2)$ where $x_2 = 0.15 + 0.50\left(1 - \frac{1}{\sqrt{3}}\right) = 0.296 \text{ m}$

$= 70(\overline{0.296}^2 - \overline{0.15}^2) =$ [illegible]

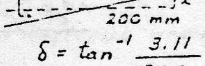
Thus $9.42 = 0.0938\,\omega^2 + 3.23 +$ [illegible]

$\omega^2 = 17.23, \; \underline{\omega = 4.15 \text{ rad/s}}$

6/125 $P = 2\pi MN/60, \; M = \frac{4500(60)}{2\pi(1725)} = 24.9 \text{ N·m}$



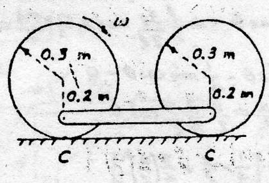
$\Sigma M_o = 0; \; 2F(0.4) - 24.9 = 0, \; F = 31.1 \text{ N}$



$F = kx; \; x = \frac{31.1}{10} = 3.11$

$\delta = \tan^{-1}\frac{3.11}{200} = \underline{0.892°, \text{ CW}}$

6/126 $v_{bar} = (0.3 - 0.2)\omega = 0.1\,\omega$ in lowest position



$\Delta V_g = -24(9.81)(0.4) = -94.2 \text{ J}$

$\Delta T_{bar} = \frac{1}{2} 24 (0.1\,\omega)^2 = 0.12\,\omega^2 \text{ J}$

$\Delta T_{wheels} = 2\left(\frac{1}{2} I_C \omega^2\right) = \frac{3}{2} 16 (0.3)^2 \omega^2 = 2.16\,\omega^2 \text{ J}$

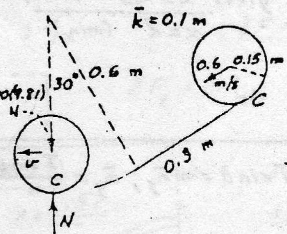
$\Delta V_g + \Delta T = 0$

Thus $-94.2 + 0.12\,\omega^2 + 2.16\,\omega^2 = 0$

$\omega^2 = 41.3, \; \underline{\omega = 6.43 \text{ rad/s}}$

6/127 $0 = \Delta V_g + \Delta T; \; \Delta V_g = -100(9.81)[0.3\sin 30° + 0.45(1 - \cos 30°)]$

$= -206 \text{ J}$



$\Delta T = \frac{1}{2} I_C (\omega_2^2 - \omega_1^2)$

$I_C = 100(\overline{0.1}^2 + \overline{0.15}^2) = 3.25 \text{ kg·m}^2$

$\omega_1 = \frac{0.6}{0.15} = 4 \text{ rad/s}, \; \omega_2 = \frac{v}{0.15}$

so $\Delta T = \frac{1}{2} 3.25 (v^2/0.0225 - 4^2)$

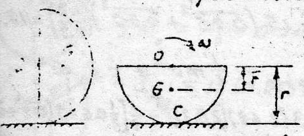
$= 72.2\,v^2 - 26 \text{ J}$

Thus $0 = -206 + 72.2\,v^2 - 26$

$v^2 = 3.22, \; v = 1.793 \text{ m/s}$

$\Sigma F_n = m\bar{a}_n; \; N - 100(9.81) = 100\frac{3.22}{0.45}, \; \underline{N = 1696 \text{ N}}$

6/128 Max. kinetic energy occurs when loss of potential energy is greatest



$\Delta V_g + \Delta T = 0;$

$-mg\dfrac{4r}{3\pi} + \dfrac{1}{2}I_C\,\omega^2 = 0$

Let m = mass; $\bar r = 4r/3\pi$

$I_C = I_G + m(r-\bar r)^2 = I_O - m\bar r^2 + m(r-\bar r)^2$

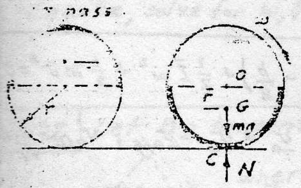
$= \frac{1}{2}mr^2 + mr^2 - 2mr\bar r = mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)$

Thus $mg\dfrac{4r}{3\pi} = \dfrac{1}{2}mr^2\left(\dfrac{3}{2} - \dfrac{8}{3\pi}\right)\omega^2,$

$\underline{\omega = 4\sqrt{\dfrac{g/r}{9\pi - 16}}}$

6/129 $\Delta V_g + \Delta T = 0;\ -mg(2\bar r) + \frac{1}{2}I_C\,\omega^2 = 0$

m = mass



$I_C = I_G + m(r-\bar r)^2$

$= I_O - m\bar r^2 + m(r-\bar r)^2$

$= mr^2 + mr^2 - 2mr\bar r$

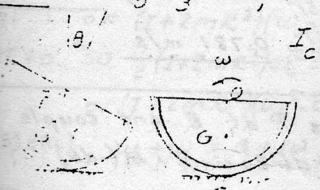
$= 2mr^2\left(1 - \frac{2}{\pi}\right)$

so $\frac{2}{2}mr^2\left(1-\frac{2}{\pi}\right)\omega^2 = 2\frac{2r}{\pi}mg,\quad \omega^2 = \dfrac{4g}{(\pi-2)r}$

$\Sigma F_n = m\bar a_n;\ N - mg = m(\bar r\omega^2),\quad N = mg\left(1 + \dfrac{2r}{\pi g}\dfrac{4g}{r(\pi-2)}\right)$

or $\underline{N = mg\left(1 + \dfrac{8}{\pi(\pi-2)}\right)}$

6/130 $I_O = \frac{2}{3}mr^2,\ \bar r = r/2;\ \bar I = \frac{2}{3}mr^2 - m\left(\frac{r}{2}\right)^2$



$I_C = \frac{2}{3}mr^2 - m\frac{r^2}{4} + m\frac{r^2}{4} = \frac{2}{3}mr^2$

$\Delta T = \frac{1}{2}I_C\omega^2 = \frac{1}{3}mr^2\omega^2$

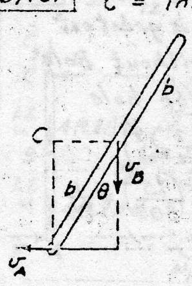
$\Delta V_g = -mg\bar r(1-\cos\theta)$

$= -\dfrac{mgr}{2}(1-\cos\theta)$

$\Delta T + \Delta V_g = 0;\ \frac{1}{3}mr^2\omega^2 = \dfrac{mgr}{2}(1-\cos\theta)$

$\underline{\omega = \sqrt{\dfrac{3g}{2r}(1-\cos\theta)}}$

6/131 C = instantaneous center



$\Delta V_g + \Delta T = 0$

$\Delta V_g = mg(b\cos\theta - b) = -mgb(1-\cos\theta)$

$\Delta T = \frac{1}{2}I_C\,\omega^2$

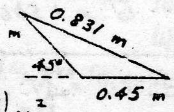
$\omega = \dfrac{v_A}{b\cos\theta},\ I_C = \dfrac{1}{12}m(2b)^2 + m(b\sin\theta)^2$

$= mb^2\left(\frac{1}{3} + \sin^2\theta\right)$

Thus $-mgb(1-\cos\theta) + \dfrac{mb^2}{2}\left(\dfrac{1}{3} + \sin^2\theta\right)\left(\dfrac{v_A}{b\cos\theta}\right)^2 = 0$

Solve for v_A & get $\quad \underline{v_A = \cos\theta\sqrt{\dfrac{6gb(1-\cos\theta)}{1 + 3\sin^2\theta}}}$

6/132 $U = \Delta T + \Delta V_g + \Delta V_e$



$U = M\theta = 18\,\frac{\pi}{4} = 14.14$ J

$\Delta T = \frac{1}{2}15(0.45\,\omega)^2 + 4\,\frac{1}{2}\left(\frac{1}{3}\,3\,\overline{0.45}^2\right)\omega^2$

$= 1.519\,\omega^2 + 0.405\,\omega^2 = 1.924\,\omega^2$ J

$\Delta V_g = -15(9.81)(0.45)(1-0.707) - 4(3)(9.81)\frac{0.45}{2}(1-0.707) = -27.2$ J

$\Delta V_e = 2\,\frac{1}{2}\,700\,(0.831 - 0.45\sqrt{2})^2 = 26.6$ J

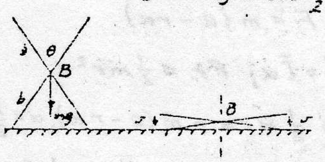
so $14.14 = 1.924\,\omega^2 - 27.2 + 26.6,\quad \omega^2 = 7.67$

$\underline{\omega = 2.77 \text{ rad/s}}$

6/133 Let ρ = mass per unit length of bar

$m = 4\rho c + 4\rho(2b) = 4\rho(c + 2b)$

$U = mgb\cos\frac{\theta}{2} = 4\rho(c+2b)gb\cos\frac{\theta}{2}$



By symmetry A & B move vertically, so each leg is rotating about its bottom end as the upper ends approach the floor.

so $\Delta T_{\text{horiz. members}} = 2\,\frac{1}{2}\rho c v^2 = \rho c v^2$

$\Delta T_{\text{legs}} = 4\,\frac{1}{2}I_{\text{end}}\,\omega^2 = 4\,\frac{1}{2}\,\frac{1}{3}\rho(2b)(2b)^2\left(\frac{v}{2b}\right)^2 = \frac{4}{3}\rho b v^2$

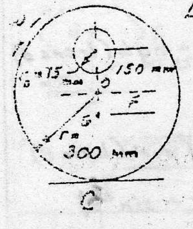
$U = \Delta T;\ 4\rho g b(c+2b)\cos\frac{\theta}{2} = \rho c v^2 + \frac{4}{3}\rho b v^2$

$\underline{v = \sqrt{12gb\,\dfrac{c+2b}{3c+4b}\cos\dfrac{\theta}{2}}}$

6/134 ω is greatest when loss of V_g is greatest

Let m = mass of disk without hole

m_o = mass remored for hole



$I_c = \frac{3}{2} m r^2 - \left(\frac{1}{2} m_o r_o^2 + m_o d^2\right)$

But $m - m_o = 75$ kg, $\frac{m_o}{m} = \left(\frac{75}{300}\right)^2 = \frac{1}{16}$

$16\, m_o - m_o = 75,$

$m_o = 5$ kg, $m = 80$ kg

$75\, \bar{r} = 80(0) - 5(-150)$, $\bar{r} = 10$ mm

Thus $I_c = \frac{3}{2} 80 (0.3)^2 - \left(\frac{1}{2} 5 [0.075]^2 + 5[0.45]^2\right)$

$= 10.80 - 1.027 = 9.77 \text{ kg·m}^2$

$\Delta V_g + \Delta T = 0;\ -75(9.81)(0.010) + \frac{1}{2}\, 9.77\, \omega^2 = 0$

$\omega^2 = 1.506$, $\underline{\omega = 1.227 \text{ rad/s}}$

6/135 $U = \Delta T$

For treads $T = 2\left(T_{hoops} + T_{top\ section}\right)$, $T_{bottom\ section} = 0$

$T_{hoops} = \frac{1}{2} I_c\, \omega^2 = \frac{1}{2}\left[2\pi r \rho (r^2 + r^2) \frac{v^2}{r^2}\right] = 2\pi \rho r v^2$

$T_{top\ section} = \frac{1}{2} m (2v)^2 = \frac{1}{2} \rho b\, 4 v^2 = 2 \rho b v^2$

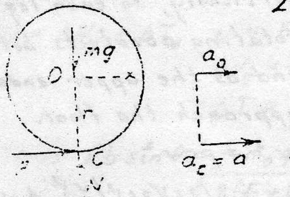
$U = M\theta = M \frac{s}{r}$

Thus $M \frac{s}{r} = 2\left[2\pi \rho r v^2 + 2\rho b v^2\right]$, $\underline{M = 4\rho \frac{r}{s} v^2 (\pi r + b)}$

6/136 $a_o = a_c - r\alpha = a - r\alpha$

$\Sigma F_x = m a_o;\ F = m(a - r\alpha)$

$\Sigma \bar{M} = \bar{I}\alpha;\ Fr = \frac{1}{2} m r^2 \alpha$



so $\frac{1}{2} m r \alpha = m(a - r\alpha)$, $\alpha = \frac{2a}{3r}$

Time t to turn through 2π is

$\theta = \frac{1}{2}\alpha t^2;\ t^2 = 2(2\pi)\frac{3r}{2a} = \frac{6\pi r}{a}$

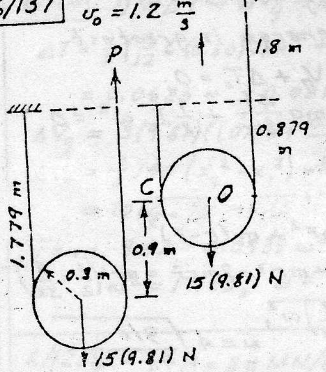
Distance mored by C is

$s = \frac{1}{2} a t^2 = \frac{1}{2} a \frac{6\pi r}{a} = 3\pi r$

Work done by F is $U = Fs = 3\pi r \left(\frac{mr}{2} \frac{2a}{3r}\right)$

$\underline{U = \pi r m a}$

6/137



$\Delta T_{pulley} = \frac{1}{2} I_c \omega^2$

$= \frac{1}{2} 15 \left(\overline{0.25}^2 + \overline{0.30}^2\right)\left(\frac{1.2}{0.3}\right)^2 = 18.30$

$\Delta T_{cable} = \frac{1}{2} m v^2 + \frac{1}{2} I_c \omega^2$

$= \frac{1}{2} 3(2.68)\left[2(1.2)\right]^2 + \frac{1}{2}\left[\frac{1}{2} 0.6\pi 3 (\overline{0.3}^2 + \overline{0.3}^2)\right] \cdot \left(\frac{1.2}{0.3}\right)^2$

$= 23.1 + 4.07 = 27.2$ J

$\Delta V_g = 15(9.81)(0.9) + 3(9.81)(0.3\pi)(0.9)$

$+ 3(9.81)\left[(0.9)(1.779) + 0.9(2.67)\right]$

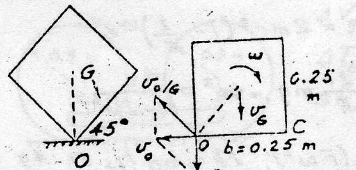
$= 275$ J

$U = \Delta T + \Delta V_g;\ 1.8\, P = 27.2 + 18.30 + 275$

$\underline{P = 178.3 \text{ N}}$

6/138 $\Delta V_g + \Delta T = 0;\ -mg\left(\frac{b}{\sqrt{2}} - \frac{b}{2}\right) + \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2} m \bar{v}^2 = 0$

$v_o = v_G + v_{o/G}$, $v_{o/G} = \frac{b}{\sqrt{2}}\omega$



From diagram

$\left(\frac{b}{\sqrt{2}}\omega\right)\frac{1}{\sqrt{2}} = v_G\ (= \bar{v})$

$b\omega = 2 v_G$

$\bar{I} = \frac{1}{6} m b^2$

Thus $\frac{mgb}{2}(\sqrt{2} - 1) = \frac{1}{2}\frac{1}{6} m b^2 \omega^2 + \frac{1}{2} m \left(\frac{b\omega}{2}\right)^2$

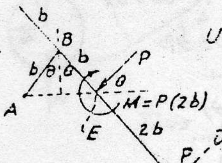
$\omega^2 = \frac{12 g}{5b}(\sqrt{2} - 1) = \frac{12(9.81)}{5(0.25)}(\sqrt{2} - 1) = 39.0$, $\underline{\omega = 6.25 \tfrac{\text{rad}}{\text{s}}}$

$v_o = |v_G| = \frac{b\omega}{2} = \frac{0.25(6.25)}{2} = \underline{0.781 \text{ m/s}}$

6/139 Replace P by force P at E and couple

$M = P(2b);\ dU = P \cos\theta\, d(-2b \sin\theta) + M(-d\theta)$

$= -2Pb \cos^2\theta - 2Pb\, d\theta$



$U = -Pb \int_{\pi/2}^{0} (2\cos^2\theta + 2)\, d\theta$

$= Pb\left[\theta + \frac{\sin 2\theta}{2} + 2\theta\right]_0^{\pi/2} = 3Pb\pi/2$

$\bar{v} = v_E = -\frac{d}{dt}(2b \sin\theta) = -2b\dot{\theta}\cos\theta$

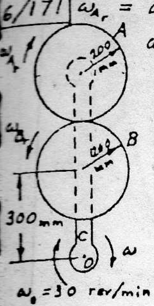
$\omega = -\dot{\theta}$ so $\bar{v} = 2b\omega \cos\theta$

$\Delta T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2} m \bar{v}^2 = \frac{1}{2}\frac{1}{12} m (4b)^2 \omega^2 + \frac{1}{2} m (2b\omega \cos\theta)^2$

$\Delta T_{\theta = 0} = \frac{2}{3} m b^2 \omega^2 + 2 m b^2 \omega^2 = \frac{8}{3} m b^2 \omega^2$

$U = \Delta T;\ \frac{3Pb\pi}{2} = \frac{8}{3} m b^2 \omega^2$, $\underline{\omega = \frac{3}{4}\sqrt{\frac{P\pi}{mb}}}$

6/171 ω_{A_r} = angular velocity of A relative to C
ω_{B_r} = " " " B " " "
$\Big\} = 1720$ rev/min



$\Sigma M_o = \dot{H}_o = 0$ so $H_o =$ const, $H_{o_{before}} = H_{o_{after}}$

$I_{A_o} = 18(0.085)^2 + 18(0.70)^2 = 0.1301 + 8.82$
$= 8.95 \ \text{kg}\cdot\text{m}^2$

$I_{B_o} = 5(0.140)^2 + 5(0.300)^2 = 0.0980 + 0.450$
$= 0.548 \ \text{kg}\cdot\text{m}^2$

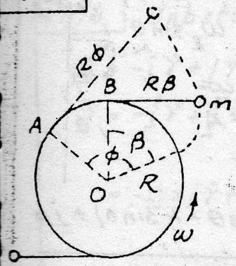
$I_{C_o} = 24(0.450)^2 = 4.86 \ \text{kg}\cdot\text{m}^2$

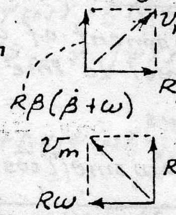
$H_{o_{before}} = (8.95 + 0.548 + 4.86)\frac{30(2\pi)}{60} = 431\frac{2\pi}{60} \ \text{kg}\cdot\text{m}^2/\text{s}$ CW

$H_{o_{after}} = [0.1301(\omega + 1720) + 8.82\,\omega]\frac{2\pi}{60}$
$+ [0.0980(\omega - 1720) + 0.450\,\omega]\frac{2\pi}{60} + 4.86\,\omega\left(\frac{2\pi}{60}\right)$

Equate, solve for ω, & get $\underline{\omega = 26.2 \text{ rev/min}}$
(same direction as ω_o)

6/172 Let B = tangent point; $\underline{v}_m = \underline{v}_B + \underline{v}_{m/B}$
where $v_B = R(\dot{\beta} + \omega)$





Note angular velocity of mB is $\omega + \dot{\beta}$

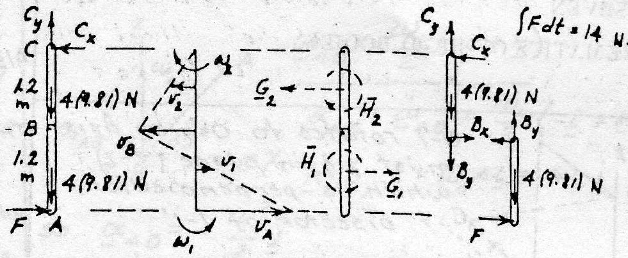
$\Delta H_o = 0$ so $(I + 2mR^2)\omega_o = I\omega + 2mR^2\omega + 2mR^2\beta^2(\omega + \dot{\beta})$
or $(I + 2mR^2)(\omega_o - \omega) = 2mR^2\beta^2(\omega + \dot{\beta})$ ----- (a)

$\Delta T = 0$ so $\frac{1}{2}(I + 2mR^2)\omega_o^2 = \frac{1}{2}I\omega^2 + 2\left(\frac{1}{2}\right)m\left(R^2\omega^2 + R^2\beta^2[\dot{\beta} + \omega]^2\right)$
or $(I + 2mR^2)(\omega_o^2 - \omega^2) = 2mR^2\beta^2(\dot{\beta} + \omega)^2$ ---- (b)

(b) ÷ (a); $\omega_o + \omega = \dot{\beta} + \omega$, $\underline{\dot{\beta} = \omega_o \text{ constant}}$

From (a), for $\beta = \phi$, $\underline{\omega_\phi = \dfrac{I + 2mR^2(1 - \phi^2)}{I + 2mR^2(1 + \phi^2)}\,\omega_o}$

6/173 $\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}4(1.2)^2 = 0.48 \ \text{kg}\cdot\text{m}^2$



$\omega_2 = v_2/0.6$, $\omega_1 = (v_1 + 2v_2)/0.6$, $m = 4$ kg

System; $\int \Sigma M_C\,dt = \Sigma \Delta H_C$; $14(2.4) = 4v_1(1.8) + 0.48\,\omega_1$
$- 4v_2(0.6) - 0.48\,\omega_2$ [a]

AB; $\int \Sigma M_C\,dt = \Delta H_C$; $14(2.4) - \int 1.2 B_x\,dt = 4v_1(1.8) + 0.48\,\omega_1$ [b]

$\int \Sigma F_x\,dt = \Delta G_x$; $14 - \int B_x\,dt = 4v_1$ [c]

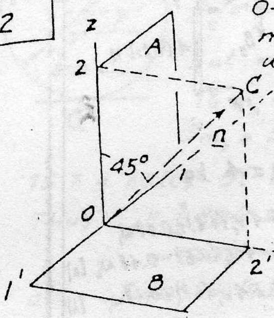
[b], [c], & ω_1 give $2v_1 + v_2 = 10.5$; [a], ω_1, & ω_2 give $5v_1 - v_2 = 21$

Combine & get $v_1 = 4.5$ m/s, $v_2 = 1.5$ m/s
& $\underline{\omega_2 = 2.50 \text{ rad/s}}$

CHAPTER SEVEN

SPACE KINEMATICS OF RIGID BODIES

7/2



0-1 rotates to 0-1' so axis must lie in plane (X-Z) which is perpendicular bisector of 1-1'

0-2 rotates to 0-2' so axis must lie in plane which is perpendicular bisector of line 2-2',

Intersection of planes is line OC defined by

unit vector $\underline{n} = \frac{\underline{i}}{\sqrt{2}} + \frac{\underline{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\underline{i} + \underline{k})$

Angle through which revolution takes place is $\underline{\Delta\theta = \pi}$

7/3 From Eq. (139) inverted

$$\{v_{XYZ}\} = [T_\psi]^{-1}[T_\theta]^{-1}\{v_{xyz}\}$$

where $v_x = 75$ mm/s, $v_y = 100$ mm/s, $v_z = 0$; $\psi = 60°$, $\theta = 30°$

$$\{v_{XYZ}\} = \begin{bmatrix} \cos\psi & -\cos\theta\sin\psi & \sin\theta\sin\psi \\ \sin\psi & \cos\theta\cos\psi & -\sin\theta\cos\psi \\ 0 & \sin\theta & \cos\theta \end{bmatrix}\{v_{xyz}\}$$

$v_X = \frac{1}{2}(75) - \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}(100) + \frac{1}{2}\frac{\sqrt{3}}{2}(0) = \underline{-75/2 \text{ mm/s}}$

$v_Y = \frac{\sqrt{3}}{2}(75) + \frac{\sqrt{3}}{2}\frac{1}{2}(100) - \frac{1}{2}\frac{1}{2}(0) = \underline{125\sqrt{3}/2 \text{ mm/s}}$

$v_Z = 0(75) + \frac{1}{2}(100) + \frac{\sqrt{3}}{2}(0) = \underline{50 \text{ mm/s}}$

7/4 $\{V_{x'y'z'}\} = [T_\phi]\{V_{xyz}\}$ where $[T_\phi] = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Also $\{V_{xyz}\} = [T_\theta][T_\psi]\{V_{XYZ}\}$ from Eq. 139

Thus $\{V_{x'y'z'}\} = [T_\phi][T_\theta][T_\psi]\{V_{XYZ}\}$

where $[T_\phi][T_\theta][T_\psi] = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$

with $a_1 = (\cos\phi\cos\psi - \sin\phi\cos\theta\sin\psi)$

$a_2 = (\cos\phi\sin\psi + \sin\phi\cos\theta\cos\psi)$

$a_3 = \sin\phi\sin\theta$

$a_4 = (-\sin\phi\cos\psi - \cos\phi\cos\theta\sin\psi)$

$a_5 = (-\sin\phi\sin\psi + \cos\phi\cos\theta\cos\psi)$

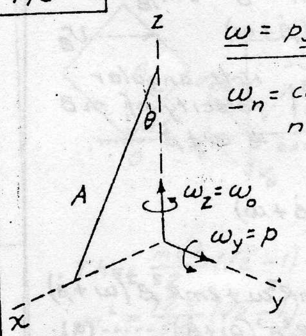
$a_6 = \cos\phi\sin\theta$

$a_7 = \sin\theta\sin\psi$

$a_8 = -\sin\theta\cos\psi$

$a_9 = \cos\theta$

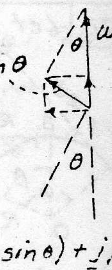
7/5



As a rigid body

$\underline{\omega} = p\underline{j} + \omega_0\underline{k}$

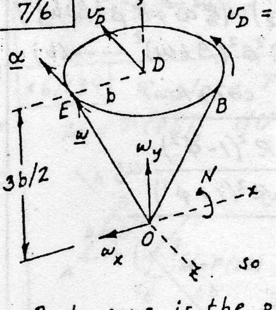
$\underline{\omega}_n$ = component of $\underline{\omega}$ normal to line A.



Thus

$\underline{\omega}_n = \omega_0\sin\theta(\underline{i}\cos\theta + \underline{k}\sin\theta) + \underline{j}p$

7/6



$v_D = \frac{3b}{2}\frac{60(2\pi)}{60} = 3\pi b$; $v_E = 0$ (Gear C fixed)

$\omega_y = \frac{v_D}{b} = 3\pi$ rad/s

$\omega_x = -\frac{60(2\pi)}{60} = -2\pi$ rad/s

$\underline{\omega} = -2\pi\underline{i} + 3\pi\underline{j} = \pi(-2\underline{i} + 3\underline{j})$ rad/s

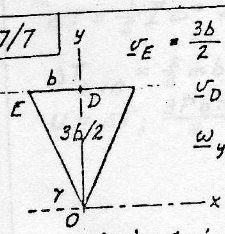
$\underline{\alpha} = \dot{\underline{\omega}}$; $\dot{\underline{i}} = \underline{0}$, $\dot{\underline{j}} = \underline{\omega}\times\underline{j} = -2\pi\underline{k}$

so $\underline{\alpha} = \underline{0} + 3\pi(-2\pi\underline{k}) = \underline{-6\pi^2\underline{k} \text{ rad/s}^2}$

Body cone is the pitch cone of gear B

Space " " " " " " C

7/7



$v_E = \frac{3b}{2}\frac{20(2\pi)}{60}(-\underline{k}) = -b\pi\underline{k}$

$v_D = \frac{3b}{2}\frac{60(2\pi)}{60}(-\underline{k}) = -3\pi b\underline{k}$

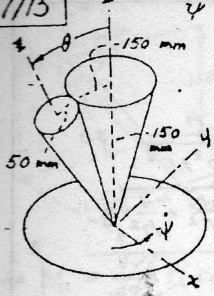
$\omega_y = \frac{v_D - v_E}{b}\underline{j} = \frac{3\pi - \pi}{b}b\underline{j} = 2\pi\underline{j}$

$\omega_x = -\frac{60(2\pi)}{60}\underline{i} = -2\pi\underline{i}$ rad/s

$\underline{\omega} = -2\pi\underline{i} + 2\pi\underline{j} = \underline{2\pi(-\underline{i} + \underline{j}) \text{ rad/s}}$

$\underline{\alpha} = \dot{\underline{\omega}} = \underline{0} + 2\pi\dot{\underline{j}} = 2\pi(-2\pi\underline{k}) = \underline{-4\pi^2\underline{k} \text{ rad/s}^2}$

◂7/15



$\dot{\psi} = \frac{2\pi}{4}$, $\theta = \tan^{-1}\frac{150}{150} + \tan^{-1}\frac{50}{\sqrt{150^2+150^2-50^2}}$

$= 45° + 13.63° = 58.63°$

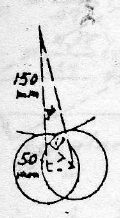
$\underline{\omega} = \underline{i}\dot{\theta} + \underline{j}\dot{\psi}\sin\theta + \underline{k}(\dot{\phi}+\dot{\psi}\cos\theta)$, $\dot{\theta}=0$

$\underline{\alpha} = \underline{\dot{\omega}} = \underline{\dot{j}}\dot{\psi}\sin\theta + \underline{\dot{k}}(\dot{\phi}+\dot{\psi}\cos\theta)$, $\ddot{\psi}=\ddot{\phi}=0$

$\underline{\dot{j}} = \underline{\Omega}\times\underline{j} = \dot{\psi}(\underline{j}\sin\theta + \underline{k}\cos\theta)\times\underline{j}$
$= -\underline{i}\dot{\psi}\cos\theta$

$\underline{\dot{k}} = \underline{\Omega}\times\underline{k} = \dot{\psi}(\underline{j}\sin\theta + \underline{k}\cos\theta)\times\underline{k} = \underline{i}\dot{\psi}\sin\theta$

so $\underline{\alpha} = -\underline{i}\dot{\psi}\cos\theta(\dot{\psi}\sin\theta) + \underline{i}\dot{\psi}\sin\theta(\dot{\phi}+\dot{\psi}\cos\theta)$
$= \underline{i}\dot{\psi}\dot{\phi}\sin\theta$



$\alpha = \frac{\pi}{2}\,3\frac{\pi}{2}\sin 58.63° = \underline{6.32 \text{ rad/s}^2}$

$150\dot{\psi} = 50\dot{\phi}$
$\dot{\phi} = 3\dot{\psi}$

◂7/16 Substitute γ for $90°-\theta$ so Eq. 138 becomes

$\underline{\omega} = -\underline{i}\dot{\gamma} + \underline{j}\dot{\psi}\cos\gamma + \underline{k}(\dot{\phi}+\dot{\psi}\sin\gamma)$

$\underline{\alpha} = -\underline{\dot{i}}\dot{\gamma} + \underline{\dot{j}}\dot{\psi}\cos\gamma - \underline{j}\dot{\psi}\dot{\gamma}\sin\gamma + \underline{\dot{k}}(\dot{\phi}+\dot{\psi}\sin\gamma)$
$+ \underline{k}(0+\dot{\psi}\dot{\gamma}\cos\gamma)$, $\ddot{\psi}=\ddot{\gamma}=\ddot{\phi}=0$

$= -\underline{\Omega}\times\underline{i}\dot{\gamma} + \underline{\Omega}\times\underline{j}\dot{\psi}\cos\gamma - \underline{j}\dot{\psi}\dot{\gamma}\sin\gamma + \underline{\Omega}\times\underline{k}(\dot{\phi}+\dot{\psi}\sin\gamma)$
$+ \underline{k}\dot{\psi}\dot{\gamma}\cos\gamma$

But angular velocity of axes is

$\underline{\Omega} = -\underline{i}\dot{\gamma} + \underline{j}\dot{\psi}\cos\gamma + \underline{k}\dot{\psi}\sin\gamma$

Substitute, simplify, & get

$\underline{\alpha} = \dot{\gamma}(\dot{\phi}-\dot{\psi}\sin\gamma)\underline{j} + \underline{i}\dot{\psi}\dot{\phi}\cos\gamma + \underline{k}\dot{\psi}\dot{\gamma}\cos\gamma$

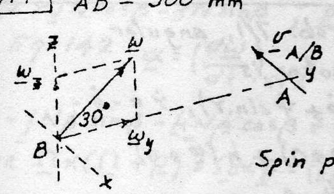
$\dot{\phi} = \frac{100(2\pi)}{60} = 10.47$ rad/s

$\dot{\psi} = \frac{20(2\pi)}{60} = 2.094$ rad/s

$= 4(10.47 - 2.094[\frac{1}{2}])\underline{j} + \underline{i}(2.094)(10.47)\frac{\sqrt{3}}{2}$
$+ (2.094)(4)\frac{\sqrt{3}}{2}\underline{k}$

$= 18.99\,\underline{i} + 37.70\,\underline{j} + 7.26\,\underline{k}$, $\alpha = \sqrt{18.99^2+37.70^2+7.26^2} = \underline{42.8 \text{ rad/s}^2}$

7/19 $\overline{AB} = 300$ mm $\omega = 40$ rad/s

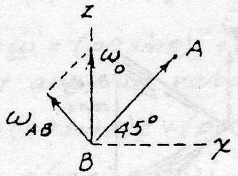


$\underline{v}_{A/B} = \underline{\omega}\times\underline{r}_{A/B} = \underline{\omega}_z\times\underline{r}_{A/B}$
$= 20\underline{k}\times300\underline{j}$

$\underline{v}_{A/B} = -6000\,\underline{i}$ mm/s

Spin $p = \omega_y = \frac{40\sqrt{3}}{2} = \underline{34.6 \text{ rad/s}}$

7/20 $\underline{\omega}_{AB}$ = component of body angular velocity normal to $\underline{r}_{A/B}$



$\underline{\omega}_{AB} = \frac{\omega_0}{2}(-\underline{i}+\underline{k})$

$\underline{\alpha}_{AB} = \underline{\dot{\omega}}_{AB} = \frac{\omega_0}{2}(-\underline{\dot{i}}+\underline{\dot{k}})$

$\underline{\dot{k}} = 0$; $\underline{\dot{i}} = \underline{\omega}_0\times\underline{i} = \omega_0\underline{j}$

so $\underline{\alpha}_{AB} = -\frac{\omega_0^2}{2}\underline{j}$

7/21 Angular velocity of rotor is

$\underline{\omega} = p\underline{k} - q\underline{i}$, $\underline{\alpha} = \underline{\dot{\omega}} = p\underline{\dot{k}} - q\underline{\dot{i}} = \underline{\Omega}\times(p\underline{k}-q\underline{i})$

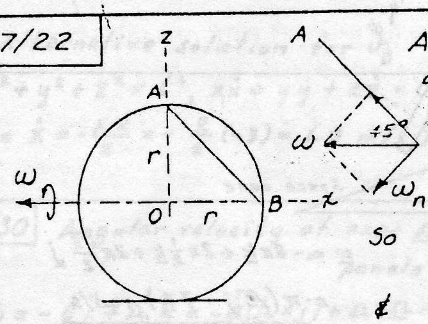
where $\underline{\Omega}$ = angular velocity of axes $= -q\underline{i}$

Thus $\underline{\alpha} = -q\underline{i}\times p\underline{k} = pq\underline{j}$

Or from Eq. 142, $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = 0 + \underline{\Omega}\times\underline{\omega}$

$= -q\underline{i}\times(p\underline{k}-q\underline{i}) = \underline{pq\underline{j}}$

7/22



Angular velocity of line AB is

$\underline{\omega}_n = \frac{\omega}{\sqrt{2}}\left(-\frac{\underline{i}}{\sqrt{2}} - \frac{\underline{k}}{\sqrt{2}}\right)$

where $\omega = v/r$

So $\underline{\omega}_n = -\frac{v}{2r}(\underline{i}+\underline{k})$

& $|\underline{\omega}_n| = \frac{v}{r\sqrt{2}}$

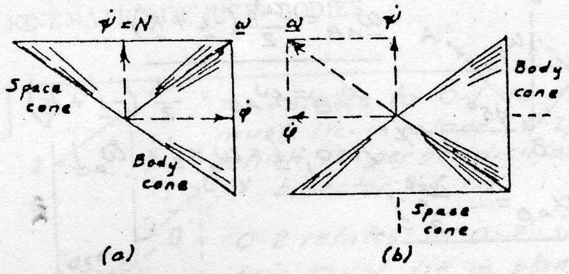
7/23 Let $\underline{\Omega}$ be the angular velocity of axes x'-y'-z' attached to sphere. From Prob. 7/22 angular velocity of line AB is

$\underline{\omega}_n = -\frac{v}{2r}(\underline{i}+\underline{k})$ & $\left(\frac{d\omega}{dt}\right)_{x'-y'-z'} = \underline{0}$. Thus from

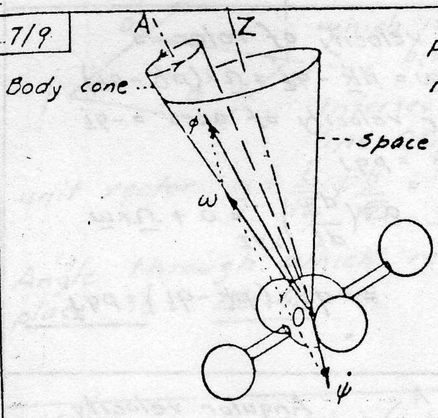
Eq. 142, $\underline{\alpha}_l = \left(\frac{d\underline{\omega}_n}{dt}\right)_{XYZ} = \underline{0} + \underline{\Omega}\times\underline{\omega}_n = -\frac{v}{r}\underline{i}\times\frac{-v}{2r}(\underline{i}+\underline{k})$

$\underline{\alpha}_l = -\frac{v^2}{2r^2}\underline{j}$

7/8 $\dot\psi = N = 10$ rad/s, $\dot\phi = 15$ rad/s

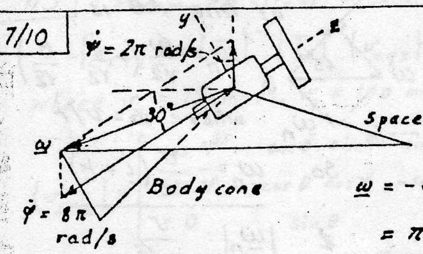


7/9



Precession is retrograde

7/10 $\dot\psi = 2\pi$ rad/s

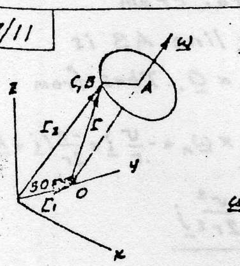


$\underline{\omega} = -8\pi\underline{k} + 2\pi\frac{1}{2}\underline{k} + 2\pi\frac{\sqrt{3}}{2}\underline{j}$

$= \pi(\sqrt{3}\,\underline{j} - 7\underline{k})$ rad/s

Body cone lies inside space cone, so retrograde precession

7/11



$\underline{a} = \frac{d}{dt}(\underline{\omega}\times\underline{r}) = \underline{\omega}\times(\underline{\omega}\times\underline{r})$; $\dot\omega = 0$

where $\underline{r} = \underline{r}_2 - \underline{r}_1$

so $\underline{r} = (75\underline{i} + 75\underline{j} + 150\underline{k}) - 50\underline{j}$

$= 75\underline{i} + 25\underline{j} + 150\underline{k}$

$\underline{\omega}\times\underline{r} = 3(3\underline{i} + 2\underline{j} + 6\underline{k})\times(75\underline{i} + 25\underline{j} + 150\underline{k})$

$= 225(2\underline{i} - \underline{k})$ mm/s

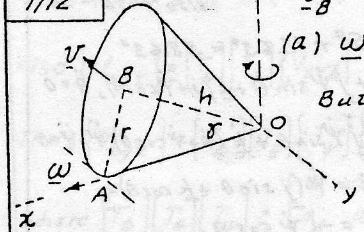
$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = 3(3\underline{i} + 2\underline{j} + 6\underline{k})\times 225(2\underline{i} - \underline{k})$

$= 675(-2\underline{i} + 15\underline{j} - 4\underline{k})$ mm/s²

$|\underline{a}_B| = a_{B_n} = 675\sqrt{2^2 + 15^2 + 4^2} = 675\sqrt{245} = 10\,570\ \frac{mm}{s^2}$

$= 10.57$ m/s²

7/12



$\underline{v}_B = -\underline{j}v$

(a) $\underline{\omega} = \frac{v}{r\cos\gamma}\underline{i}$

But $\cos\gamma = \frac{h}{\sqrt{h^2 + r^2}}$ so

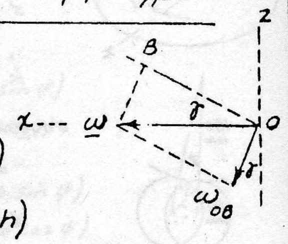
$\underline{\omega} = v\sqrt{\frac{1}{r^2} + \frac{1}{h^2}}\,\underline{i}$

(b) $|\underline{\omega}_{OB}| = \omega\sin\gamma = \frac{v}{r}\tan\gamma$

$= \frac{v}{h}$

so $\underline{\omega}_{OB} = \frac{v}{h}(\underline{i}\sin\gamma - \underline{k}\cos\gamma)$

$= \frac{v}{h\sqrt{r^2 + h^2}}(\underline{i}r - \underline{k}h)$



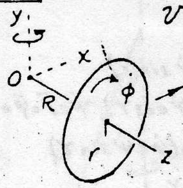
(c) $\underline{\omega}_{OA} = \underline{0}$ since $\underline{v}_A = \underline{0}$

(d) $\underline{\alpha} = \dot{\underline{\omega}} = \frac{v}{r\cos\gamma}\dot{\underline{i}}$ but $\dot{\underline{i}} = -\frac{v}{h\cos\gamma}\underline{j}$ so

$\underline{\alpha} = -\frac{v^2}{h^2}\left(\frac{r}{h} + \frac{h}{r}\right)\underline{j}$

7/13 Axes revolve about y-axis

$v = R\Omega_y = R\frac{2\pi}{T}$; $\dot\phi = \frac{v}{r} = \frac{R}{r}\frac{2\pi}{T}$



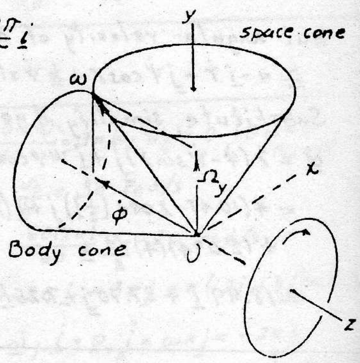
$\underline{\omega}$ = angular velocity of disk

$= \frac{2\pi}{T}\underline{j} - \frac{R}{r}\frac{2\pi}{T}\underline{k} = \frac{2\pi}{T}(\underline{j} - \frac{R}{r}\underline{k})$

$\underline{\alpha} = \dot{\underline{\omega}} = \frac{2\pi}{T}(\dot{\underline{j}} - \frac{R}{r}\dot{\underline{k}}) = \frac{2\pi}{T}(0 - \frac{R}{r}\frac{2\pi}{T}\underline{i})$

where $\dot{\underline{k}} = \underline{\Omega}\times\underline{k} = \Omega_y\underline{j}\times\underline{k} = \frac{2\pi}{T}\underline{i}$

Thus $\underline{\alpha} = -(\frac{2\pi}{T})^2\frac{R}{r}\underline{i}$



7/14 From Sample Prob. 7/1, angular velocity $\underline{\omega}$ of rotor is

$\underline{\omega} = -\underline{i}\dot\gamma + \underline{j}\dot\psi\cos\gamma + \underline{k}(\dot\phi + \dot\psi\sin\gamma)$, $\dot\gamma = -\dot\theta$

$\underline{\alpha} = \dot{\underline{\omega}} = -\underline{i}\ddot\gamma + \underline{0} + \dot{\underline{k}}(\dot\phi + 0) + \underline{k}(\ddot\phi + 0)$

where $\dot{\underline{i}} = \underline{0}$, $\dot\psi = 0$

Thus $\underline{\alpha} = -15\underline{i} + \underline{\Omega}\times\underline{k}\left(\frac{30[2\pi]}{60}\right) + \underline{k}(10 + 0)$

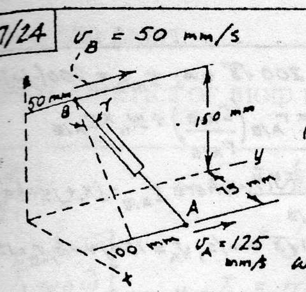
$= -15\underline{i} + (-\dot\gamma\underline{i}\times\underline{k})(\pi) + 10\underline{k}$; $\underline{\Omega} = -\dot\gamma\underline{i}$

$= -15\underline{i} + (-12\pi)(-\underline{j}) + 10\underline{k}$

$= -15\underline{i} + 12\pi\underline{j} + 10\underline{k}$ rad/s²

so $\alpha = \sqrt{15^2 + (12\pi)^2 + 10^2} = 41.8$ rad/s²

7/24 $v_B = 50$ mm/s



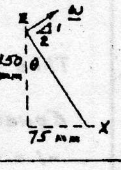
$v_A = v_B + v_{A/B}$, $v_{A/B} = (125-50)\underline{j} = 75\underline{j}$ mm/s

$(v_{A/B})_{\text{normal to }AB} = 75\cos\gamma$

$= 75\sqrt{75^2+150^2}/\sqrt{75^2+150^2+50^2}$

$= 225\sqrt{5}/7$ mm/s

$\omega_n = \frac{225\sqrt{5}}{7}/175 = \frac{9\sqrt{5}}{49}$ rad/s



so $\underline{\omega}_n = \frac{9\sqrt{5}}{49}(\underline{i}\cos\theta + \underline{k}\sin\theta)$

$= \frac{9\sqrt{5}}{49}\left(\frac{2}{\sqrt{5}}\underline{i} + \frac{1}{\sqrt{5}}\underline{k}\right) = \underline{\frac{9}{49}(2\underline{i}+\underline{k})\text{ rad/s}}$

7/25 Angular velocity of axes is $\underline{\Omega} = p\underline{k}$

" " " A " $\underline{\omega} = \underline{\Omega} - \dot\beta\underline{i}$

$\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}} - \ddot\beta\underline{i} - \dot\beta\,\underline{\Omega}\times\underline{i}$

$= \underline{0} - \ddot\beta\underline{i} - \dot\beta p\underline{j}$

(a) before; $\dot\beta\, d\dot\beta = \ddot\beta\, d\beta$, $\ddot\beta = \dot\beta\frac{d\dot\beta}{d\beta} = \left(2\frac{2\pi}{360}\right)\frac{2}{18}$
$= 0.00388$ rad/s²

$\underline{\alpha} = -0.00388\,\underline{i} - \frac{\pi}{900}\underline{j} = \underline{-3.88\underline{i} - 3.49\underline{j}\text{ mrad/s}^2}$

(b) after; $\ddot\beta = 0$, $\underline{\alpha} = \underline{-3.49\underline{j}\text{ mrad/s}^2}$

7/26 Angular velocity $\underline{\omega}$ of A may have a component along x-axis and along z-axis but none about y-axis. Thus $\underline{\omega}\cdot\underline{j} = 0$

But a vector along $\underline{j}$-direction is $\underline{h}\times\underline{n}$ or $\underline{h}\times(\underline{r}\times\underline{h})$; Magnitude is immaterial.

Hence $\underline{\omega}\cdot\underline{h}\times(\underline{r}\times\underline{h}) = 0$ is the provision that $\underline{\omega}$ will not have a component along y.

7/27 $\underline{\Omega}$ = angular velocity of axes x-y-z

$\underline{\omega}$ = " " " simulator $= \underline{\Omega} + \underline{p}$

Let N = angular velocity of frame = 0.2 rad/sec const.

$p = 0.9$ rad/s const., $\dot\beta = 0.15$ rad/s const.

$\underline{\Omega} = \underline{i}\dot\beta + \underline{j}N\cos\beta - \underline{k}N\sin\beta$; $\underline{p} = p\underline{k}$

From Eq. 142 $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} + \underline{\Omega}\times\underline{\omega}$

$\underline{\alpha} = \left(0 - \underline{j}N\dot\beta\sin\beta - \underline{k}N\dot\beta\cos\beta + 0\right) + \underline{\Omega}\times(\underline{\Omega}+\underline{p})$

where $\underline{\Omega}\times(\underline{\Omega}+\underline{p}) = \underline{\Omega}\times\underline{p} = (\underline{i}\dot\beta + \underline{j}N\cos\beta - \underline{k}N\sin\beta)\times p\underline{k}$

$= \underline{i}Np\cos\beta - \underline{j}p\dot\beta$

So $\underline{\alpha}_{\beta=0} = \underline{i}Np - \underline{j}p\dot\beta - \underline{k}N\dot\beta$

$= 0.2(0.9)\underline{i} - 0.9(0.15)\underline{j} - 0.2(0.15)\underline{k}$ rad/s²

$= \underline{0.18\underline{i} - 0.135\underline{j} - 0.03\underline{k}\text{ rad/s}^2}$

7/28 Angular velocity of drum is

$\underline{\omega} = (-p\cos\theta)\underline{i} + \dot\theta\underline{j} + (p\sin\theta + \Omega)\underline{k}$; From Eq. 142

$\underline{\alpha} = \dot{\underline{\omega}} = (p\dot\theta\sin\theta)\underline{i} + (p\dot\theta\cos\theta)\underline{k} + \underline{\Omega}\times\underline{\omega}$

But angular velocity of axes is $\underline{\Omega} = \Omega\underline{k}$, so

$\underline{\alpha} = (p\dot\theta\sin\theta)\underline{i} + (p\dot\theta\cos\theta)\underline{k} - (p\Omega\cos\theta)\underline{j} - \Omega\dot\theta\underline{i}$

$= \underline{\dot\theta(p\sin\theta - \Omega)\underline{i} - (p\Omega\cos\theta)\underline{j} + (p\dot\theta\cos\theta)\underline{k}}$

7/29 $\overline{OB} = \sqrt{7^2-2^2-3^2} = 6$ m; $\underline{v}_A = -3\underline{i}$ m/s

$\underline{v}_B = \underline{v}_A + \underline{\omega}_n\times\underline{r}_{B/A}$, $\underline{r}_{B/A} = -2\underline{i} - 3\underline{j} + 6\underline{k}$ m

so $v_B\underline{k} = -3\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -2 & -3 & 6 \end{vmatrix}$; equate coefficients of $\underline{i}, \underline{j}, \underline{k}$ & get

$1 = 2\omega_y + \omega_z$, $\omega_z = -3\omega_x$, $v_B = -3\omega_x + 2\omega_y$

Now $\underline{\omega}_n$ is ⊥ to AB, so $\underline{\omega}_n\cdot\underline{r}_{B/A} = 0$ which gives $-2\omega_x - 3\omega_y + 6\omega_z = 0$. Eliminate ω's & get $\underline{v}_B = 1.0\underline{k}\,\frac{m}{s}$

Then $\omega_x = -3/49$ rad/s, $\omega_y = 20/49$ rad/s, $\omega_z = 9/49$ rad/s

$\underline{\omega}_n = \frac{1}{49}(-3\underline{i} + 20\underline{j} + 9\underline{k})$ rad/s

(Alternative solution for v_B

$x^2+y^2+z^2 = l^2$, $x\dot x + y\dot y + z\dot z = 0$, $\dot y = 0$, $\dot x = -3$ m/s

So $\dot z = -\frac{x\dot x}{z} = -\frac{2}{6}(-3) = 1.0$ m/s)

7/30 Angular velocity of axes $\underline{\Omega} = \Omega\underline{k}$

" " panels $\underline{\omega} = -\dot\theta\underline{j} + \Omega\underline{k}$

$\dot{\underline{\omega}} = -\dot\theta\dot{\underline{j}} + \Omega\dot{\underline{k}} = -\dot\theta(\underline{\Omega}\times\underline{j}) + \Omega(\underline{\Omega}\times\underline{k}) = \underline{\Omega}\times\underline{\omega} = \Omega\dot\theta\underline{i}$

$= \frac{1}{2}\frac{1}{4}\underline{i} = \underline{\frac{1}{8}\underline{i}\text{ rad/s}^2}$

$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}}\times\underline{r}_{A/O} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/O}) + 2\underline{\Omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_O = \dot{\underline{\Omega}} = \underline{0}$, $\underline{\Omega}\times\underline{r}_{A/O} = \frac{1}{2}\underline{k}\times(-0.3\underline{i} + 2.4\underline{j} + 0.3\sqrt{3}\underline{k})$
$= -1.2\underline{i} - 0.15\underline{j}$ m/s

$\underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/O}) = \frac{1}{2}\underline{k}\times(-1.2\underline{i} - 0.15\underline{j}) = 0.075\underline{i} - 0.6\underline{j}$ m/s²

$2\underline{\Omega}\times\underline{v}_{rel} = 2\frac{1}{2}\underline{k}\times\left(-\frac{0.6}{4}\frac{\sqrt{3}}{2}\underline{i} - \frac{0.6}{4}\frac{1}{2}\underline{k}\right) = -0.075\sqrt{3}\underline{j}$ m/s²

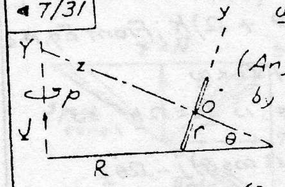
$\underline{a}_{rel} = 0.6\left(\frac{1}{4}\right)^2\left(\frac{1}{2}\underline{i} - \frac{\sqrt{3}}{2}\underline{k}\right) = 0.01875\underline{i} - 0.01875\sqrt{3}\,\underline{k}$ m/s²

$\underline{a}_A = (0.075+0.01875)\underline{i} + (-0.6-0.075\sqrt{3})\underline{j} - 0.01875\sqrt{3}\,\underline{k}$ m/s²

$= \underline{0.0938\,\underline{i} - 0.730\,\underline{j} - 0.0325\,\underline{k}\text{ m/s}^2}$

with $a_A = 0.737$ m/s²

◄ 7/31



$\underline{\omega} = \underline{J}p + \frac{Rp}{r}\underline{k} = (p\cos\theta)\underline{j} + \left(p\sin\theta + \frac{Rp}{r}\right)\underline{k}$

(Angle $d\phi$ measured in x-y-z turned by wheel in time dt is $d\phi = \frac{R(p\,dt)}{r}$ so $\dot{\phi} = \frac{Rp}{r}$)

so $\underline{\omega} = p\left[\underline{j}\cos\theta + \underline{k}\left(\sin\theta + \frac{R}{r}\right)\right]$

Angular velocity of axes is $\underline{\Omega} = \underline{J}p$ so

$\underline{\omega} = \underline{\Omega} + \left(\frac{Rp}{r}\right)\underline{k}$; Now use $\left(\frac{d[\,]}{dt}\right)_{XYZ} = \left(\frac{d[\,]}{dt}\right)_{xyz} + \underline{\Omega}\times[\,]$

noting $\underline{\Omega}$ is constant in XYZ & xyz.

Thus $\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \dot{0} + \underline{\Omega}\times\left[\underline{\Omega} + \frac{Rp}{r}\underline{k}\right]$

$= \left[(p\cos\theta)\underline{j} + (p\sin\theta)\underline{k}\right]\times\frac{Rp}{r}\underline{k}$; $\underline{\alpha} = \frac{Rp^2}{r}\cos\theta\,\underline{i}$

or merely $\underline{\alpha} = \dot{\underline{\omega}} = 0 + \frac{Rp}{r}\dot{\underline{k}} = \frac{Rp}{r}(\underline{\Omega}\times\underline{k})$, etc.

◄ 7/32 Angular vel. of xyz is $\underline{\Omega} = \underline{i}q\sin\theta - \underline{j}\dot{\theta} + \underline{k}q\cos\theta$

$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}}\times\underline{r}_{A/O} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/O}) + 2\underline{\Omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_O = -0.15(8^2)\underline{j} = -9.6\underline{j}$ m/s², $\underline{r}_{A/O} = 0.15\underline{i} - 0.1\underline{k}$ m

$\dot{\underline{\Omega}} = \underline{\Omega}\times\underline{\Omega} + \underline{i}q\dot{\theta}\cos\theta + \underline{0} - \underline{k}q\dot{\theta}\sin\theta$ by Eq. 142

$= q\dot{\theta}(\underline{i}\cos\theta - \underline{k}\sin\theta) = 40(\sqrt{3}\,\underline{i} - \underline{k})$ rad/s²

$\dot{\underline{\Omega}}\times\underline{r}_{A/O} = 40(0.1\sqrt{3} - 0.15)\underline{j} = 0.928\underline{j}$ m/s²

$\underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/O}) = -25.0\underline{i} + 0.928\underline{j} + 15.76\underline{k}$ m/s²

$2\underline{\Omega}\times\underline{v}_{rel} = 2(4\underline{i} - 10\underline{j} + 4\sqrt{3}\,\underline{k})\times(3\underline{j}) = 24(\underline{k} - \sqrt{3}\,\underline{i})$ m/s²

$a_{rel} = 0.1(30)^2\underline{k} = 90\underline{k}$ m/s²

Substitute in $\underline{a}_A$ & get $\begin{cases} \underline{a}_A = -66.5\underline{i} - 7.74\underline{j} + 129.8\underline{k} \text{ m/s}^2 \\ a_A = \sqrt{66.5^2 + 7.74^2 + 129.8^2} \\ = \underline{146.0 \text{ m/s}^2} \end{cases}$

Angular vel. of rotor is

$\underline{\omega} = \underline{\Omega} + \underline{i}p$

$\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}} + \dot{\underline{i}}p,\ \dot{p} = 0$

$= \dot{\underline{\Omega}} + \underline{\Omega}\times\underline{i}p = 40(\sqrt{3}\,\underline{i} - \underline{k}) + (4\underline{i} - 10\underline{j} + 4\sqrt{3}\,\underline{k})\times 30\underline{i}$

$= \underline{20(2\sqrt{3}\,\underline{i} + 6\sqrt{3}\,\underline{j} + 13\underline{k})\text{ rad/s}^2}$

with $\alpha = 20\sqrt{12 + 108 + 169} = \underline{340 \text{ rad/s}^2}$

◄ 7/33 $\overline{AB}^2 = 3(200)^2$, $\overline{AB} = 200\sqrt{3}$ mm; $\underline{r}_{A/B} = 200(-\underline{i} + \underline{j} + \underline{k})$

$\underline{v}_A = \underline{v}_B + \frac{d}{dt}(\underline{r}_{A/B})$; $\frac{d}{dt}(\underline{r}_{A/B}) = \dot{r}_{A/B}\left(\frac{\underline{r}_{A/B}}{r_{A/B}}\right) + \underline{\omega}_n\times\underline{r}_{A/B}$

$\dot{r}_{A/B} = \frac{d}{dt}\sqrt{x_1^2 + y_1^2 + z_1^2} = \frac{x_1\dot{x}_1 + y_1\dot{y}_1}{r_{A/B}}$ where $\underline{r}_{A/B} = \underline{i}x_1 + \underline{j}y_1 + \underline{k}z_1$, $\dot{z}_1 =$

$= \frac{(-200)(-150) + (200)(150)}{200\sqrt{3}} = 100\sqrt{3}$ mm/s; $\underline{v}_A = 150\underline{j}$ mm/s, $\underline{v}_B = 150\underline{i}$

Thus $150\underline{j} = 150\underline{i} + 100\sqrt{3}\,\frac{-\underline{i} + \underline{j} + \underline{k}}{\sqrt{3}} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -200 & 200 & 200 \end{vmatrix}$

Equate coefficients & get

$\left.\begin{aligned} -1 &= 4\omega_y - 4\omega_z \\ -1 &= 4\omega_x + 4\omega_z \\ -1 &= 2\omega_x + 2\omega_y \end{aligned}\right\}$; Also $\underline{\omega}\cdot\underline{r}_{A/B} = 0$ for $\underline{\omega} \perp \underline{r}_{A/B}$ so $-\omega_x + \omega_y + \omega_z = 0$ - - - - - (1) Combine (1) with any two of the $\underline{i}, \underline{j}, \underline{k}$ eqs. & get

$\omega_x = -1/4$ rad/s, $\omega_y = -1/4$ rad/s, $\omega_z = 0$

Hence $\underline{\underline{\omega}_n = -\frac{1}{4}(\underline{i} + \underline{j})\text{ rad/s}}$ with $\omega_n = \sqrt{2}/4$ rad/s

◄ 7/34 Note $\underline{\omega}_A\times\underline{r}_{A/B} \equiv \underline{\omega}\times\underline{r}_{A/B}$ of Prob. 7/33, so the equations

$\begin{Bmatrix} -1 = 4\omega_{A_y} - 4\omega_{A_z} \\ -1 = 4\omega_{A_x} + 4\omega_{A_z} \\ -1 = 2\omega_{A_x} + 2\omega_{A_y} \end{Bmatrix}$

may be written directly from the solution for Prob. 7/33 where $\underline{\omega}_A$ replaces $\underline{\omega}$. Also from results of Prob. 7/26, the angular velocity $\underline{\omega}_A$ obeys $\underline{\omega}_A\cdot\underline{j}\times(\underline{r}_{A/O}\times\underline{j}) = 0$ which gives $\underline{\omega}_A\cdot 200(-\underline{i} + \underline{k}) = 200(-\omega_{A_x} + \omega_{A_z}) = 0$. Combine with two of above three equations & get

$\omega_{A_x} = -1/8$ rad/s, $\omega_{A_y} = -3/8$ rad/s, $\omega_{A_z} = -1/8$ rad/s

so $\underline{\underline{\omega}_A = -\frac{1}{8}(\underline{i} + 3\underline{j} + \underline{k})\text{ rad/s}}$

with $\omega_A = \frac{1}{8}\sqrt{1^2 + 3^2 + 1^2} = \sqrt{11}/8$ rad/s

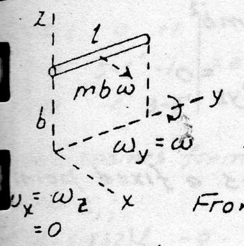
CHAPTER EIGHT

SPACE KINETICS OF RIGID BODIES

8/2

$I_{xx} = \frac{1}{12}ml^2 + m\left(\frac{l^2}{4}+b^2\right)$

$= \frac{1}{3}ml^2 + mb^2$



$I_{yy} = mb^2, \quad I_{zz} = \frac{1}{3}ml^2$

$I_{xy} = I_{xz} = 0, \quad I_{yz} = \frac{1}{2}mbl$

From Eq. 147,

$\underline{H} = \underline{i}(0) + \underline{j}(0 + mb^2\omega - 0) + \underline{k}\left(0 - \frac{1}{2}mbl\omega + 0\right)$

$= \underline{mb^2\omega\left(\underline{j} - \frac{1}{2}\frac{l}{b}\underline{k}\right)}$

8/3 Let $m' = m/3$

$I_{xx} = \frac{1}{3}m'b^2 + \frac{1}{12}m'b^2 + \frac{5}{4}m'b^2 = \frac{5}{3}m'b^2 = \frac{5}{9}mb^2$

$I_{yy} = \frac{1}{3}m'b^2 + m'b^2 + \frac{1}{12}m'b^2 + \frac{5}{4}m'b^2 = \frac{8}{3}m'b^2 = \frac{8}{9}mb^2$

$I_{zz} = \frac{1}{3}m'b^2 + \frac{1}{12}m'b^2 + \frac{5}{4}m'b^2 + 2m'b^2 = \frac{11}{3}m'b^2 = \frac{11}{9}mb^2$

$I_{xy} = \frac{1}{2}m'b^2 + m'b^2 = \frac{3}{2}m'b^2 = \frac{1}{2}mb^2$

$I_{xz} = \frac{1}{2}m'b^2 = \frac{1}{6}mb^2$

$I_{yz} = \frac{1}{2}m'b^2 = \frac{1}{6}mb^2$

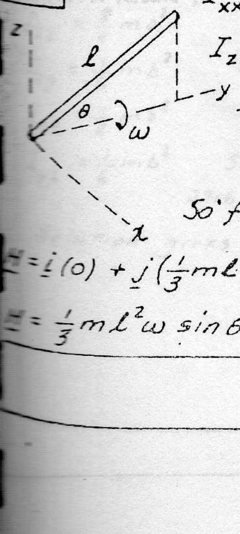
Dir. cosines for OM are $l = m = n = 1/\sqrt{3}$

Substitute in Eq. 151

$I_M = mb^2\frac{1}{3}\left[\frac{5}{9} + \frac{8}{9} + \frac{11}{9} - \frac{2}{2} - \frac{1}{3} - \frac{1}{3}\right] = \underline{\frac{1}{3}mb^2}$

8/4

$I_{xx} = \frac{1}{3}ml^2, \quad I_{yy} = \frac{1}{3}m(l\sin\theta)^2$



$I_{zz} = \frac{1}{3}m(l\cos\theta)^2$

$I_{xy} = I_{xz} = 0; \quad I_{yz} = \int yz\, dm$

$= \frac{1}{3}ml^2\sin\theta\cos\theta$

So from Eq. 147

$\underline{H} = \underline{i}(0) + \underline{j}\left(\frac{1}{3}ml^2\sin^2\theta\ \omega\right) + \underline{k}\left(-\frac{1}{3}ml^2\sin\theta\cos\theta\ \omega\right)$

$\underline{H} = \underline{\frac{1}{3}ml^2\omega\sin\theta\ (\underline{j}\sin\theta - \underline{k}\cos\theta)}$

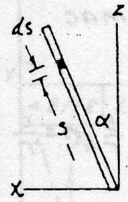
8/5 $I_{xx} = I_{yy} = I_{zz} = 2\left(\frac{2}{5}mr^2 + mb^2\right) + \frac{2}{5}mr^2$

$= m\left(\frac{6}{5}r^2 + 2b^2\right)$

$I_{xy} = I_{yz} = I_{xz} = 0$

Eq. 153 for principal moment of Inertia I is $\left[m\left(\frac{6}{5}r^2 + 2b^2\right) - I\right]^3 = 0$, $I_1 = I_2 = I_3 = m\left(\frac{6}{5}r^2 + 2b^2\right)$ & $q_1 = q_2 = q_3 = 1/\sqrt{m\left(\frac{6}{5}r^2 + 2b^2\right)}$, so ellipsoid of inertia is a sphere of radius $q = q_1 = q_2 = q_3$ & I is the same for all axes through its center O.

8/6 $H_x = I_{xx}\omega_x - I_{xz}\omega_z, \quad \omega_x = \omega, \quad \omega_z = p, \quad I_{xy} = 0$



$I_{xx} = \frac{1}{3}mh^2$

$I_{xz} = \int_0^l (s\sin\alpha)(s\cos\alpha)\rho\, ds = \frac{\rho\sin\alpha\cos\alpha}{3}l^3$

$= \frac{1}{3}mrh$

$\underline{H_x = \frac{mh}{3}(h\omega - rp)}$

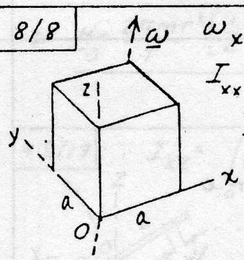
8/7 $H_z = -I_{xz}\omega_x + I_{zz}\omega_z, \quad I_{yz} = 0, \quad \omega_x = \omega, \quad \omega_z = p$

From Prob. 8/6, $I_{xz} = \frac{1}{3}mrh$; Also $I_{zz} = \frac{1}{3}mr^2$

$H_z = \frac{mr}{3}(rp - h\omega)$

8/8

$\omega_x = \omega_y = \omega_z = \omega/\sqrt{3}$



$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}ma^2$

$I_{xy} = I_{xz} = I_{yz} = \frac{1}{4}ma^2$

$H_x = H_y = H_z = \frac{2}{3}ma^2\frac{\omega}{\sqrt{3}} - 2\frac{1}{4}ma^2\frac{\omega}{\sqrt{3}}$

$= \frac{ma^2\omega}{6\sqrt{3}}$

$\underline{\underline{H} = \frac{ma^2\omega}{6\sqrt{3}}(\underline{i} + \underline{j} + \underline{k}), \quad H = \frac{ma^2\omega}{6}}$

8/9 $\omega_x = \omega_y = 0, \quad \omega_z = \omega$

$I_{xz} = 0, \quad I_{yz} = 0 + m\left(\frac{4r}{3\pi}\right)\left(c + \frac{b}{2}\right); \quad I_{zz} = \frac{1}{2}mr^2$

So $\underline{H} = -I_{yz}\omega_z\underline{j} + I_{zz}\omega_z\underline{k}$

$\underline{\underline{H} = mr\omega\left[\frac{-2(2c+b)}{3\pi}\underline{j} + \frac{r}{2}\underline{k}\right]}$

8/10 Let Ω = angular velocity of x-y-z about z_0

Axes; $\Omega_x = -\Omega \sin\theta$, $\Omega_y = \dot\theta = 0$, $\Omega_z = \Omega\cos\theta$, $\Omega = 2\pi f$

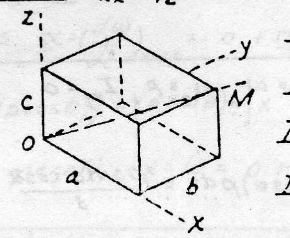
Body; $\omega_x = -\Omega \sin\theta$, $\omega_y = 0$, $\omega_z = \Omega\cos\theta + p$

$H_x = I_{xx}\omega_x = mk'^2(-2\pi f \sin\theta)$, $H_y = I_{yy}\omega_y = 0$,

$H_z = I_{zz}\omega_z = mk^2(2\pi f\cos\theta + p)$

$\underline{H} = 2\pi m f(-k'^2 \sin\theta\, \underline{i} + k^2\cos\theta\, \underline{k}) + mk^2 p\, \underline{k}$

8/11 $I_{xx} = \frac{1}{12}m(b^2+c^2) + m\left(\frac{b^2}{4}+\frac{c^2}{4}\right) = \frac{1}{3}m(b^2+c^2)$



$I_{yy} = \frac{1}{3}m(a^2+c^2)$

$I_{zz} = \frac{1}{3}m(a^2+b^2)$

$I_{xy} = \frac{1}{4}mab$, $I_{xz} = \frac{1}{4}mac$

$I_{yz} = \frac{1}{4}mbc$

Direction cosines of OA are $l = \frac{a}{r}$, $m = \frac{b}{r}$, $n = \frac{c}{r}$ where $r^2 = a^2+b^2+c^2$.

From Eq. 151,

$$I_M = \frac{1}{3}m\frac{1}{r^2}\left([b^2+c^2]a^2 + [a^2+c^2]b^2 + [a^2+b^2]c^2\right)$$
$$-2\frac{1}{4}m\frac{1}{r^2}\left(ab[ab] + ac[ac] + bc[bc]\right)$$

$$I_M = \frac{1}{6}m\,\frac{a^2b^2+a^2c^2+b^2c^2}{a^2+b^2+c^2}$$

8/12 $H_{xyz} = \bar{H} + \bar{r}\times m\bar{v}$

$\bar{H} = \underline{i}\,\bar{I}_{xx}\omega_x + \underline{j}\,\bar{I}_{yy}\omega_y + \underline{k}\,\bar{I}_{zz}\omega_z$

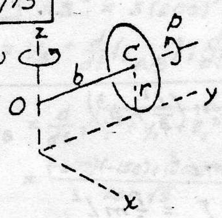
$= \underline{i}\left(\frac{1}{12}mb^2 + \frac{1}{4}mr^2\right)\omega + \underline{j}\,\frac{1}{2}mr^2 p + 0$

$\bar{r} = h\underline{k} - \frac{b}{2}\underline{j}$, $\bar{v} = -h\omega\underline{j} - \frac{b}{2}\omega\underline{k}$

$\bar{r}\times\bar{v} = \underline{i}\left(h^2 + \frac{b^2}{4}\right)\omega$

so
$$\begin{cases} H_x = \left(\frac{b^2}{3} + \frac{r^2}{4} + h^2\right)m\omega \\ H_y = \frac{1}{2}mr^2 p \\ H_z = 0 \end{cases}$$

8/13



$\underline{v}_C = -b\omega\,\underline{i}$

$I_{O_{xx}} = \frac{1}{4}mr^2 + mb^2$

$I_{O_{yy}} = \frac{1}{2}mr^2$

$I_{O_{zz}} = \frac{1}{4}mr^2 + mb^2$

$I_{O_{xy}} = I_{O_{xz}} = I_{O_{yz}} = 0$

$\omega_x = 0$, $\omega_y = -\frac{b\omega}{r}$, $\omega_z = \omega$

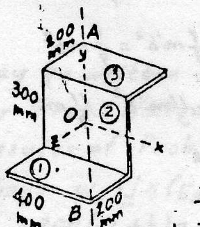
Eq. 147 holds for point O as a fixed point on axis of disk, so

$\underline{H}_O = (0)\underline{i} + \frac{1}{2}mr^2\left(\frac{-b\omega}{r}\right)\underline{j} + \left(\frac{1}{4}mr^2 + mb^2\right)\omega\underline{k}$

$= mr^2\omega\left(-\frac{1}{2}\frac{b}{r}\underline{j} + \left[\frac{1}{4} + \frac{b^2}{r^2}\right]\underline{k}\right)$

8/14 $m_3 = m_1 = 9.43$ kg

$m_2 = 14.14$ kg



(kg·

	1	2	3	Tota
I_{xx}	0.338	0.1061	0.338	0.7
I_{yy}	0.251	0.1886	0.251	0.69
I_{zz}	0.338	0.295	0.338	0.97
I_{xy}	0	0	0	0
I_{xz}	0	0	0	
I_{yz}	-0.141	0	-0.141	-0.2

$\overline{OB} = \sqrt{\overline{0.2}^2 + \overline{0.15}^2 + \overline{0.2}^2} = 0.320$ m

$l = 0.2/0.320 = 0.625$, $m = -0.15/0.320 = -0.469$, $n = l = 0.6$

From Eq. 151,

$I = 0.782(0.625)^2 + 0.691(-0.469)^2 + 0.970(0.625)^2$
$-2(-0.283)(-0.469)(0.62$

$= 0.305 + 0.1518 + 0.379 - 0.1656$

$I = 0.670$ kg·m²

8/15

$I_{xx} = m(b^2+2b^2+b^2) = 4b^2m$ $I_{xy} = 0$

$I_{yy} = m(b^2+b^2+b^2+2b^2) = 5b^2m$ $I_{xz} = -mb^2$

$I_{zz} = m(b^2+b^2+b^2) = 3b^2m$ $I_{yz} = mb^2$

Eq. 153 with $I' = I/mb^2$ gives

$$\begin{vmatrix} 4-I' & 0 & +1 \\ 0 & 5-I' & -1 \\ +1 & -1 & 3-I' \end{vmatrix} = 0$$

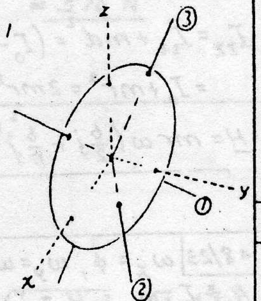
Expansion yields $I'^3 - 12I'^2 + 45I' - 51 = 0$

Solution gives $\begin{cases} I_1 = 5.532\ mb^2 \text{ max} \\ I_2 = 4.347\ mb^2 \\ I_3 = 2.121\ mb^2 \text{ min} \end{cases}$

Dir. cosines from Eq. 154

For I_1;

$$\left.\begin{aligned} &(4-5.532)l_1 - 0 + n_1 = 0 \\ &0 + (5-5.532)m_1 - n_1 = 0 \\ &l_1 - m_1 + (3-5.532)n_1 = 0 \end{aligned}\right\} \& \ l_1^2+m_1^2+n_1^2 = 1 \text{ give } l_1 = -0.293,\ m_1 = 0.844,\ n_1 = -0.449$$



Similarly for I_2 & I_3;

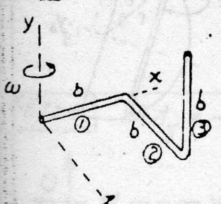
$\begin{cases} l_2 = 0.844 \\ m_2 = 0.449 \\ n_2 = 0.293 \end{cases}$ $\begin{cases} l_3 = -0.449 \\ m_3 = 0.293 \\ n_3 = 0.844 \end{cases}$

$\omega_x = \omega_y = 0$, $\omega_z = \omega$

$H_x = -I_{xz}\omega_z$; $H_y = -I_{yz}\omega_z$; $H_z = I_{zz}\omega_z$

$\underline{H} = mb^2\omega(\underline{i} - \underline{j} + 3\underline{k})$

8/16 $m_0 = m/3$



	①	②	③
I_{xx}	0	$\frac{1}{3}m_0b^2$	$\frac{4}{3}m_0b^2$
I_{yy}	$\frac{1}{3}m_0b^2$	$\frac{4}{3}m_0b^2$	$2m_0b^2$
I_{zz}	$\frac{1}{3}m_0b^2$	m_0b^2	$\frac{4}{3}m_0b^2$
I_{xy}	0	0	$\frac{1}{2}m_0b^2$
I_{xz}	0	$\frac{1}{2}m_0b^2$	m_0b^2
I_{yz}	0	0	$\frac{1}{2}m_0b^2$

For system

$I_{xx} = \frac{5}{9}mb^2$

$I_{yy} = \frac{11}{9}mb^2$

$I_{zz} = \frac{8}{9}mb^2$

$I_{xy} = \frac{1}{6}mb^2$

$I_{xz} = \frac{1}{2}mb^2$

$I_{yz} = \frac{1}{6}mb^2$

Eq. 153 gives with $I' = I/mb^2$

$$\begin{vmatrix} \frac{5}{9}-I' & -\frac{1}{6} & -\frac{1}{2} \\ -\frac{1}{6} & \frac{11}{9}-I' & -\frac{1}{6} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{8}{9}-I' \end{vmatrix} = 0$$

Simplifies to

$2916I^3 - 7776I^2 + 5647I - 671 = 0$

Solution gives $I_1 = 0.145\ mb^2$ min

$I_2 = 1.299\ mb^2$ max

$I_3 = 1.223\ mb^2$ intermediate

8/17 From Prob. 8/16 $I_3 = 1.223\ mb^2$

Dir. cosines for I_3 satisfy Eq. 154. Thus

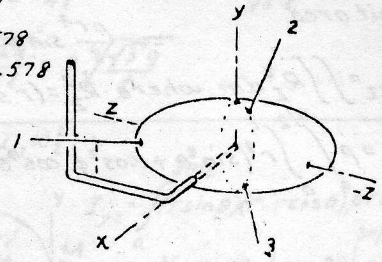
$$\left.\begin{aligned} &(5/9 - 1.223)l_3 - \tfrac{1}{6}m_3 - \tfrac{1}{2}n_3 = 0 \\ &-\tfrac{1}{6}l_3 + (\tfrac{11}{9} - 1.223)m_3 - \tfrac{1}{6}n_3 = 0 \\ &-\tfrac{1}{2}l_3 - \tfrac{1}{6}m_3 + (\tfrac{8}{9} - 1.223)n_3 = 0 \end{aligned}\right\} \& \ l_3^2+m_3^2+n_3^2 = 1$$

sol. gives $l_3 = 1/\sqrt{3}$, $m_3 = n_3 = -1/\sqrt{3}$

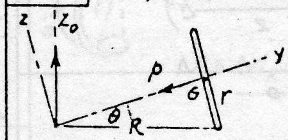
Similarly for I_1 & I_2 which give

$l_1 = 0.790$, $m_1 = 0.212$, $n_1 = 0.578$

$l_2 = 0.212$, $m_2 = 0.790$, $n_2 = -0.578$



8/18 Let p = spin about y relative to x-y-z



$p = \frac{R\omega}{r}$; Let ω_y' = total ang. vel. about y

$\omega_x = 0$, $\omega_y' = \omega\sin\theta - p = (\frac{r}{R} - \frac{R}{r})\omega$

$\omega_z = \omega\cos\theta = \omega\sqrt{1 - r^2/R^2}$

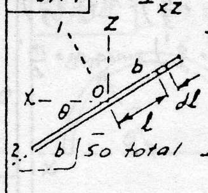
$H_y = I_{yy}\omega_y'$; $H_z = I_{zz}\omega_z$; $I_{xy} = I_{xz} = I_{yz} = 0$

$I_{zz} = \bar{I}_{zz} + md^2 = \frac{1}{4}mr^2 + m(R^2 - r^2) = m(R^2 - \frac{3}{4}r^2)$

So $\underline{H}_0 = \frac{1}{2}mr^2(\frac{r}{R} - \frac{R}{r})\omega\underline{j} + m(R^2 - \frac{3}{4}r^2)\omega\sqrt{1 - \frac{r^2}{R^2}}\,\underline{k}$

or $\underline{H}_0 = \frac{2\pi mr^2}{\tau}\left[\frac{1}{2}(\frac{r}{R} - \frac{R}{r})\underline{j} + (\frac{R^2}{r^2} - \frac{3}{4})\sqrt{1 - \frac{r^2}{R^2}}\,\underline{k}\right]$ where $\omega = 2\pi/\tau$

8/19 $I_{xz} = \int_{-b}^{0}(-l\cos\theta)(l\sin\theta)2c\rho\,dl = -\frac{2}{3}\rho cb^3\sin 2\theta$



$I_{yz} = I_{xy} = 0$ by symmetry

$I_{zz} = \bar{I}_{zz} + md^2$ (per panel)

So total $I_{zz} = 2\left\{\frac{2cb\rho}{12}(c^2 + [2b\cos\theta]^2) + 2cb\rho(a + \frac{c}{2})^2\right\}$

$\underline{\omega} = \omega\underline{k}$ $= 4cb\rho\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}$

$H_0 = -I_{xz}\omega_z\underline{i} + I_{zz}\omega_z\underline{k}$; $m = 4cb\rho$ (total)

$H_0 = \frac{m}{3}b^2\omega\sin\theta\cos\theta\,\underline{i} + m\omega(\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac)\underline{k}$

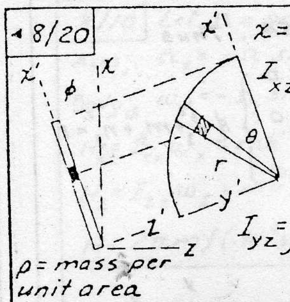
By symm. principal axes are 0-1, 0-2, 0-y

& $I_1 = 4\rho bc\{(b^2+c^2)/3 + a^2 + ac\}$ (max.)

$I_2 = 4\rho bc\{c^2/3 + a^2 + ac\}$ (intermediate)

$I_3 = I_{yy} = \frac{4}{3}\rho b^3c$ (min.)

◂8/20 $x = x'\cos\phi = r\cos\theta\cos\phi,\ z = -r\cos\theta\sin\phi$



ρ = mass per unit area

$$I_{xz} = \int xz\,dA\,\rho = \rho\int_0^{\pi/2}\int_0^r -\frac{r^2}{2}\sin 2\phi\cos^2\theta\, r\,dr\,d\theta$$

$$= -\frac{\rho\pi r^4}{32}\sin 2\phi = -\frac{mr^2}{8}\sin 2\phi$$

$$I_{yz} = \int yz\,dA\,\rho = \rho\int_0^{\pi/2}\int_0^r -\frac{r^2}{2}\sin\phi\sin 2\theta\, r\,dr\,d\theta$$

$$= -\frac{\rho r^4}{8}\sin\phi = -\frac{mr^2}{2\pi}\sin\phi$$

$I_{zz} = \iint R_z^2\,dm$ where $R_z^2 = (r\sin\theta)^2 + (r\cos\phi\cos\theta)^2$

$$= \rho\int_0^{\pi/2}\int_0^r r^2(\sin^2\theta + \cos^2\phi\cos^2\theta)\,r\,dr\,d\theta = \frac{\rho\pi r^4}{16}(1+\cos^2\phi)$$

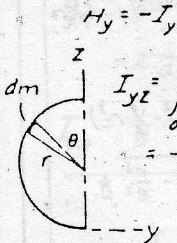
$$= \frac{mr^2}{4}(1+\cos^2\phi)$$

$$\underline{H} = -I_{xz}\omega_z\underline{i} - I_{yz}\omega_z\underline{j} + I_{zz}\omega_z\underline{k}$$

$$= \frac{mr^2\omega}{2}\left(\frac{\sin 2\phi}{4}\underline{i} + \frac{\sin\phi}{\pi}\underline{j} + \frac{1+\cos^2\phi}{2}\underline{k}\right)$$

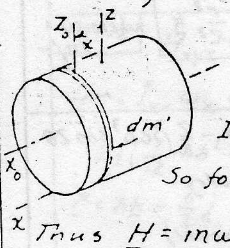
◂8/21 $H_x = 0$ since $\omega_x = \omega_y = 0$ & $I_{xz} = 0$

$H_y = -I_{yz}\omega_z, \quad H_z = I_{zz}\omega_z$



$$I_{yz} = \int_0^{\pi}(-r\sin\theta)(r + r\cos\theta)\,\rho r\,d\theta$$

$$= -\rho r^3(-\cos\theta)\Big|_0^{\pi} - \frac{\rho r^3}{4}(-\cos 2\theta)\Big|_0^{\pi} = -2\rho r^3 = -2\frac{mr^2}{\pi}$$



For complete shell $dI_{zz} = dI_{z_0z_0} + x^2dm'$

$dI_{zz} = \frac{1}{2}r^2dm' + x^2dm'$

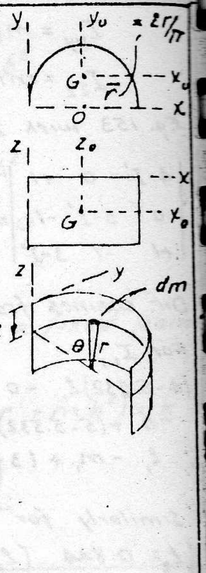
$= (\frac{1}{2}r^2 + x^2)\rho\,dx$, ρ = mass/length $= m'/2b$

$$I_{zz} = \left(\frac{1}{2}r^2x + x^3/3\right)_{-b}^{b}\rho = 2b\rho\left(\frac{r^2}{2}+\frac{b^2}{3}\right) = m'\left(\frac{r^2}{2}+\frac{b^2}{3}\right)$$

So for half shell $I_{zz} = m\left(\frac{r^2}{2}+\frac{b^2}{3}\right)$ since $m = \frac{m'}{2}$

Thus $\underline{H} = m\omega\left(\frac{2r^2}{\pi}\underline{j} + \left[\frac{r^2}{2}+\frac{b^2}{3}\right]\underline{k}\right)$

◂8/22 $\omega_x = \omega_y = 0,\ \omega_z = \omega;$



$H_x = -I_{xz}\omega_z, \quad H_y = -I_{yz}\omega_z, \quad H_z = I_{zz}\omega_z$

$$I_{xz} = \bar{I}_{xz} + md_xd_z = 0 + m(r)(-b/2) = -\frac{mrb}{2}$$

$dI_{yz} = (r\sin\theta)(-z)\rho r\,d\theta\,dz$

$$I_{yz} = -\rho r^2\frac{z^2}{2}\Big]_0^{-b}(-\cos\theta)\Big]_0^{\pi} = -\rho r^2b^2 = -\frac{mrb}{\pi}$$

(or more simply $I_{yz} = \bar{I}_{yz} + md_yd_z = 0 + m\left(\frac{2r}{\pi}\right)\left(-\frac{b}{2}\right)$)

$$I_{zz} = \bar{I}_{zz} + md^2 = (I_0 - m\bar{r}^2) + m(r^2 + \bar{r}^2)$$

$$= I_0 + mr^2 = 2mr^2$$

$$\underline{H} = mr\omega\left(\frac{b}{2}\underline{i} + \frac{b}{\pi}\underline{j} + 2r\underline{k}\right)$$

◂8/23 $\omega_x = \dot\phi,\ \omega_y = \omega\sin\phi,\ \omega_z = \omega\cos\phi;\ I_{xz} = I_{xy} = 0$

$H_x = I_{xx}\omega_x, \quad H_y = I_{yy}\omega_y - I_{yz}\omega_z, \quad H_z = -I_{yz}\omega_y + I_{zz}\omega_z$

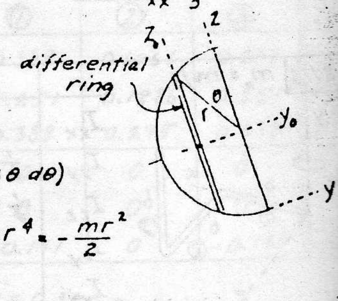
$I_{xx} = \bar{I}_{xx} + m'r^2$ for complete shell of mass m'

$= \frac{2}{3}m'r^2 + m'r^2 = \frac{5}{3}m'r^2$. So for half shell $m = \frac{m'}{2}$

& $I_{xx} = \frac{5}{3}mr^2$

By symmetry $I_{yy} = I_{xx}$

$I_{zz} = \frac{2}{3}mr^2$



For differential ring,

$dI_{yz} = dI_{y_0z_0} + dm(-r\sin\theta)(r)$

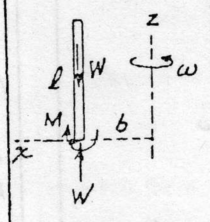
$$I_{yz} = 0 - \int_0^{\pi/2} r^2\sin\theta\,(2\pi\rho r^2\cos\theta\,d\theta)$$

$$= -\frac{\pi\rho r^4}{2}(-\cos 2\theta)\Big]_0^{\pi/2} = -\pi\rho r^4 = -\frac{mr^2}{2}$$

Thus

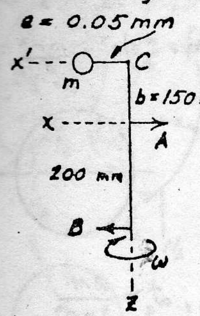
$$\underline{H} = mr^2\left[\frac{5\dot\phi}{3}\underline{i} + \omega\left(\frac{5}{3}\sin\phi + \frac{1}{2}\cos\phi\right)\underline{j} + \omega\left(\frac{1}{2}\sin\phi + \frac{2}{3}\cos\phi\right)\underline{k}\right]$$

8/25 $\Sigma M_y = -I_{xz}\omega_z^2; \quad -M = -m\frac{b\ell}{2}\omega^2$



$$M = \frac{mb\ell}{2}\omega^2$$

8/26 $\Sigma M_y = -I_{yz}\dot\omega_z - I_{xz}\omega_z^2$, $\dot\omega_z = 0$



$\omega_z = \omega = 10\,000\frac{2\pi}{60} = 1047$ rad/s

$I_{xz} = -mbe = -6(0.15)(50)(10^{-6})$

$= -45(10^{-6})$ kg·m^2

Thus $B(0.20) = 45(10^{-6})(1047)^2$, $\underline{B = 247\text{ N}}$

For origin of coordinates x'-y'-z at C,

$\Sigma M_{y'} = 0$ since $I_{x'z} = 0$, so that

$0.35B - 0.15A = 0$, $A = \frac{0.35}{0.15}247$

$= \underline{576\text{ N}}$

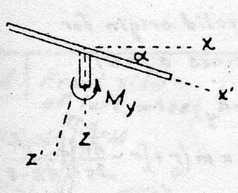
8/27 $\Sigma M_{A_x} = I_{yz}\omega_z^2$; $\Sigma M_{A_y} = -I_{xz}\omega_z^2$

$I_{yz} = (\rho b)b\frac{b}{2} + (\rho b)bb + (\rho b)b\frac{3b}{2} = 3\rho b^3$

$I_{xz} = (\rho b)\frac{b}{2}b + (\rho b)b\frac{3b}{2} = 2\rho b^3$

$M_x = 3\rho b^3\omega^2$, $M_y = -2\rho b^3\omega^2$, $M = \sqrt{M_x^2 + M_y^2} = \underline{\sqrt{13}\rho b^3\omega^2}$

8/28 $\Sigma M_y = -I_{xz}\omega_z^2$; $I_{xz} = \int(x'\cos\alpha)(x'\sin\alpha)\,dm$

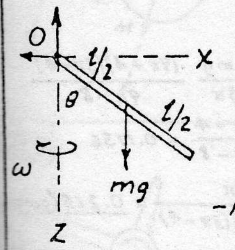


$= \frac{\sin 2\alpha}{2}I_{yy}$

$I_{yy} = \frac{1}{4}mr^2$

so $M_y = \underline{-\frac{\sin 2\alpha}{8}mr^2\omega^2}$

8/29 $\Sigma M_y = -I_{yz}\dot\omega_z - I_{xz}\omega_z^2$; $\dot\omega_z = \dot\omega = 0$



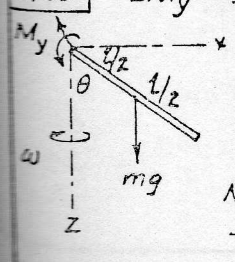
$I_{xz} = \int xz\,dm = \int xz\rho\,dl$

$= \rho\int_0^{l\cos\theta} z^2\tan\theta\frac{dz}{\cos\theta} = \frac{1}{6}ml^2\sin 2\theta$

$-mg\frac{l}{2}\sin\theta = -\frac{1}{6}ml^2\sin 2\theta\ \omega^2$

$\underline{\theta = 0}$, $\underline{\theta = \cos^{-1}\frac{3g}{2l\omega^2}}$, $\underline{\omega_{min} = \sqrt{\frac{3g}{2l}}}$

8/30 $\Sigma M_y = -I_{yz}\dot\omega_z - I_{xz}\omega_z^2$, $\dot\omega_z = 0$, $\omega_z = \omega$

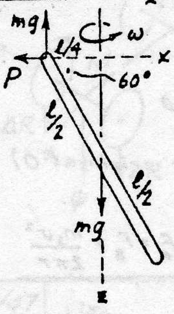


From Prob. 8/29, $I_{xz} = \frac{1}{6}ml^2\sin 2\theta$

$M_y - mg\frac{l}{2}\sin\theta = -\frac{ml^2}{6}\sin 2\theta\ \omega^2$

$\underline{M_y = \frac{mgl}{2}\sin\theta\left(1 - \frac{2l}{3g}\omega^2\cos\theta\right)}$

8/31 $\Sigma F_x = m\bar a_x$; $P = 0$; $\Sigma M_y = -I_{xz}\omega_z^2$



$I_{xz} = \int xz\,dm = \int_{-l/4}^{l/4} x\sqrt{3}\left(\frac{l}{4} + x\right)\rho\,dx$

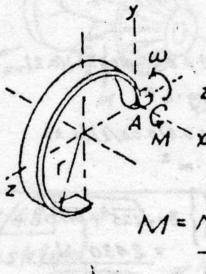
where ρ = mass/(x-component of length)

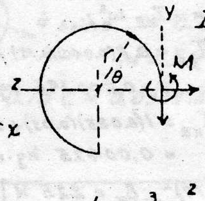
$I_{xz} = \frac{\sqrt{3}}{48}ml^2$ $m = \rho l/2$

so $-mg\frac{l}{4} = -\frac{\sqrt{3}}{48}ml^2\omega^2$, $\omega^2 = \frac{12g}{\sqrt{3}l}$

$\therefore \omega = \sqrt{\frac{12g}{\sqrt{3}l}} = \underline{2\sqrt{\frac{\sqrt{3}g}{l}}}$

8/32 At A $M_x = I_{yz}\omega_z^2$





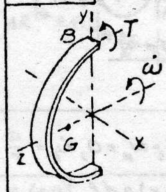
$I_{yz} = \int_0^{3\pi/2}(r\sin\theta)(r - r\cos\theta)\rho r\,d\theta$

$= \rho r^3\left[-\cos\theta + \frac{1}{4}\cos 2\theta\right]_0^{3\pi/2}$

$= \frac{1}{2}\rho r^3$

$M = M_x = \underline{\frac{1}{2}\rho r^3\omega^2}$

8/33 For entire ring section $\Sigma M_z = I_{zz}\dot\omega$



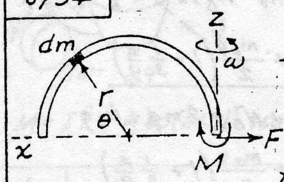
where $I_{zz} = \frac{3}{4}\frac{1}{2}(2\pi r\rho)r^2 = \frac{3\pi\rho r^3}{4}$

So $\dot\omega = \frac{4M}{3\pi\rho r^3}$

For 1/2 ring, $\Sigma M_z = I_{zz}\dot\omega$; Force at B is zero since $\bar a = 0$; Thus

$\Sigma M_z = T = \frac{\pi\rho r^3}{2}\frac{4M}{3\pi\rho r^3} = \underline{\frac{2}{3}M}$

8/34 $\Sigma M_y = -I_{xz}\omega_z^2$



$I_{xz} = \int xz\,dm = \int_0^{\pi}(r + r\cos\theta)(r\sin\theta)\rho r\,d\theta$

where ρ = mass/unit length

$I_{xz} = \rho r^3\left[-\cos\theta - \frac{1}{4}\cos 2\theta\right]_0^{\pi}$

$= 2\rho r^3 = \frac{2}{\pi}mr^2$

$-M = -\frac{2}{\pi}mr^2\omega^2$, $\underline{M = \frac{2mr^2\omega^2}{\pi}}$

8/35 With G as origin,

$$I_{xz} = \int_0^{\pi/2} (r\sin\theta)(-r + r\cos\theta)\rho r\,d\theta + \int_0^{\pi/2} (-r\sin\theta)(r - r\cos\theta)\rho r\,d\theta$$

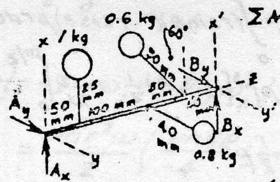
$$= -\rho r^3 = -\frac{m_2 r^2}{\pi} \quad \text{(same as for origin at } O\text{)}$$

$$\left.\begin{aligned} &\Sigma M_y = -I_{xz}\omega_z^2;\ F_A r + F_B r = \frac{m_2 r^2}{\pi}\frac{v^2}{r^2} \\ &\Sigma F_x = 0;\ F_A - F_B = 0 \end{aligned}\right\} \underline{F_A = F_B = \frac{m_2 v^2}{2\pi r}}$$

8/36 $\omega_z = 3600\frac{2\pi}{60} = 377$ rad/s

$\Sigma M_x = I_{yz}\omega_z^2$; (neglect gravity unbalance)

$\Sigma M_y = -I_{xz}\omega_z^2$;



$$I_{yz} = 0.8(0.040)(0.15) + 0.6(-0.05\tfrac{\sqrt{3}}{2})(0.2) = -0.000396\ \text{kg}\cdot\text{m}^2$$

$$I_{xz} = 1(0.025)(0.05) + 0.6(0.05/2)(0.2) = 0.00425\ \text{kg}\cdot\text{m}^2$$

$$\left.\begin{aligned} &-0.25 B_y = -396(10^{-6})(377)^2,\ B_y = 225\ \text{N} \\ &0.25 B_x = -4250(10^{-6})(377)^2,\ B_x = -2420\ \text{N} \end{aligned}\right\} B = \sqrt{225^2 + 2420^2} = \underline{2430\ \text{N}}$$

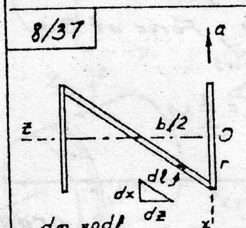
For origin at B with x'-y'-z,

$$I_{y'z} = 0.8(0.04)(-0.10) + 0.6(-0.05\tfrac{\sqrt{3}}{2})(-0.05) = -1901(10^{-6})\ \text{kg}\cdot\text{m}^2$$

$$I_{x'z} = 1(0.025)(-0.2) + 0.6(0.05/2)(-0.05) = -5750(10^{-6})\ \text{kg}\cdot\text{m}^2$$

$$\left.\begin{aligned} &0.25 A_y = -1901(10^{-6})(377)^2,\ A_y = -1081\ \text{N} \\ &-0.25 A_x = +5750(10^{-6})(377)^2,\ A_x = -3270\ \text{N} \end{aligned}\right\} A = \underline{3440\ \text{N}}$$

8/37

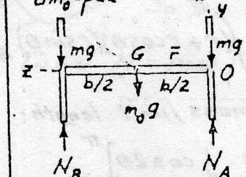


$\Sigma M_x = -I_{xz}\dot\omega_z + I_{yz}\omega_z^2$; $\omega_x = 0$, $\dot\omega_z = \frac{a}{r}$

$$I_{xz} = \int xz\,dm = \int x\frac{b}{2r}(r - x)\rho\,dl$$

$$= \frac{bl\rho}{4r^2}\int_{-r}^{r} x(r-x)\,dx = -\frac{1}{6}m_0 br$$

so $m_0 g\frac{b}{2} + (mg - N_B)b = +\frac{1}{6}m_0 br\frac{a}{r}$



$$\underline{N_B = mg + \frac{m_0 g}{2}\left(1 - \frac{1}{3}\frac{a}{g}\right)}$$

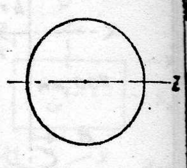
$\Sigma F_y = 0$; $N_A + N_B = 2mg + m_0 g$

$$\underline{N_A = mg + \frac{m_0 g}{2}\left(1 + \frac{1}{3}\frac{a}{g}\right)}$$

(Note: ΣM_x of Eq. 165 is valid here since $\underline{a}_0 \times m\underline{\bar r}$ term of Eq. 100 has no x-component.

◀8/38 $\omega_z = 0$, $\dot\omega_z \neq 0$ (Sample Prob. 8/24), $I_{xz} = -m_2 r^2/\pi$)

$$\Sigma M_z = I_{zz}\dot\omega_z;\ I_{zz} = \frac{2m_1}{2}r^2 + \frac{m_2}{2}r^2 = \left(m_1 + \frac{m_2}{2}\right)r^2 = \frac{mr^2}{2}$$



where $(I_z)_{\text{complete hoop}} = \frac{1}{2}m'r^2 = \frac{1}{2}2m_2 r^2$

$\therefore (I_z)_{\text{two } \frac{1}{4}\text{-hoops}} = \frac{m_2}{2}r^2$

$$\left.\begin{aligned} &\text{so } M + F_A r - F_B r = \frac{mr^2}{2}\dot\omega_z \\ &\Sigma F_x = m\bar a_x;\ F_B - F_A = mr\dot\omega_z \end{aligned}\right\} \frac{M}{r} = \frac{3}{2}mr\dot\omega_z,\ \dot\omega_z = \frac{2M}{3mr^2}$$

$$\Sigma M_x = -I_{xz}\dot\omega_z;\ mgr - N_A(2r) = -\left(-\frac{m_2 r^2}{\pi}\right)\frac{2M}{3mr^2}$$

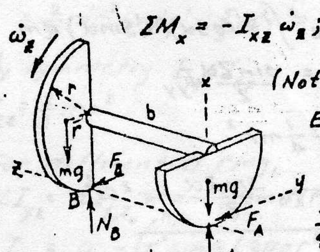
$\Sigma F_y = 0$; $N_A + N_B = mg$

which give $\left[\begin{aligned} N_A &= \frac{mg}{2} - \frac{Mm_2}{3\pi mr} \\ N_B &= \frac{mg}{2} + \frac{Mm_2}{3\pi mr} \end{aligned}\right]$

(Note: ΣM_x of Eq. 165 is valid here since the term $\underline{a}_0 \times m\underline{\bar r}$ of Eq. 100 has no x-component)

◀8/39 $\Sigma M_z = I_{zz}\dot\omega_z$; $-mg\frac{4r}{3\pi} = \left(3 - \frac{8}{3\pi}\right)mr^2\dot\omega_z$, $\dot\omega_z = \frac{-4g/r}{9\pi - 8}$

$\Sigma M_x = -I_{xz}\dot\omega_z$; $F_B b = -mrb\frac{-4g/r}{9\pi - 8}$, $F_B = \frac{4mg}{9\pi - 8}$



(Note: A is valid origin for Eq. 165 since $a_A = 0$)

$\Sigma F_y = m\bar a_y$;

$$\frac{4mg}{9\pi - 8} + F_A = m\left(r + \left[r - \frac{4r}{3\pi}\right]\right)\frac{4g/r}{9\pi - 8}$$

$$F_A = \frac{4mg}{9\pi - 8}\left(1 - \frac{4}{3\pi}\right)$$

$\bar r = 4r/3\pi$

$I_{zz_B} = \frac{3}{2}mr^2$

$I_{zz_A} = \bar I + m(r - \bar r)^2 = \frac{1}{2}mr^2 - m\bar r^2 + m(r - \bar r)^2 = \left(\frac{3}{2} - \frac{8}{3\pi}\right)mr^2$

$I_{zz_{A+B}} = \left(3 - \frac{8}{3\pi}\right)mr^2$

$I_{xz} = mrb$

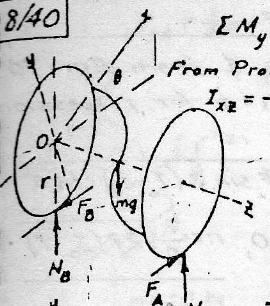
Similarly

$$N_A = mg,\quad N_B = \frac{mg}{3\pi}\,\frac{(9\pi + 4)(3\pi - 4)}{9\pi - 8}$$

$$f_A = \frac{F_A}{N_A} = \frac{4}{3\pi}\,\frac{3\pi - 4}{9\pi - 8} = 0.1136$$

$$f_B = \frac{F_B}{N_B} = \frac{12\pi}{(9\pi + 4)(3\pi - 4)} = \underline{0.215}\ \text{(governs)}$$

8/40



$\Sigma M_y = -I_{xz}\,\omega_z^2$; $N_A \sin\theta\,(2r) + F_A\cos\theta\,(2r)$

From Prob. 8/24 $I_{xz} = -\dfrac{m_z r^2}{\pi}$

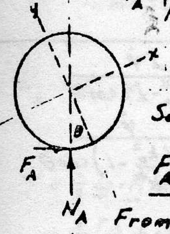
$-mgr\sin\theta = \dfrac{m_z r^2}{\pi}\,\dfrac{v^2}{r^2}$ or

$N_A\sin\theta + F_A\cos\theta - \dfrac{mg}{2}\sin\theta = \dfrac{m_z v^2}{2\pi r}$ --- (a)

$\Sigma M_x = I_{yz}\,\omega_z^2 = 0$;

$-N_A\cos\theta\,(2r) + F_A\sin\theta\,(2r) + mgr\cos\theta = 0$

or $N_A\cos\theta - F_A\sin\theta - \dfrac{mg}{2}\cos\theta = 0$ --- (b)

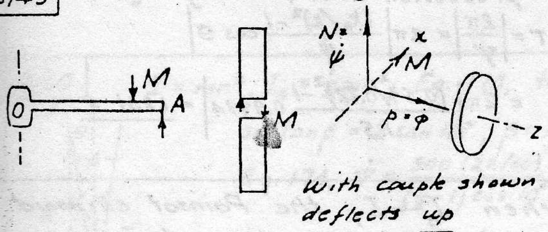


Solve (a) & (b) for N_A & F_A & get

$\underline{F_A = \dfrac{m_z v^2}{2\pi r}\cos\theta}$, $\underline{N_A = \dfrac{mg}{2} + \dfrac{m_z v^2}{2\pi r}\sin\theta}$

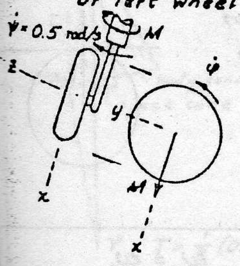
From $\Sigma F_x = \Sigma F_y = 0$, $\underline{F_B = F_A}$; $\underline{N_B = \dfrac{mg}{2} - \dfrac{m_z v^2}{2\pi r}\sin\theta}$

8/43



With couple shown and A deflects up

8/44



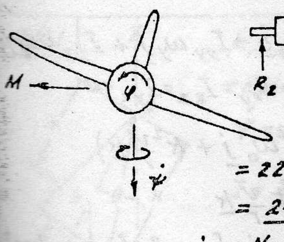
Eq. 168, $M = I\dot\phi\dot\psi$

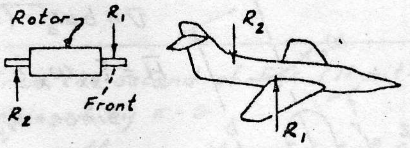
$\dot\phi = \dfrac{v}{r} = \dfrac{225}{3.6}/0.4 = 156.2$ rad/s

$M = 31(0.25)^2(156.2)(0.5)$

$= \underline{151.4\ \text{N}\cdot\text{m}}$

8/45





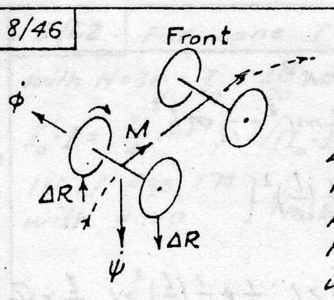
$M = I\dot\phi\dot\psi$

$= 225\,(0.25)^2(18\,000)\dfrac{2\pi}{60}\;\dfrac{1000/3.6}{3000}$

$= \underline{2450\ \text{N}\cdot\text{m}}$

Nose tends to rise

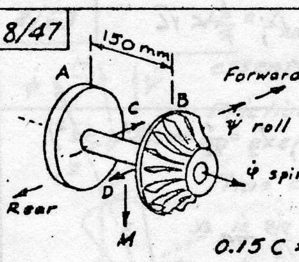
8/46



Change in normal force is ΔR

Triad ψ, ϕ, M as shown ΔR is down under right rear wheel to give proper direction of M. Hence <u>decrease</u> of force under right rear wheel.

8/47



$\dot\varphi = 20\,000\,\dfrac{2\pi}{60} = 2094$ rad/s

$\dot\psi = 2$ rad/s

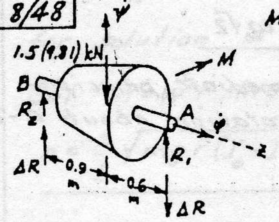
$I = 3.5(0.079)^2 + 2.4(0.071)^2$

$= 0.0339\ \text{kg}\cdot\text{m}^2$

$M = I\dot\varphi\dot\psi$,

$0.15\,C = 0.0339(2094)(2)$, $\underline{C = D = 948\ \text{N}}$

8/48



$M = I\dot\phi\dot\psi$

$1.5\,\Delta R = 1.5\,(0.225)^2\,6000\,\dfrac{2\pi}{60}\,\dfrac{22(1.852)}{3.6(400)}$

$\Delta R = 0.900$ kN

Also static reactions are

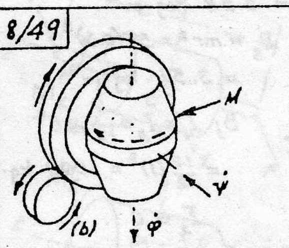
$R_2 = \dfrac{0.6}{1.5}\,1.5(9.81) = 5.89$ kN,

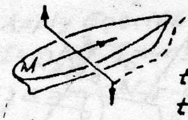
$R_1 = 8.83$ kN

Thus $A = 8.83 - 0.90 = \underline{7.93\ \text{kN}}$

$B = 5.89 + 0.90 = \underline{6.79\ \text{kN}}$

8/49





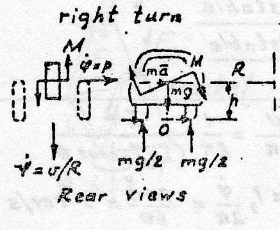
M needed on structure of ship to counteract roll to port (left).

Reaction on gyro is opposite to M on ship. Proper direction of triad shown – requiring rotation (b) of motor.

$M = I\dot\phi\dot\psi = 80(1.45)^2\,960\,\dfrac{2\pi}{60}\,0.320 = \underline{5410\ \text{kN}\cdot\text{m}}$

8/50

Assume right turn



$m\bar a = mv^2/R$ $\quad\Sigma M_O = m\bar a h$

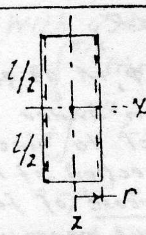
$M + 0 = mv^2h/R$

$M = I\dot\phi\dot\psi = m_0 k^2 p\,v/R$

so $mv^2\dfrac{h}{R} = m_0 k^2 p\,\dfrac{v}{R}$

$\underline{p = \dfrac{m}{m_0}\,\dfrac{vh}{k^2}}$ opposite direction to wheels

8/51



$I = I_{zz} = mr^2$

$I_o = I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$

$\frac{I_o}{I} = \frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2$

Direct precession if $\frac{I_o}{I} > 1$; $\frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2 > 1$, $\frac{l}{r} > \sqrt{6}$

Retrograde " " $\frac{I_o}{I} < 1$; $\frac{l}{r} < \sqrt{6}$

8/52 $I_1 = \frac{1}{12}m(a^2 + \overline{2a}^2)$, $I_2 = \frac{1}{12}m(a^2 + \overline{3a}^2)$

$I_3 = \frac{1}{12}m(\overline{2a}^2 + \overline{3a}^2)$

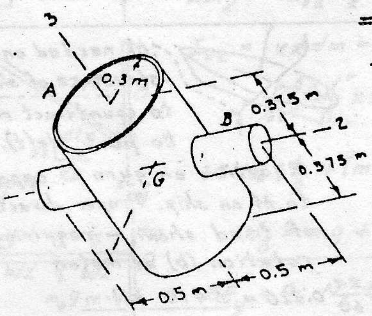
$I_1 = \frac{5}{12}ma^2$, $I_2 = \frac{5}{6}ma^2$, $I_3 = \frac{13}{12}ma^2$

$T_{0-1} = T_{0-2}$; $\frac{1}{2}I_2\omega_o^2 = \frac{1}{2}I_1\omega^2$, $\omega = \omega_o\sqrt{I_2/I_1}$

so $\omega = \omega_o\sqrt{\frac{5}{6}ma^2 \Big/ \frac{5}{12}ma^2} = \omega_o\sqrt{2}$

Axis 0-2 is axis of intermediate principal moment of inertia so rotation about 0-2 is unstable.

8/53 A; $I_1 = I_2 = m\left(\frac{r^2}{2} + \frac{l^2}{12}\right) = 39\left(\frac{\overline{0.3}^2}{2} + \frac{\overline{0.75}^2}{12}\right)$



$= 3.58 \text{ kg}\cdot\text{m}^2$

$I_3 = mr^2 = 39(0.3)^2$

$= 3.51 \text{ kg}\cdot\text{m}^2$

B; $I_1 = I_3 = \frac{1}{12}ml^2$

$= \frac{12}{12}(1)^2 = 1.000 \text{ kg}\cdot\text{m}^2$

$I_2 \approx 0$

So $I_1 = 3.58 + 1.00 = 4.58 \text{ kg}\cdot\text{m}^2$ (max)

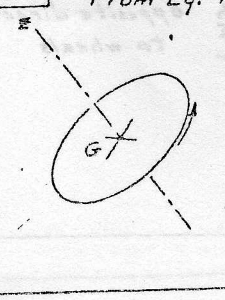
$I_2 \cong 3.58 + 0 = 3.58 \text{ kg}\cdot\text{m}^2$ (min)

$I_3 = 3.51 + 1.00 = 4.51 \text{ kg}\cdot\text{m}^2$ (intermed)

Rotation about axes 1 & 2 stable

" " axis 3 unstable

8/54 From Eq. 174 $f = \frac{\dot\psi}{2\pi} = \frac{1}{2\pi}\frac{I\dot\varphi}{(I - I_o)\cos\theta}$



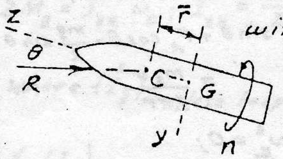
$\cos\theta \approx 1$, $\frac{\dot\varphi}{2\pi} = \frac{300}{60} = 5$ rev/s

$\frac{I}{I_o - I} = \frac{mr^2}{\frac{1}{2}mr^2 - mr^2} = -2$

So $f = 2(5) = 10$ Hz

Retrograde precession

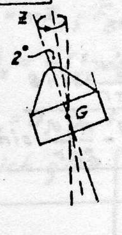
8/55 $\Sigma M_x = M = R\bar{r}\sin\theta$ & from Eq. 170 minimum $n = \dot\varphi$ for precession with $\theta \to 0$ is



$n = \frac{2}{I}\sqrt{R\bar{r}\sin\theta\,(I_o - I)\,\text{ctn}\,\theta}$

& for $\theta = 0$, $n = \frac{2}{I}\sqrt{R\bar{r}(I_o - I)}$

8/56 For zero moment, $\dot\psi = \frac{I\dot\varphi}{(I_o - I)\cos\theta}$



where $k = 0.72$ m

$k_o = 0.54$ m

$\dot\varphi = 1.5$ rad/s

$\theta = 2°$

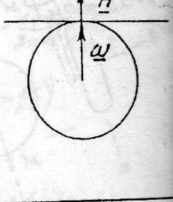
$= \frac{\dot\varphi}{\left(\frac{k_o^2}{k^2} - 1\right)\cos\theta}$

$(I = k^2 m) > (I_o = k_o^2 m)$ so retrograde precession with $\dot\varphi$ in negative z-dir.

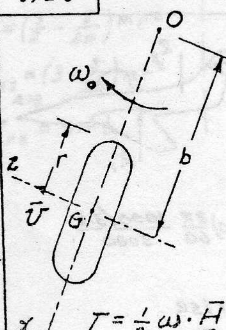
Period $T = \left|\frac{2\pi}{\dot\psi}\right| = 2\pi\left|\frac{(k_o/k)^2 - 1}{\dot\varphi}\cos\theta\right|$

$= 2\pi\left|\frac{(0.54/0.72)^2 - 1}{1.5}0.9994\right| = 1.831$ s

8/57 When $I_o \to I$, the Poinsot ellipsoid approaches a sphere & $\underline{\omega}$ & $\underline{\bar{H}}$ must be collinear. Thus body spins about one axis with no precession. Space & body cones degenerate into the common line which is the invariable line.



8/58 $\omega_z = \frac{v}{r}$, $\omega_x = 0$, $\omega_y = \omega_o$



$\bar{v} = b\omega_o$ (rel. to airplane)

$\bar{H} = I_{xx}\omega_x\underline{i} + I_{yy}\omega_y\underline{j} + I_{zz}\omega_z\underline{k}$

since $I_{xy} = I_{xz} = I_{yz} = 0$

$\bar{H} = m\left(k_o^2\omega_o\underline{j} + k^2\frac{v}{r}\underline{k}\right)$

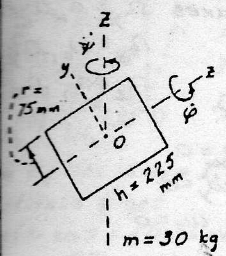
$\underline{\omega} = \omega_o\underline{j} + \frac{v}{r}\underline{k}$

$T = \frac{1}{2}\underline{\omega}\cdot\bar{H} + \frac{1}{2}\bar{v}\cdot\underline{G} = \frac{1}{2}\left[mk_o^2\omega_o^2 + mk^2\frac{v^2}{r^2}\right]$

$+ \frac{1}{2}b\omega_o\, m b\omega_o$

So $T = \frac{1}{2}m\left[(k_o^2 + b^2)\omega_o^2 + k^2\frac{v^2}{r^2}\right]$

8/59 $I_{zz} = \frac{1}{2}mr^2 = \frac{1}{2}30(0.075)^2 = 0.0844$ kg·m²



$I_{xx} = I_{yy} = \frac{1}{4}mr^2 + \frac{1}{12}mh^2$

$= \frac{30}{4}\left[(0.075)^2 + \frac{1}{3}(0.225)^2\right] = 0.1688$ kg·m²

$\dot\psi = \frac{150(2\pi)}{60} = 15.71$ rad/s $= 5\pi$ rad/s

$\dot\phi = 300\frac{2\pi}{60} = 10\pi$ rad/s

$\omega_x = \dot\theta = 0,\ \omega_y = \dot\psi\sin\theta = \frac{5\sqrt{3}}{2}\pi$ rad/s

$\omega_z = \dot\phi + \dot\psi\cos\theta = \frac{25\pi}{2}$ rad/s

$\bar H_x = I_{xx}\omega_x = 0;\ \bar H_y = I_{yy}\omega_y = 0.1688\frac{5\sqrt{3}}{2}\pi$ kg·m²/s

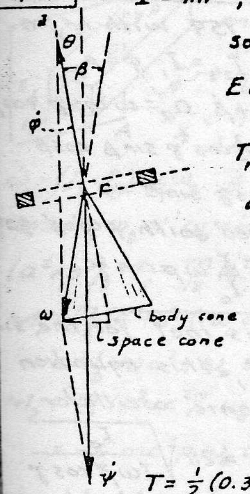
$\bar H_z = I_{zz}\omega_z = 0.0844\frac{25\pi}{2}$ kg·m²/s

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H} = \frac{1}{2}\left[\omega_x H_x + \omega_y H_y + \omega_z H_z\right]$

$= \frac{1}{2}\left[0 + 0.1688\left(\frac{5\sqrt{3}}{2}\pi\right)^2 + 0.0844\left(\frac{25\pi}{2}\right)^2\right]$

$= \frac{0.0844}{8}(25\pi^2)(31) = \underline{80.7\ \text{J}}$

8/60 $I = mr^2,\ I_o = \frac{1}{2}mr^2$; Eq. 173, $\tan\theta = \frac{1}{2}\tan\beta$



so $\tan\beta = 2\tan 10°,\ \beta = 19.43°$

Eq. 174, $\dot\psi = \frac{300(2\pi/60)}{(\frac{1}{2}-1)\cos 10°} = -63.8$ rad/s

$T_{rotation} = \frac{1}{2}\underline{\omega}\cdot\underline{\bar H} = \frac{1}{2}(I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + I_{zz}\omega_z^2)$

$\omega_x = \Omega_x = \dot\theta = 0;\ \omega_y = \Omega_y = \dot\psi\sin\theta$

$= -63.8\sin 10° = -11.08$ rad/s

$\omega_z = \dot\phi - |\dot\psi|\cos\theta = 31.4 - 63.8\cos 10°$

$= -31.4$ rad/s

$I_{xx} = I_{yy} = I_o = \frac{1}{2}mr^2 = \frac{1}{2}8(0.3)^2$

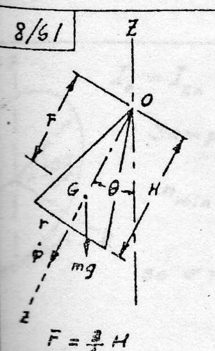
$= 0.360$ kg·m²

$I_{zz} = I = mr^2 = 0.720$ kg·m²

$T = \frac{1}{2}(0.360)(0^2 + \overline{11.08}^2 + 2[31.4]^2)$

$= \underline{377\ \text{J}}$

8/61



$F = \frac{3}{4}H$

$= \frac{3}{4}(3r) = \frac{9}{4}r$

$= 225$ mm

From the second of Eqs. 171 with θ replaced by π − θ

$\dot\psi = \frac{M}{I\dot\phi\sin\theta}$; $M = mg\bar r\sin\theta = \frac{9}{4}mgr\sin\theta$

$I = \frac{3}{10}mr^2,\ \dot\phi = 2700\frac{2\pi}{60} = 283$ rad/s

$\dot\psi = \frac{\frac{9}{4}mgr\sin\theta}{\frac{3}{10}mr^2(283)\sin\theta} = \frac{15}{2}\frac{9.81}{(0.1)(283)}$

$= 2.60$ rad/s

Result independent of θ

Period $\tau = \frac{2\pi}{\dot\psi} = \frac{2\pi}{2.60} = \underline{2.41\ \text{s}}$

8/62 For cone $I_o = I_{xx} = \frac{3}{5}mH^2 + \frac{3}{20}mr^2$

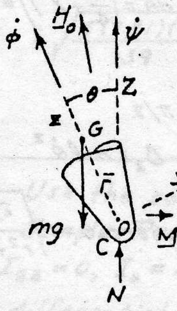
with $H = 3r$, $I_o = \frac{111}{20}mr^2$; $I = I_{zz} = \frac{3}{10}mr^2$

$I_o - I = \frac{21}{4}mr^2$, $I/(I_o - I) = 2/35$

1st of Eqs. 171 with θ → 0 } $\dot\psi_{Fast} = \frac{I\dot\phi}{|(I_o - I)\cos(\pi-\theta)|} = \frac{2}{35}2700$

$= \underline{154\ \text{rev/min}}$ (neg. Z-dir.)

8/63



$H_x = I_{xx}\omega_x = I_{xx}\dot\theta$, small so $\underline{H}_o$ is essentially in y-z plane

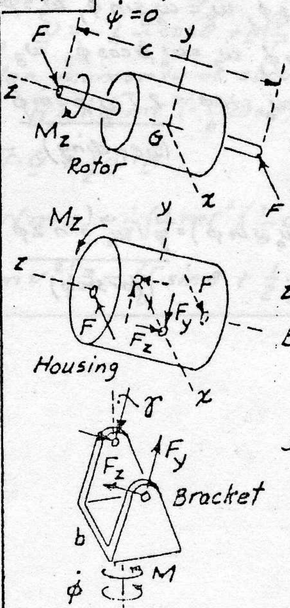
Friction force acts into paper at C so exerts moment $\underline{M}_f$ as shown. Thus change in $\underline{H}_o$ due to $\underline{M}_f$ is in direction of $\underline{M}_f$, hence toward Z-axis. Thus axis rises.

From Eq. 170 min. value of $n = \dot\phi$ for solution is $n = \dot\phi = \frac{2}{I}\sqrt{M_o(I_o - I)\,ctn\,\theta}$

But $M = W\bar r\sin\theta$, so for θ → 0

$n = \frac{2}{I}\sqrt{mg\bar r(I_o - I)}$, (assumes $I_o > I$)

8/64 $\dot\theta = 0$, $\dot\phi = p =$ const., M_z = torque of electric field to maintain $\dot\phi$ constant (ω_z is not constant)



Apply Eqs. 166 with $I = mk_z^2$, $I_o = mk_x^2$, $\gamma = \pi/2 - \theta$

x; $0 = mk_x^2(-\ddot\gamma - 0) + 0$, so $\ddot\gamma = 0$

y: $F_C = F_z b = \frac{mk_x^2}{\cos\gamma}\frac{d}{dt}(\dot\psi\cos^2\gamma) + mk_z^2\dot\gamma\omega_z$

$= mk_x^2\ddot\psi\cos\gamma$

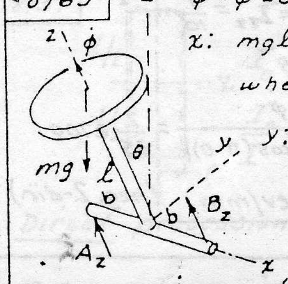
z; $-M_z = -F_y b = mk_z^2(\ddot\psi\sin\gamma + 0)$

But $M = F_z b\cos\gamma - F_y b\sin\gamma$

$= mk_x^2\ddot\psi\cos^2\gamma + mk_z^2\sin^2\gamma$

So $\ddot\psi = \frac{M/m}{k_x^2\cos^2\gamma + k_z^2\sin^2\gamma}$

◄ 8/65



$\dot\psi = \ddot\psi = 0$; From moment equations Eqs 166

x: $mgl \sin\theta = m\left(\frac{r^2}{4}+\ell^2\right)\ddot\theta$ ------- (a)

where $I_o = I_{xx} = \frac{1}{4}mr^2 + m\ell^2$

y: $-b(-A_z+B_z) = -\frac{1}{2}mr^2\dot\phi\dot\theta$ ----- (b)

where $I = \frac{1}{2}mr^2$

z: $0 = I\ddot\phi$ where $\omega_z = 0+\dot\phi$ --- (c)

From (c), $\dot\phi$ = constant

From (a) with $\dot\theta\, d\dot\theta = \ddot\theta\, d\theta$, $\int_0^{\pi/2} gl\sin\theta\, d\theta = \left(\frac{r^2}{4}+\ell^2\right)\int_0^{\dot\theta}\dot\theta\, d\dot\theta$

which gives $\dot\theta^2 = 8gl/(r^2+4\ell^2)$

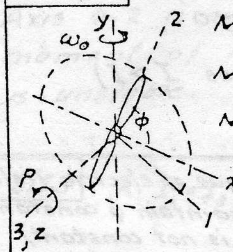
From (b) $-A_z + B_z = \frac{1}{2}m\frac{r^2}{b}\dot\phi\dot\theta$ for $\theta = \pi/2$

Also for $\theta = \pi/2$, $\Sigma F_z = m\bar a_z$ gives $-A_z - B_z = m\ell\dot\theta^2$

Solve & get
$$\left.\begin{aligned} A_z &= \frac{-m\dot\theta}{2}\left(\frac{r^2}{2b}\dot\phi + \ell\dot\theta\right) \\ B_z &= \frac{m\dot\theta}{2}\left(\frac{r^2}{\cdot}\dot{} \; \ell\dot\theta\right)\end{aligned}\right\} \text{where } \dot\theta = 2\sqrt{\frac{2g\ell}{r^2+4\ell^2}}$$

◄ 8/66

From Eqs. 158 with 1,2,3 sub. for x,y,z,



$M_1 = I_1\dot\omega_1 - (I_2-I_3)\omega_2\omega_3$

$M_2 = I_2\dot\omega_2 - (I_3-I_1)\omega_3\omega_1$

$M_3 = I_3\dot\omega_3 - (I_1-I_2)\omega_1\omega_2$

$I_1 = I_3 = I$; $I_2 = 0$

$\omega_1 = -\omega_o\cos\phi$, $\omega_2 = \omega_o\sin\phi$, $\omega_3 = p = \dot\phi$

$\dot\omega_1 = \omega_o p\sin\phi$, $\dot\omega_2 = \omega_o p\cos\phi$, $\dot\omega_3 = 0$

Thus

$M_B = M_1 = I\omega_o p\sin\phi + Ip\omega_o\sin\phi = \underline{2I\omega_o p\sin\phi}$

(bending)

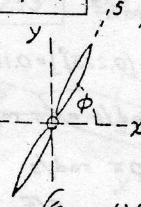
$M_2 = 0 - 0 = 0$

$M_z = M_3 = 0 - I(-\omega_o\cos\phi)(\omega_o\sin\phi) = \underline{\frac{1}{2}I\omega_o^2\sin 2\phi}$

(torsion)

◄ 8/67

x-y-z fixed to aircraft



Angular velocity of axes $\Omega_x = 0, \dot\Omega_x = 0$
$\Omega_y = \omega_o, \dot\Omega_y =$
$\Omega_z = 0, \dot\Omega_z =$

Angular velocity of propeller $\omega_x = 0$, $\dot\omega_x = 0$
$\omega_y = \omega_o$, $\dot\omega_y = 0$
$\omega_z = p = \dot\phi$, $\dot\omega_z = 0$

$I_{xy} = \int(s\cos\phi)(s\sin\phi)\,dm$
$= \frac{1}{2}\sin 2\phi\int s^2 dm = \frac{I}{2}\sin 2\phi$

$I_{yy} = \int s^2\cos^2\phi\, dm = I\cos^2\phi$

$H_x = I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z = 0 - \frac{I}{2}\sin 2\phi\,\omega_o + 0$, $\dot H_x = -I\omega_o p\cos$

$H_y = -I_{xy}\omega_x + I_{yy}\omega_y - I_{yz}\omega_z = 0 + I\omega_o\cos^2\phi + 0$, $\dot H_y = -I\omega_o p\sin 2\phi$

$H_z = -I_{xz}\omega_x - I_{yz}\omega_y + I_{zz}\omega_z = 0+0+Ip$; $\dot H_z = 0$

$\Sigma M_x = \dot H_x - H_y\Omega_z + H_z\Omega_y = -I\omega_o p\cos 2\phi - 0 + I\omega_o p = 2I\omega_o p\sin$

$\Sigma M_y = \dot H_y - H_z\Omega_x + H_x\Omega_z = -I\omega_o p\sin 2\phi + 0$.

$\Sigma M_z = \dot H_z - H_x\Omega_y + H_y\Omega_x = 0 + \frac{1}{2}I\omega_o^2\sin 2\phi + 0$

$M_1 = M_x\sin\phi - M_y\cos\phi = 2I\omega_o p\sin\phi(\sin^2\phi + \cos^2\phi) = \underline{2I\omega_o p\sin}$

$M_3 = \Sigma M_z = \frac{1}{2}I\omega_o^2\sin 2\phi$

◄ 8/68

The second of Eqs. 159 with $M_y = 0$ & with $I_{xx} = I_{yy} = I_o$, $I_{zz} = I$, $\dot\phi = p$,

$\Omega_x = -\omega\cos\gamma\sin\beta$, $\Omega_y = \omega\sin\gamma + \dot\beta$, $\Omega_z = \omega\cos\gamma\cos\beta$

gives $0 = I_o(0+\ddot\beta) + (I - I_o)\omega^2\cos^2\gamma\sin\beta\cos\beta$
$+ Ip\omega\cos\gamma\sin\beta = 0$

Neglect term in ω^2 compared with $p\omega$ & get

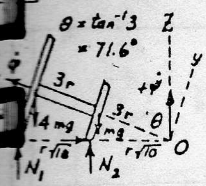
$\ddot\beta + K^2\sin\beta = 0$ where $K^2 = \frac{I}{I_o}\omega p\cos\gamma$

which is the same form as that for the sim pendulum. With β small, this equation describes the linear harmonic oscillator & has a period $\tau = \frac{2\pi}{K} = 2\pi\sqrt{\frac{I_o}{I\omega p\cos\gamma}}$

Thus gyro axis will oscillate about the north direction. With some damping it will point north.

8/69

$\tau = 2\pi/\dot{\psi}$ = const. $\quad \dot{\varphi} = -2r\sqrt{10}\,\dot{\psi}/2r = -\dfrac{2\pi\sqrt{10}}{\tau}$



(Seen by allowing ground to rotate under axle at rate $\dot{\psi}$)

$I = I_{zz} = \frac{1}{2}4m(2r)^2 + \frac{1}{2}mr^2 = \frac{17}{2}mr^2$

$I_{xx} = I_0 = \frac{1}{4}4m(2r)^2 + 4m(6r)^2 + \frac{1}{4}mr^2 + m(3r)^2 = \frac{629}{4}mr^2$

From Eq. 166 $\Sigma M_x = I_0\ddot{\theta} + (I - I_0)\dot{\psi}^2\sin\theta\cos\theta + I\dot{\varphi}\dot{\psi}\sin\theta$

Substitute & get

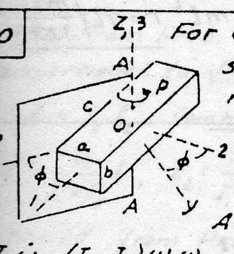
$$mg(3r\sin\theta + 4(6r)\sin\theta) - r\sqrt{10}(2N_1 + N_2)$$
$$= 0 + \left(\frac{17}{2} - \frac{629}{4}\right)mr^2\left(\frac{2\pi}{\tau}\right)^2\frac{3}{\sqrt{10}}\frac{1}{\sqrt{10}} - \frac{17}{2}mr^2\frac{2\pi\sqrt{10}}{\tau}\frac{2\pi}{\tau}\frac{3}{\sqrt{10}}$$
$$= -\frac{280.5\,mr^2\pi^2}{\tau^2} \text{ or}$$

$$27\,mg\sin\theta - \sqrt{10}(2N_1 + N_2) = -\frac{280.5\,mr\pi^2}{\tau^2}$$

Also $\Sigma F_z = 0$; $N_1 + N_2 = 5\,mg$

Solve simultaneously, with $\tau = 4$ s, $r = 0.15$ m, $g = 9.81\ \text{m/s}^2$ & get $\underline{N_1 = 3.94\,mg}$, $\underline{N_2 = 1.06\,mg}$

8/70



For constant ω_0, M applied to shaft about x-axis equals moment applied to block by shaft A-A.

Use Euler's Eqs 158 where 1, 2, 3 replace x, y, z

Angular velocity of block is $\underline{\omega}$

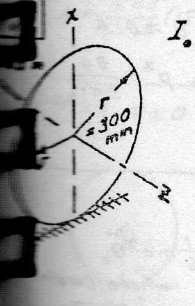
$M_1 = I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3$ & $M_2 = I_2\dot{\omega}_2 - (I_3 - I_1)\omega_3\omega_1$

$\omega_1 = \omega_0\cos\phi$, $\omega_2 = -\omega_0\sin\phi$, $\omega_3 = \dot{\phi} = p$

$I_1 = \frac{m}{12}(a^2 + b^2)$, $I_2 = \frac{m}{12}(b^2 + c^2)$, $I_3 = \frac{m}{12}(a^2 + c^2)$

$$M_1 = \frac{m}{12}(a^2 + b^2)(-\omega_0\dot{\phi}\sin\phi) - \frac{m}{12}(b^2 - a^2)(-\dot{\phi}\omega_0\sin\phi)$$
$$= -\frac{1}{6}ma^2p\omega_0\sin\phi$$
$$M_2 = \frac{m}{12}(b^2 + c^2)(-\omega_0\dot{\phi}\cos\phi) - \frac{m}{12}(c^2 - b^2)(\dot{\phi}\omega\cos\phi)$$
$$= -\frac{1}{6}mc^2p\omega_0\cos\phi$$

$$M = M_1\cos\phi - M_2\sin\phi$$
$$= \frac{1}{12}m(c^2 - a^2)p\omega_0\sin 2\phi$$



$I_0 = I_{xx} = I_{yy} = \frac{1}{2}mr^2$, $I = I_{zz} = mr^2$

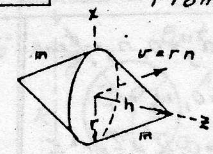
Sample Prob. 8/71,

$$n_{min} = \sqrt{\frac{\frac{1}{2}mr^2\,mgr}{mr^2(mr^2 + mr^2)}} = \frac{1}{2}\sqrt{\frac{g}{r}}$$

so $v = rn = \frac{1}{2}\sqrt{gr} = \frac{1}{2}\sqrt{9.81(0.3)}$
$= \underline{0.858\ \text{m/s}}$

8/73

From Sample Prob. 8/71, cones will be stable if



$$n > n_{min} = \sqrt{\frac{I_{xx}\,mgr}{I_{zz}(I_{zz} + mr^2)}}$$

where $I_{xx} = 2\frac{m}{20}(3r^2 + 2h^2)$, $I_{zz} = 2\frac{3}{10}mr^2$ } m = mass of each cone

$h = 600$ mm
$r = 300$ mm

Substitute & get

$$v = r\sqrt{\frac{\frac{1}{10}(3r^2 + 2h^2)2gr}{\frac{3}{5}r^2(\frac{3}{5} + 2)r^2}} = \sqrt{\frac{5gr}{39}\left(3 + \frac{2h^2}{r^2}\right)}$$
$$= \sqrt{\frac{5(9.81)(0.3)}{39}\left(3 + \frac{2(0.6)^2}{(0.3)^2}\right)} = \underline{2.04\ \text{m/s}}$$

8/74

Use the coordinates and notation of Sample Prob. 8/71

$I = I_{zz} = 0$, $I_0 = I_{xx} = I_{yy} = \frac{1}{12}mL^2$

The differential equation (j) becomes

$$m\left(\frac{L^2}{12} + r^2\right)\ddot{\theta} - mgr\,\theta = 12\frac{r^2}{L^2}Cn$$

Coeff. of θ is negative, hence motion grows with time & is $\underline{\text{unstable}}$.

8/75

For disk, $\omega_x = -\frac{v}{b}\cos\theta$, $\omega_y = 0$, $\omega_z = \frac{v}{r} - \frac{v}{b}\sin\theta$

So angular velocity is

$$\underline{\omega} = -\underline{i}\frac{v}{b}\cos\theta + \underline{k}\left(\frac{v}{r} - \frac{v}{b}\sin\theta\right)$$

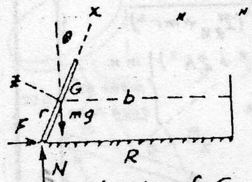
Angular momentum about center is

$$\underline{H} = \underline{i}\frac{1}{4}mr^2\left(-\frac{v}{b}\cos\theta\right) + \underline{k}\frac{1}{2}mr^2\left(\frac{v}{r} - \frac{v}{b}\sin\theta\right)$$

$$T = \frac{1}{2}\underline{\bar{v}}\cdot\underline{G} + \frac{1}{2}\underline{\omega}\cdot\underline{H}$$
$$= \frac{1}{2}v(mv) + \frac{1}{2}\left(\frac{1}{4}mr^2\right)\left(\frac{v}{b}\cos\theta\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r} - \frac{v}{b}\sin\theta\right)^2$$
$$= \frac{1}{4}mv^2\left[3 - \frac{2r}{b}\sin\theta + \frac{1}{2}\frac{r^2}{b^2}(1 + \sin^2\theta)\right]$$

◂ 8/76 Ang. vel. axes $\Omega_x = -\frac{v}{b}\cos\theta,\ \Omega_y = 0,\ \Omega_z = -\frac{v}{b}\sin\theta$

" " disk $\omega_x = -\frac{v}{b}\cos\theta,\ \omega_y = 0,\ \omega_z = \frac{v}{r}$



$\bar{H}_x = \frac{1}{4}mr^2\left(-\frac{v}{b}\cos\theta\right),\ \bar{H}_y = 0,\ \bar{H}_z = \frac{1}{2}mr^2\frac{v}{r}$

Substitute in Eq. 156 & get

$$\begin{cases} 0 = 0 - 0 + 0 \\ Nr\sin\theta - Fr\cos\theta = 0 - \frac{1}{2}mrv\left(-\frac{v}{b}\cos\theta\right) + \frac{1}{4}mr^2\frac{v}{b}\cos\theta\,\frac{v}{b}\sin\theta \\ 0 = 0 - 0 + 0 \end{cases}$$

v = velocity of G

$\mathbf{v} = v\mathbf{j}$ (into paper)

$I = I_{zz} = \frac{1}{2}mr^2$

$I_{xx} = I_{yy} = \frac{1}{4}mr^2$

Also the force eqs. are $F = m\frac{v^2}{b}$; $N = mg$; $0 = 0$

Substitution gives

$mgr\sin\theta - m\frac{v^2}{b}r\cos\theta = \frac{1}{4}m\frac{v^2}{b}r\cos\theta\left(2 + \frac{r}{b}\sin\theta\right)$

which simplifies to

$$\tan\theta = \frac{v^2}{2gb}\left(3 + \frac{r}{2b}\sin\theta\right)$$

◂ 8/77 Use the notation & equations (a) & (b) in Sample Prob. 8/71 with $\dot\theta$, $\dot\varphi$, θ small. Neglect products $\theta^2, \dot\theta^2, \dot\varphi\dot\theta, \dot\varphi\theta, \dot\theta\theta$. Eqs. (a) & (b) of Prob. 8/71 become for θ small

$$0 = I_0\ddot\psi,\quad \dot\psi = n \text{ constant} \qquad (c)$$

$$Nr\theta - F_2 r = I_0\ddot\theta - I\omega_z n + I_0 n^2\theta \qquad (d)$$

$$F_1 r = I\dot\omega_z \qquad (e)$$

$$N - mg + F_2\theta = -mr(0) \qquad (f)$$

$$-F_1 = mr(\dot\omega_z + n\dot\theta) \qquad (g)$$

$$(N - mg)\theta - F_2 = mr(-\ddot\theta + n\omega_z) \qquad (h)$$

(e) & (g); $(I + mr^2)\dot\omega_z + mr^2 n\dot\theta = 0$ which is integrated to give $(I + mr^2)\omega_z + mr^2 n\theta = 0$ since $\omega_z = 0$ when $\theta = 0$

Combine with combination of (d) & (h) to get

$(I_0 + mr^2)\ddot\theta + ([I_0 + mr^2]n^2 - mgr)\theta = 0$

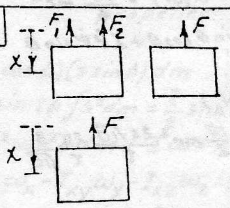
Stable if coeff. of θ is positive. Thus $n > \sqrt{\dfrac{mgr}{I_0 + mr^2}}$

But for $I_0 = \frac{1}{4}mr^2$, $n > 2\sqrt{\dfrac{g}{5r}}$

CHAPTER NINE

VIBRATION AND TIME RESPONSE

9/2



(a) $F = F_1 + F_2$

$kx = k_1x + k_2x,\quad k = k_1 + k_2$

(b) $F = F_1 = F_2$

$x_1 = \frac{F_1}{k_1},\ x_2 = \frac{F_2}{k_2},\ x = \frac{F}{k}$

$x = x_1 + x_2;\quad \frac{F}{k} = \frac{F_1}{k_1} + \frac{F_2}{k_2};\quad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

9/3 $M = I\ddot\theta,\ -\frac{JG}{L}\theta = I\ddot\theta,\ \ddot\theta + \frac{JG}{IL}\theta = 0,$

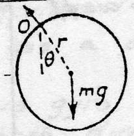
$\tau = 2\pi/p = 2\pi\sqrt{\frac{IL}{JG}}$; for $I = \frac{1}{12}ml^2$, $\tau = \pi l\sqrt{\frac{mL}{3JG}}$

9/4 From Prob. 9/3 $\tau = 2\pi\sqrt{\frac{IL}{JG}}$

so $\frac{\tau_1^2}{\tau_2^2} = \frac{IL/JG}{(I + 2mr^2)L/JG} = \frac{I}{I + 2mr^2}$

Solve for I & get $I = \dfrac{2mr^2}{(\tau_2/\tau_1)^2 - 1}$

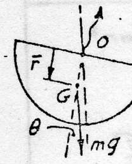
9/5 $\Sigma M_0 = I_0\ddot\theta;\ -mgr\sin\theta = 2mr^2\ddot\theta$



For θ small, $\ddot\theta + \frac{g}{2r}\theta = 0$

$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{2r}{g}}$

9/6 $\Sigma M_0 = I_0\ddot\theta;\ -mg\frac{4r}{3\pi}\sin\theta = \frac{1}{2}mr^2\ddot\theta$



$F = \frac{4r}{3\pi}$

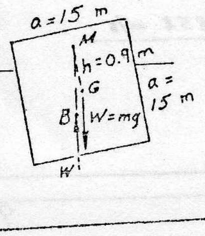
$I_0 = \frac{1}{2}mr^2$

For small θ, $\sin\theta \approx \theta$ &

$\ddot\theta + \frac{8g}{3\pi r}\theta = 0,\ p^2 = \frac{8g}{3\pi r}$

$f = \frac{p}{2\pi} = \frac{1}{\pi}\sqrt{\frac{2g}{3\pi r}}$

9/7 $\bar{I} = \frac{1}{12}m(a^2 + a^2) = \frac{1}{6}ma^2$



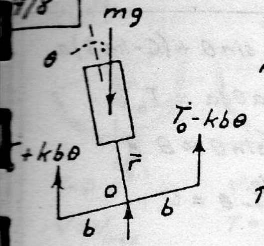
$\Sigma\bar{M} = \bar{I}\ddot\theta;\ -mgh\theta = \frac{1}{6}ma^2\ddot\theta$

where θ=

so $\ddot\theta + \frac{6gh}{a^2}\theta = 0,\ p = \frac{1}{a}\sqrt{6gh}$

$\tau = \frac{2\pi}{p} = \frac{2\pi a}{\sqrt{6gh}} = \frac{2\pi(15)}{\sqrt{6(9.81)(0.9)}} = 12$

9/8



$\Sigma M_O = I_O \alpha$; for θ small

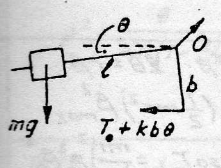
$$mg\bar r\theta + (T_o - kb\theta)b - (T_o + kb\theta)b = I\ddot\theta$$

or $\ddot\theta + \frac{2kb^2 - mg\bar r}{I}\theta = 0$

$$\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{I}{2kb^2 - mg\bar r}}$$

Note: instability if $k < \frac{W\bar r}{2b^2}$

9/9 For equil. with $\theta = 0$, $T_o b = mgl$, $T_o = \frac{mgl}{b}$



$$\Sigma M_O = I_O\ddot\theta$$

$$mgl\cos\theta - (mg\tfrac{l}{b} + kb\theta)b\cos\theta = ml^2\ddot\theta$$

$\ddot\theta + \frac{kb^2}{ml^2}\theta = 0$ for $\cos\theta \approx 1$

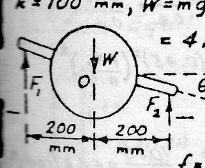
$p^2 = \frac{kb^2}{ml^2}$ so $f = \frac{p}{2\pi} = \frac{1}{2\pi}\frac{b}{l}\sqrt{\frac{k}{m}}$

9/10 From Prob. 9/3, the period for each rotor is proportional to $\sqrt{Il}$, so with $\tau_1 = \tau_2$, there results $I_1 l_1 = I_2 l_2$. Combination with $l = l_1 + l_2$ gives $l_1 = \frac{I_2}{I_1 + I_2} l$, $l_2 = \frac{I_1}{I_1 + I_2} l$. Thus for I_1 or I_2 the period from Prob. 9/3 is

$$\tau = 2\pi\sqrt{\frac{I_1 I_2}{I_1 + I_2}\frac{l}{JG}}$$

9/11 $F_1 = 422/2 - 2k(0.2\theta) = 211 - 0.4k\theta$; $F_2 = 211 + 0.4k\theta$

$\bar k = 100$ mm, $W = mg = 43(9.81) = 422$ N



$\Sigma M_O = I_O\ddot\theta$;

$$(211 - 0.4k\theta)0.2 - (211 + 0.4k\theta)0.2 = 43(0.1)^2\ddot\theta$$

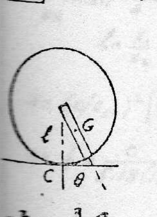
$$\ddot\theta + \frac{0.16}{0.43}k\theta = 0$$

$f = \frac{360}{60} = 6$ Hz; $f = \frac{p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{0.16}{0.43}k}$

$k = \frac{0.43}{0.16}(12\pi)^2 = 3820$ N/m

9/12 $V = 0$ for $\theta = 0$; $V_{\theta_o} = V_{max} = mg\frac{l}{2}(1 - \cos\theta_o)$

$$= mg\frac{l}{2}\left(1 - \left[1 - \frac{\theta_o^2}{2} + \dots\right]\right)$$

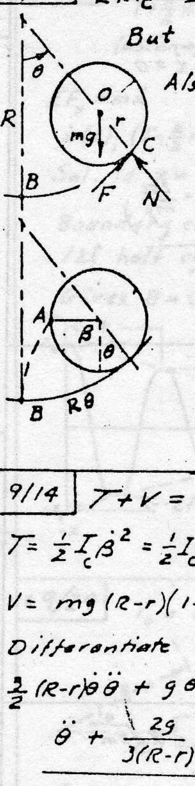


$T_{max_{\theta=0}} = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\frac{1}{3}ml^2(\theta_o p)^2$, $\omega = \theta_o p$

$V_{max} = T_{max}$; $\frac{mgl\theta_o^2}{4} = \frac{ml^2\theta_o^2 p^2}{6}$

$p^2 = \frac{3}{2}\frac{g}{l}$, $\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{2l}{3g}}$

9/13 $\Sigma M_C = I_C\alpha$; $-mgr\sin\theta = I_C\ddot\beta$



But $R\theta = r(\beta + \theta)$ or $\beta = \frac{R-r}{r}\theta$

Also $\sin\theta \approx \theta$, $I_C = \frac{3}{2}mr^2$

so $-mgr\theta = \frac{3}{2}mr^2\frac{R-r}{r}\ddot\theta$

$$\ddot\theta + \frac{2g}{3(R-r)}\theta = 0$$

Thus $\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{3(R-r)}{2g}}$

9/14 $T + V = E$, const. ; $\beta = \frac{R-r}{r}\theta$

$$T = \frac{1}{2}I_C\dot\beta^2 = \frac{1}{2}I_C\left(\frac{R-r}{r}\right)^2\dot\theta^2 = \frac{3}{4}m(R-r)^2\dot\theta^2$$

$V = mg(R-r)(1-\cos\theta)$; so $\frac{3}{4}(R-r)\dot\theta^2 + g(1-\cos\theta) = \frac{E}{m(R-r)}$

Differentiate with time t get

$\frac{3}{2}(R-r)\dot\theta\ddot\theta + g\dot\theta\sin\theta = 0$; $\dot\theta \neq 0$ & for θ small,

$\ddot\theta + \frac{2g}{3(R-r)}\theta = 0$ $f = p/2\pi = \frac{1}{2\pi}\sqrt{\frac{2g}{3(R-r)}}$

9/15 With $m = 0$, Eq. 178 becomes $\dot x = -\frac{k}{c}x$

or $\int_{x_o}^{x}\frac{dx}{x} = \int_0^t -\frac{k}{c}dt$; $\ln\frac{x}{x_o} = -\frac{k}{c}t$, $x = x_o e^{-k/c\, t}$

9/16 $x_1 = 4.65$ mm, $x_2 = 4.30$ mm, $m = 1.10$ kg

$\ln\frac{x_1}{x_2} = b\tau = \frac{c}{2m}\frac{1}{f}$, $c = 2mf\ln\frac{x_1}{x_2}$

Thus $c = 2(1.10)(10)\ln\frac{4.65}{4.30} = 1.722$ N·s/m

9/17 For critical damping $c = c_{cr} = 2\sqrt{km}$

& $b = c/2m = \sqrt{k/m}$

So $x = e^{-bt}(A_1 + A_2 t)$; $x = x_o$ & $\dot x = 0$ when $t = 0$

which gives $A_1 = x_o$, $A_2 = bx_o = \sqrt{k/m}\,x_o$

Thus $x_1 = x_o e^{-\sqrt{\frac{k}{m}}t_1}\left(1 + \sqrt{\frac{k}{m}}t_1\right)$ for $x = x_1$, $t = t_1$

9/18 $E = (T+V)_1 - (T+V)_2$

But at x_1 & x_2, $T_1 = T_2 = 0$ since $\dot{x} = 0$

Thus $E = V_1 - V_2 = \frac{1}{2}k(x_1^2 - x_2^2)$

But $x_1/x_2 = e^{b\tau}$

so $E = \frac{1}{2}kx_1^2(1 - e^{-2b\tau}) = \frac{1}{2}kx_1^2(1 - e^{-\frac{2\pi c}{mg}})$

where $g = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$

9/19 $\frac{x_7}{x_{19}} = \frac{e^{-bt_1}}{e^{-b(t_1+12\tau)}}$ where $12 = 19 - 7$

$\ln\frac{x_7}{x_{19}} = 12b\tau = 12\frac{2\pi\eta}{\sqrt{1-\eta^2}}$ where $\eta = c/c_{cr}$

Given $\frac{x_7}{x_{19}} = 30$, so $\ln 30 = 3.4012$

$3.4012 = \frac{24\pi\eta}{\sqrt{1-\eta^2}}$, $\left(\frac{24\pi}{3.4012}\right)^2 = \frac{1}{\eta^2} - 1$

$\eta = c/c_{cr} = \underline{0.0451}$

9/20 From Eq. 180 with $b = c/2m$, $c_{cr} = 2\sqrt{km}$

$\frac{c}{2m}\tau = 2\pi\frac{c/c_{cr}}{\sqrt{1-(c/c_{cr})^2}}$, $\sqrt{1-(c/c_{cr})^2} = \frac{4\pi m}{c_{cr}\tau}$

Solve for c & get $c = 2\sqrt{km}\sqrt{1 - \frac{4\pi^2 m}{k\tau^2}}$

$c = 2\sqrt{850(1)}\sqrt{1 - \frac{4\pi^2(1)}{850(0.32)^2}} = \underline{43.1 \text{ N·s/m}}$

$c_{cr} = 2\sqrt{850(1)} = \underline{58.3 \text{ N·s/m}}$

9/21 For the half cycle from $x = 0.30$ mm to $x = -0.15$ mm,

$\ln\frac{|x_1|}{|x_2|} = \frac{b\tau}{2} = \frac{\pi\delta}{\sqrt{1-\delta^2}}$ From Eq. 180; $b = \frac{c}{2m}$, $\eta = \frac{c}{2\sqrt{km}}$

$\frac{0.300}{0.150} = \frac{0.150}{h}$, $\underline{h = 0.075 \text{ mm}}$

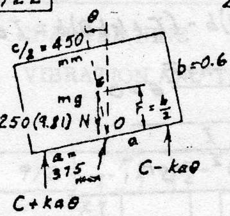
$\ln\frac{0.300}{0.150} = 0.693 = \frac{c}{4(4)}(3.8 - 3.2)2$; $\underline{c = 9.24 \text{ N·s/m}}$

$0.693 = \frac{\pi\eta}{\sqrt{1-\eta^2}}$, $\eta = 0.215 = \frac{9.24}{2\sqrt{k(4)}}$

Solve for k & get $\underline{k = 115.0 \text{ N/m}}$

9/22 $\Sigma M_o = I_o\ddot{\theta}$; $mg\frac{b}{2}\sin\theta + (C - ka\theta)a$

$-(C + ka\theta)a = I_o\ddot{\theta}$



For small θ, $\sin\theta \approx \theta$ &

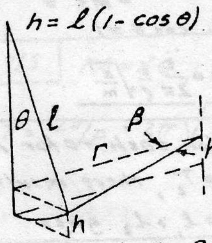
$\ddot{\theta} + \frac{2ka^2 - mgb/2}{I_o}\theta = 0$

Coeff. of θ must be (+) for harmonic oscillation.

Thus $k_{min} = \frac{mgb}{4a^2} = \frac{250(9.81)(0.6)}{4(0.375)^2} = \underline{2616 \text{ N/m}}$

9/23 $l\sin\theta = 2r\sin\frac{\beta}{2}$ & for small β, $l\theta = r\beta$

$h = l(1 - \cos\theta) \approx l\theta^2/2$; $T_{max} = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\dot{\beta}^2_{max}$



But $\dot{\beta}_{max} = p\beta_o$, so $T_{max} = \frac{mr^2}{4}p^2\beta_o^2$

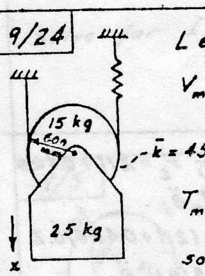
Also $V_{max} = mgh = \frac{mgl\theta_o^2}{2} = \frac{mgr^2\beta_o^2}{2l}$

$V_{max} = T_{max}$; $\frac{mgr^2\beta_o^2}{2l} = \frac{mr^2p^2\beta_o^2}{4}$

& $p^2 = \frac{2g}{l}$, $\tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{l}{2g}} = \pi\sqrt{2l/g}$

independent of r

9/24 Let $V = 0$ when $x = 0$; x_o = max displacement



$V_{max} = V_g + V_e = -40(9.81)x_o + 20(9.81)(2x_o)$

$+ \frac{1}{2}(540)(2x_o)^2$

$= 1080\,x_o^2$

$T_{max} = \Sigma\frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\Omega^2$, $\Omega = \frac{v}{0.6} = \frac{px_o}{0.6}$

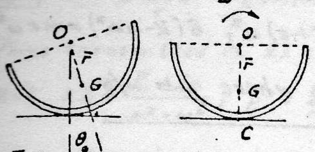
so $T_{max} = \frac{1}{2}40p^2x_o^2 + \frac{1}{2}15(0.45)^2\left(\frac{px_o}{0.6}\right)^2$

$= 24.2\,p^2x_o^2$

$T_{max} = V_{max}$; $24.2p^2x_o^2 = 1080x_o^2$, $p^2 = \frac{1080}{24.2} = 44.6\left(\frac{rad}{s}\right)^2$

$f = \frac{p}{2\pi} = \frac{6.68}{2\pi} = \underline{1.063 \text{ Hz}}$

9/25



$T=0$ | $V=0$

$V=V_{max}$ | $T=T_{max}$

$V_{max} = mg\bar{r}(1-\cos\theta_0)$
$\quad = mg\frac{2r}{\pi}(1-\cos\theta_0)$

For small angles,
$\cos\theta_0 \approx 1-\theta_0^2/2$
& $V_{max} = mgr\,\theta_0^2/\pi$

$$T_{max} = \tfrac{1}{2}I_C\omega^2 = \tfrac{1}{2}(I_G+m[r-\bar{r}]^2)\omega^2 = \tfrac{1}{2}(I_0-m\bar{r}^2+m(r-\bar{r})^2)\omega^2$$
$$= mr^2\left(1-\frac{2}{\pi}\right)\theta_0^2p^2 \text{ where } \omega=\theta_0 p$$
$$T_{max} = V_{max};\ mr^2\left(1-\frac{2}{\pi}\right)\theta_0^2p^2 = mgr\frac{\theta_0^2}{\pi}$$
$$p^2 = \frac{g}{\pi r(1-2/\pi)} \ \&\ \tau = \frac{2\pi}{p} = 2\pi\sqrt{\frac{(\pi-2)r}{g}}$$

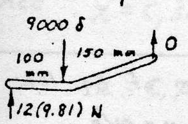
9/26

Let y_0 = max. vertical deflection (up) above neutral position, m

δ = spring deflection in neutral position, m

$$(T_{max})_{y=0} = \Delta V,\quad \Delta V = \Delta V_g + \Delta V_e$$
$$= 24(9.81)y_0 + 2\frac{9000}{2}\left(\delta-\frac{150}{250}y_0\right)^2 - 2\tfrac{1}{2}9000\,\delta^2$$
$$= 235y_0 + 3240y_0^2 - 10\,800\,y_0\,\delta$$

But from equil. of a lower member,



$\Sigma M_0 = 0;\ 9000\,\delta(150) - 12(9.81)(250) = 0,\ \delta = 21.8$ mm

so $\Delta V = 235y_0 + 3240y_0^2 - 235y_0 = 3240y_0^2$

& $T_{max} = \frac{1}{2}24(\dot{y}_{max})^2 = 12\dot{y}_{max}^2$

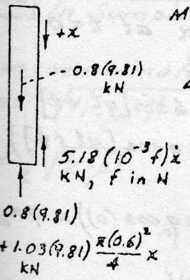
But $\dot{y}_{max} = y_0 p$ (p = circular frequency)

so $12y_0^2p^2 = 3240y_0^2$, $p=\sqrt{270} = 16.43$ rad/s

$f = \frac{p}{2\pi} = \underline{2.62\ \text{Hz}}$

9/27

Equil. $0.8(9.81) = 1.03(9.81)\frac{\pi(0.6)^2}{4}h$, h = equil. submerged depth = 2.75 m



Mean submerged area of vertical surface is
$A = \pi(0.6)(2.75) = 5.18\ \text{m}^2$

$$\Sigma F_x = m\ddot{x};\ -1.03(9.81)\frac{\pi(0.6)^2}{4}x - 5.18f\dot{x}(10^{-3}) = 0.8\ddot{x}$$

Damping coefficient is $c = 5.18(10^{-3})f$

Critical damping coefficient is

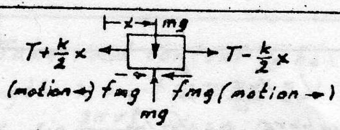
$$c_{cr} = 2\sqrt{km} = 2\sqrt{\frac{1.03(9.81)\pi(0.6)^2}{4}0.8} = 3.02\ \text{kN·s/m}$$

$$\ln\frac{x_1}{x_2} = 2\pi\frac{c/c_{cr}}{\sqrt{1-(c/c_{cr})^2}};\ \ln\frac{0.3}{0.3-0.1} = 0.4055$$

so $(c/c_{cr})^2\left[\left(\frac{2\pi}{0.4055}\right)^2+1\right] = 1$, $(c/c_{cr})^2 = \frac{1}{241}$, $c = \frac{3.02}{\sqrt{241}} = 0.1947$ kN·s/m

so $f = \frac{c}{5.18(10^{-3})} = \frac{0.1947}{0.00518} = \underline{37.6\ \text{N·s/m}^3}$

9/28



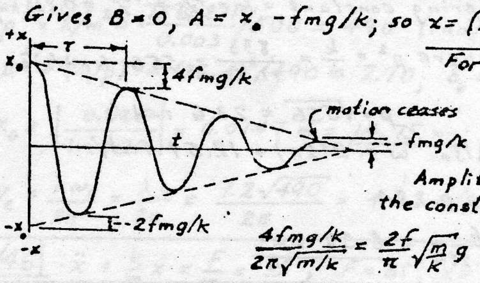
$\Sigma F_x = m\ddot{x}$

$\dot{x}(\pm);\ (T-\frac{k}{2}x)-(T+\frac{k}{2}x) \mp fmg = m\ddot{x};\ \ \ddot{x}+\frac{k}{m}x = \mp fg$

Sol. is $x = A\cos pt + B\sin pt \mp \frac{fmg}{k}$, $p=\sqrt{\frac{k}{m}}$

Boundary conditions, 1st half cycle: $\dot{x}=0$ when $t=0$; $x=x_0$ " " ; (+) sign for fmg/k

Gives $B=0$, $A = x_0 - fmg/k$; so $x = \left(x_0-\frac{fmg}{k}\right)\cos pt + \frac{fmg}{k}$ For 1st half cycle



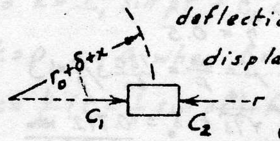
$\tau = 2\pi\sqrt{\frac{m}{k}}$

Amplitude decreases at the constant time rate of

$$\frac{4fmg/k}{2\pi\sqrt{m/k}} = \frac{2f}{\pi}\sqrt{\frac{m}{k}}\,g$$

9/29

r_0 = radial distance at rest; δ = steady-state deflection at speed ω; x = added displacement due to vibration.



C = compression for rest condition

For steady state, $\Sigma F_r = ma_r$
$C_1 - C_2 = ma_r$;
$(C-\frac{k}{2}\delta)-(C+\frac{k}{2}\delta) = -m(r_0+\delta)\omega^2$
$k\delta = m(r_0+\delta)\omega^2$ -------- (a)

During vibration $\Sigma F_r = ma_r$; $C_1 - C_2 = m(\ddot{r}-r\omega^2)$

$$[C-\tfrac{k}{2}(\delta+x)]-[C+\tfrac{k}{2}(\delta+x)] = m\left[\frac{d^2}{dt^2}(r_0+\delta+x)-(r_0+\delta+x)\omega^2\right]$$

$-k\delta - kx = m\ddot{x} - m(r_0+\delta)\omega^2 - mx\omega^2$ since r_0 & δ are constant

Substitute (a) & get $\ddot{x}+\left(\frac{k}{m}-\omega^2\right)x = 0$

So $f = \frac{p}{2\pi} = \underline{\frac{1}{2\pi}\sqrt{\frac{k}{m}-\omega^2}}$ For $\omega^2 = \frac{k}{m}$, $\ddot{x}=0$ & $\dot{x}$ = const

For $\omega^2 > \frac{k}{m}$, $\ddot{x} - [\ ]x = 0$ & solution is hyperbolic function

9/31

From Fig. 90 for $\eta = 0.2$ & $x_0/\delta_0 = 2$,

$\mu < 0.78$ or $\mu > 1.11$ where $\mu = f/f_0$, $f_0 = 4$ Hz

Thus $f < [0.78(4) = 3.12]$ Hz

$f > [1.11(4) = 4.44]$ Hz

9/32

$\omega = 2\pi f = 2\pi(5) = 31.42$ rad/s

$p^2 = k/m = \frac{3600}{0.25} = 14\,400\ (\text{rad/s})^2$

$p = 120.0$ rad/s, $\omega/p = 0.262$; hence it is small & instrument acts like an accelerometer

& $a_0 \approx x_0p^2 = 0.0052(14\,400) = 74.9\ \text{m/s}^2 = \underline{7.6\,g}$

9/33 $\ddot{x} + \frac{k}{m}x = 0, \quad J = \dddot{x} = -\frac{k}{m}\dot{x}$

But $x = x_0 \sin pt$ where $p = \sqrt{k/m}$

so $J = -\frac{k}{m}x_0\sqrt{\frac{k}{m}}\cos pt, \quad J_0 = -\left(\frac{k}{m}\right)^{3/2}x_0$

$\underline{|x_0| = \left(\frac{m}{k}\right)^{3/2}J_0}$

9/34 Equiv. spring constant $= k = P/\delta = 10/12 = 0.833$ kN/m

$x_0 = \frac{\delta_0}{1-(\omega/p)^2}$ where $p^2 = \frac{k}{m} = \frac{833}{1.5} = 556\ (\text{rad/s})^2$

$p = \sqrt{556} = 23.6$ rad/s

Also $\omega = 2(2\pi) = 12.57$ rad/s

$(\omega/p)^2 = \frac{16\pi^2}{556} = 0.284$

Thus $x_0 = \frac{6}{1-0.284} = \underline{8.38 \text{ mm}}$

9/35 For seismic instrument, Sample Prob. 9/30 gives $\frac{x_0}{\delta_0} = \frac{\mu^2}{\sqrt{(1-\mu^2)^2+(2\eta\mu)^2}}$, $x_0 = 0.75$ mm

$\eta = 0.5$

$\mu = \frac{\omega}{p} = \frac{180}{60}\,3/3 = 3$

So $\frac{0.75}{\delta_0} = \frac{3^2}{\sqrt{(1-3^2)^2+(2[0.5]3)^2}} = \frac{9}{\sqrt{73}}, \quad \delta_0 = \underline{0.712 \text{ mm}}$

9/36 $\frac{x_0}{\delta_0} = \frac{1}{1-(\omega/p)^2}$ where $\omega = 2\pi f$

$p^2 = k/m = \frac{4(7.2[10^3])}{43} = 670\ (\text{rad/s})^2$

$|x_0/\delta_0| = \frac{0.15}{0.10} = 1.5$

$\left(\frac{\omega}{p}\right)^2 = \frac{4\pi^2 f^2}{670} = 0.0589 f^2$

For $\omega < p$, $1.5 = \frac{1}{1-0.0589f^2}$, $f = 2.38$ Hz

For $\omega > p$, $-1.5 = \frac{1}{1-0.0589f^2}$, $f = 5.32$ Hz

Prohibited range $\underline{2.38 < f < 5.32 \text{ Hz}}$

9/37 With negligible damping,

$x_0/\delta_0 = \frac{1}{|1-(\omega/p)^2|}$ & $p^2 = k/m = k/24$, k in N/m

where $x_0 = 0.30$ mm, $\delta_0 = 0.60$ mm

$\omega = 2\pi f = 2\pi(4) = 8\pi$ rad/s

For $x_0/\delta_0 = 1/2 < 1$, $1-(\omega/p)^2$ is negative

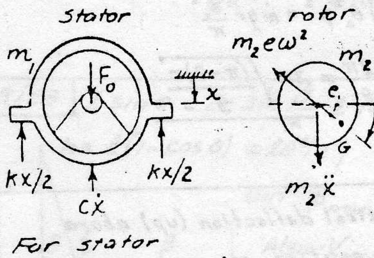
Thus $1 - \frac{(8\pi)^2}{k/24} = -2; \quad \underline{k = 5050 \text{ N/m}}$

9/38 $F_n = m\bar{a}_n; \quad k\delta = m(\delta+e)\omega^2; \quad \delta(k-m\omega^2) = me\omega^2$

$\delta = \frac{\frac{m}{k}e\omega^2}{1-\frac{m}{k}\omega^2} = \underline{e\frac{\mu^2}{1-\mu^2} \text{ where } \mu = \omega/p}$

$\underline{\omega_c = p = \sqrt{k/m}}$

9/39 For simplicity assume rotation about vertical axis



F_0 = reaction to vertical components of forces on rotor $= m_2(e\omega^2 \sin\omega t - \ddot{x})$

For stator

$\Sigma F_x = m\ddot{x}; \quad -c\dot{x} - kx + m_2(e\omega^2\sin\omega t - \ddot{x}) = m_1\ddot{x}$

so $\ddot{x} + \frac{c}{m_1+m_2}\dot{x} + \frac{k}{m_1+m_2}x = \frac{m_2 e\omega^2}{m_1+m_2}\sin\omega t$

which is identical with Eq. 181a when m_1+m_2 is used for m.

9/40 From the steady-state term of Eq. 182 & $\delta_0 = F_0/k = m'e\omega^2/k$ & $p^2 = k/m$, $\mu = \omega/p$

$\frac{x_0}{\delta_0} = \frac{x_0 m p^2}{m'e\omega^2} = \frac{m}{m'}\frac{x_0}{e}\frac{1}{\mu^2} = \frac{1}{\sqrt{(1-\mu^2)^2+(2\eta\mu)^2}}$

Thus $\frac{mx_0}{m'e} = \frac{\mu^2}{\sqrt{(1-\mu^2)^2+(2\eta\mu)^2}}$ same as Sample Prob. 9/30

9/41 $\int F\,dt = I = m\Delta v, \quad I = m\dot{x}$ at $t=0$

After impulse Eq. 179 holds &

$x = x_0 e^{-bt}\sin(qt+\phi), \quad \dot{x} = x_0 e^{-bt}(-b\sin[qt+\phi] + q\cos[qt+\phi])$

$t=0, \ x=0; \quad 0 = x_0 \sin\phi, \quad \phi = 0$

" $\quad \dot{x} = I/m, \quad I/m = x_0(-b\sin(0) + q\cos(0)) = qx_0$

so $x_0 = \frac{I}{mq}$ & $\underline{x = \frac{I}{mq}e^{-bt}\sin qt}$

where $b = c/2m$

$q = \sqrt{k/m - (c/2m)^2}$

9/42 $F_o = 2m_oe\omega^2 = 2(1)(0.012)(1800\,\frac{2\pi}{60})^2 = 853$ N

Force transmitted is $kx = 1500$ N

But $kx = \frac{F_o\,k/m}{p^2-\omega^2}$, $p^2 = \frac{k}{m}$ so $\frac{kx}{F_o} = \frac{1}{1-(\frac{\omega}{p})^2}$

Thus $\frac{1500}{853} = \left|\frac{1}{1-(\frac{\omega}{p})^2}\right|$ or $\left|1-(\frac{\omega}{p})^2\right| = 0.568$

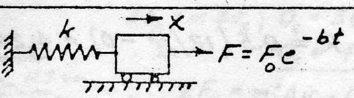
$(\frac{\omega}{p})^2 = 1.568$ or 0.432, $p^2 = \frac{\omega^2}{1.568}$ or $p^2 = \frac{\omega^2}{0.432}$

$\omega^2 = \left[1800\,\frac{2\pi}{60}\right]^2 = 35\,500\ (\text{rad/s})^2$

$p^2 = \frac{35\,500}{1.568} = 22\,650$, $p^2 = \frac{35\,500}{0.432} = 82\,340$

$k = p^2 m = p^2(10) = \underline{226.5\text{ kN/m}}$ or $\underline{823.4\text{ kN/m}}$

9/43



Diff. eq. is $\ddot{x} + \frac{k}{m}x = \frac{F_o}{m}e^{-bt}$; $x = x_c + x_p$

$x_c = C_1 \sin pt + C_2 \cos pt$ where $p^2 = k/m$

Try $x_p = Be^{-bt}$, substitution gives

$Bb^2e^{-bt} + \frac{k}{m}Be^{-bt} = \frac{F_o}{m}e^{-bt}$, so $B = \frac{F_o}{mb^2+k}$

Hence $x = C_1 \sin pt + C_2 \cos pt + \frac{F_o}{mb^2+k}e^{-bt}$

$\dot{x} = C_1 p \cos pt - C_2 p \sin pt - \frac{F_o b}{mb^2+k}e^{-bt}$

$x = 0$ & $\dot{x} = 0$ when $t = 0$; give $C_2 = -\frac{F_o}{mb^2+k}$, $C_1 = \frac{F_o b/p}{mb^2+k}$

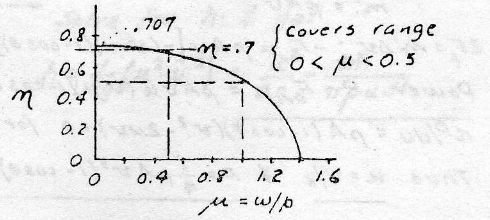
& $x = \frac{F_o}{mb^2+k}\left[\frac{b}{p}\sin pt - \cos pt + e^{-bt}\right]$

9/44 In Sample Prob. 9/30 $x_o = \frac{1}{p^2}|\ddot{\delta}|_{max}$

when denominator in expression for x_o/δ_o is unity

Thus $\sqrt{(1-\mu^2)^2 + (2\eta\mu)^2} = 1$ which gives

$\eta = \frac{1}{2}\sqrt{2-\mu^2}$



9/45 Take road contour to be $x = x_o \sin \omega t$

Wavelength $\lambda = vT = v\frac{2\pi}{\omega}$ so $\omega = 2\pi v/\lambda$

& $x = x_o \sin\frac{2\pi v t}{\lambda}$. Equivalent system is that of Fig. 84 b & Eq. 175 b with $c = 0$.

From Eq. 182 with $p = \sqrt{k/m}$, $\mu = \omega/p$, $F_o \equiv k\delta_o$

steady-state portion yields $x_o = \frac{\delta_o}{1-\mu^2}$

$\omega = 2\pi\,\frac{25}{3.6}/1.2 = 36.4$ rad/s

$p^2 = k/m = \frac{75(9.81)}{0.003}/500 = 490\ (\text{rad/s})^2$

$\mu^2 = (\omega/p)^2 = (36.4)^2/490 = 2.70$; $\delta_o = \frac{0.050}{2} = 0.025$ m

$x_o = \left|\frac{0.025}{1-2.70}\right| = 0.0147\text{ m} = \underline{14.75\text{ mm}}$

$v_c = \frac{\lambda\omega_c}{2\pi} = \frac{\lambda p}{2\pi} = \frac{1.2\sqrt{490}}{2\pi} = 4.23\text{ m/s} = \underline{15.23\text{ km/h}}$

9/46 $\ddot{x} + \frac{k}{m}x = \frac{F}{m} = \frac{bt}{m}$, $F = bt$, $x = x_c + x_p$

$x_c = C_1 \sin pt + C_2 \cos pt$; try $x_p = At$ & get $A = b/k$

so $x = C_1 \sin pt + C_2 \cos pt + \frac{bt}{k}$

$\dot{x} = 0$, $x = 0$ when $t = 0$ gives $C_1 = -\frac{b}{kp}$, $C_2 = 0$

$x = \frac{b}{k}(t - \frac{1}{p}\sin pt)$ for $0 < t < \frac{3}{4}$ s; $p = \sqrt{k/m} = \sqrt{90/0.75}$
$= 10.95$ rad/s

$x = \frac{6.25}{3/4}\,\frac{1}{90}\left(t - \frac{1}{10.95}\sin 10.95t\right)$ m

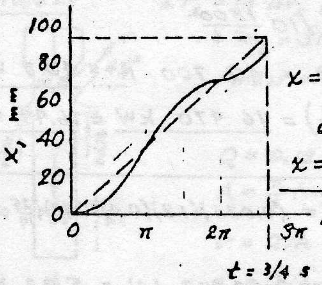
where $b = \frac{6.25}{3/4} = 8.33$ N/s

so

$x = 0.0926\left(t - \frac{1}{10.95}\sin 10.95t\right)$ m

or

$x = 92.6\left(t - \frac{1}{10.95}\sin 10.95t\right)$ mm



9/47 Energy loss during motion dx is

$dE = (c\dot{x})\,dx = c\dot{x}^2 dt$

& over one complete cycle is $E = c\int_0^{2\pi/\omega}\dot{x}^2 dt$

where $x = \frac{F_o/k}{\sqrt{(1-\mu^2)^2 + (2\eta\mu)^2}}\sin(\omega t - \phi)$

Let $x = A\sin(\omega t - \phi)$ so $\dot{x} = A\omega\cos(\omega t - \phi)$

$E = cA^2\omega^2\int_0^{2\pi/\omega}\cos^2(\omega t - \phi)\,dt = cA^2\omega\left[\frac{\omega t - \phi}{2} - \frac{1}{4}\sin 2(\omega t - \phi)\right]_0^{2\pi/\omega}$

$= cA^2\omega\,[\pi]$

Power $P = $ (Energy loss per cycle)(cycles per sec.)

$= E\frac{\omega}{2\pi} = cA^2\omega^2/2$

or $P = \dfrac{F_o^2 c\omega^2/(2k^2)}{(1-\mu^2)^2 + (2\eta\mu)^2}$

CHAPTER TEN

DYNAMICS OF NONRIGID SYSTEMS

10/4 Thrust $T = m'(u - v)$

$= 30(40 - 10(1.852)/3.6) = 1046$ N

For constant hull speed $\underline{R = T = 1046 \text{ N}}$

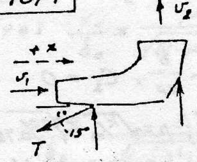
10/5 Vertical thrust $= m'\Delta v_y = 2(60+1)(600)$

$= 73\,200$ N

$\Sigma F_y = ma_y;\; 73.2 - 7(9.81) = 7a,\; \underline{a = 0.647 \text{ m/s}^2}$

10/6 $\Sigma M = m'(v_2 d_2 - v_1 d_1),\; M = mr\omega(r) = \underline{mr^2\omega}$

10/7



$\Sigma F_x = m'\Delta v_x$

$-T\cos 15° = (43 + 0.8)(0 - 720)$

$T = 32\,600 \text{ N} = \underline{32.6 \text{ kN}}$

10/8 Thrust $T = m_g' u - m_a' v = D$, drag

$T = D = (110 + 0.97)\,780 - 110\,\dfrac{1500}{3.6}$

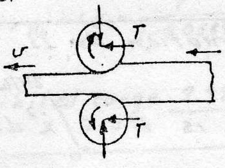
$= 86\,600 - 45\,800 = 40\,700 \text{ N} = \underline{40.7 \text{ kN}}$

$P = Tv,\; P = 40.7\left(\dfrac{1500}{3.6}\right) = \underline{16\,970 \text{ kW}} = \underline{16.97 \text{ MW}}$

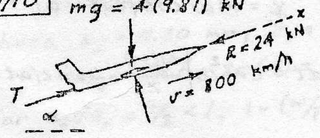
10/9 $\Sigma F = m'\Delta v;\; 2T = (0.025)(1.2)(0.4)(7.83)\left(\frac{25}{19}0.4 - 0.4\right)$

$\rho_{steel} = 7.83$ t/m³

$T = 0.00593 \text{ kN} = \underline{5.93 \text{ N}}$



10/10 $mg = 4(9.81)$ kN



$T = m_a'(u - v) + m_f' u$

$= 90(600 - 800/3.6) + 0.9(600)$

$= 34\,500$ N

$\Sigma F_x = 0;\; 34.5 - 24 - 4(9.81)\sin\alpha = 0$

$\sin\alpha = 0.2686,\; \underline{\alpha = 15.58°}$

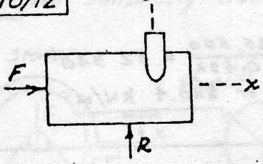
10/11 $kx = m'\Delta v,\; 15(150) = 1000\,\dfrac{\pi(0.030)^2}{4}\,v^2$

$\underline{v = 56.4 \text{ m/s}}$

$\Sigma M_A = m'vd;\; M = 15(150)(15\sin 75° - 4.8\cos 75°)$

$= 2250(13.25) = \underline{29\,800 \text{ N·m}}$

10/12



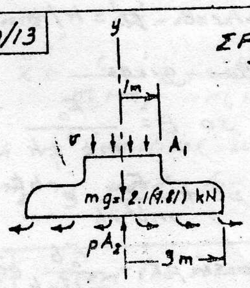
$m' = 30/60 = 0.5$ t/s

$\Sigma F_x = m'\Delta v_x$

$F = 0.5(15/3.6 - 0) = \underline{2.08 \text{ kN}}$

$\Sigma F_y = m'\Delta v_y;\; R = 0.5(12/\sqrt{2} - 0) = \underline{4.24 \text{ kN}}$

10/13



$\Sigma F_y = m'\Delta v_y;\; m' = \rho A_1 v$

$\Delta v_y = 0 - (-v) = v$

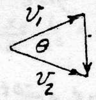
so $pA_2 - mg = \rho A_1 v^2$

$p(\pi 3^2) - 2.1(9.81)$

$= 1.217(10^{-3})(\pi 1^2)(45)^2$

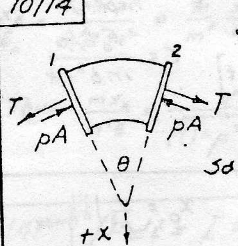
$\underline{p = 1.002 \text{ kPa}}$

10/14



$\therefore \Delta v_x = 2v\sin\frac{\theta}{2};\; m' = \rho A v$

$\Sigma F_x = m'\Delta v_x$ (entire system)



$2T\sin\frac{\theta}{2} - 2pA\sin\frac{\theta}{2} = \rho A v\left(2v\sin\frac{\theta}{2}\right)$

So $T - pA = \rho A v^2,\; \underline{T = A(p + \rho v^2)}$

10/15 $\Delta v_x = -(v - u)(1 - \cos\theta)$ See Sample Prob. 10/1

$m' = \rho A v$

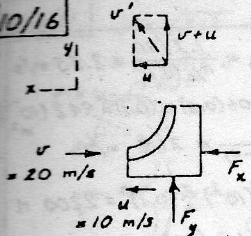
$\Sigma F_x = m'\Delta v_x;\; -F_{av} = \rho A v\left[-(v-u)(1-\cos\theta)\right]$

Power $P = F_{av}\, u = \rho A v u (v - u)(1 - \cos\theta)$

$dP/du = \rho A(1 - \cos\theta)(v^2 - 2uv) = 0$ for max. P

Thus $\underline{u = v/2}$ & $\underline{P = \frac{1}{4}\rho A v^3 (1 - \cos\theta)}$

10/16



Final velocity of diverted water is $\underline{v}' = \underline{u} + \underline{v}_{rel}$

where $v_{rel} = v + u$

$m' = \rho A v_{rel}$

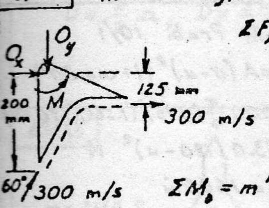
$= 1000 \frac{\pi}{4}(0.025)^2(20+10)$

$= 14.73$ kg/s

$\Sigma F_x = m'\Delta v_x$; $F_x = 14.73(10 - [-20]) = \underline{442 \text{ N}}$

$\Sigma F_y = m'\Delta v_y$; $F_y = 14.73(30 - 0) = \underline{442 \text{ N}}$

10/17 $m' = 3$ kg/s



$\Sigma F_x = m'\Delta v_x$; $O_x = 3(300 - 300 \cos 60°)$

$\underline{O_x = 450 \text{ N}}$

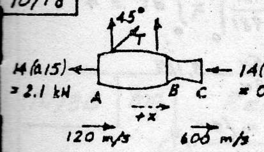
$\Sigma F_y = m'\Delta v_y$; $O_y = 3(0 - [-300 \sin 60°])$

$\underline{O_y = 779 \text{ N}}$

$\Sigma M_o = m'(v_2 d_2 - v_1 d_1)$; $M = 3(300[0.125] - 300 \frac{0.200}{2})$

$\underline{M = 22.5 \text{ N·m}}$

10/18

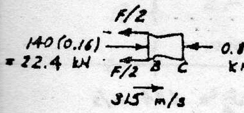


$\Sigma F_x = m'\Delta v_x$

$-2.1 - 0.84 + T \cos 45°$

$= [30.6(600) - 30(120)](10^{-3})$

$\underline{T = 25.0 \text{ kN}}$



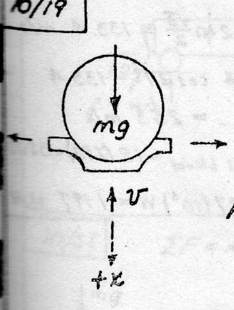
$\Sigma F_x = m'\Delta v_x$

$22.4 - 0.84 - F$

$= 30.6(600 - 315)(10^{-3})$

$\underline{F = 12.84 \text{ kN}}$

10/19



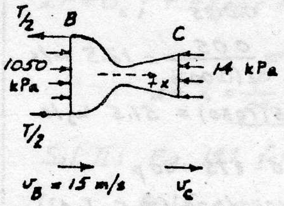
$\Sigma F_x = m'\Delta v_x$

$mg = \rho \frac{\pi d^2}{4} u (0 - [-v])$; But for water with energy conserved, $u^2 - v^2 = 2gh$

Solve for h & get

$h = \frac{1}{2g}(u^2 - v^2) = \underline{\frac{u^2}{2g} - \frac{2m}{\pi \rho u d^2}}$

10/20 $A_C = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$, $A_B = 4A_C = 0.0314 \text{ m}^2$



$m' = \rho A v$

$m'_B = 13.5(0.0314)(15)$

$= 6.36$ kg/s

& at C, $6.36 = 1.217(0.00785)v_C$

$v_C = 666$ m/s

$\Sigma F_x = m'\Delta v_x$; $-T + 1050(0.0314) - 14(0.00785)$

$= 6.36(666 - 15)(10^{-3})$

$\underline{T = 28.7 \text{ kN}}$

10/21 $v = 250/3.6 = 69.4$ m/s

With reversers, $T_R = m'_g u \sin 20° + m'_a v$

so $T_R = (40 + 0.6)(600)(0.3420) + 40(69.4)$

$= 8330 + 2780 = 11\,100$ N

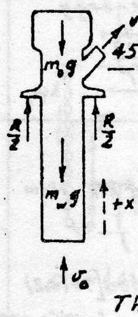
Without reversers, $T = m'_g u - m'_a v$

$T = (40 + 0.6)(600) - 40(69.4)$

$= 24\,400 - 2780 = 21\,600$ N

$n = \frac{T_R}{T} = \frac{11\,100}{21\,600} = \underline{0.515}$

10/22 $\Sigma F_x = m'\Delta v_x$, $R - m_o g - m_w g = \rho Q(v \cos 45° - v_o)$



$m_o = 310$ kg

mass of water $m_w = \rho V = 1000 \frac{\pi}{4}(0.2)^2(6)$

$= 188.5$ kg

$Q = 0.125 \text{ m}^3/\text{s}$

$A = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$, $A_o = \frac{\pi}{4}(0.25)^2 = 0.0491 \text{ m}^2$

$v = Q/A = 0.125/0.00785 = 15.92$ m/s

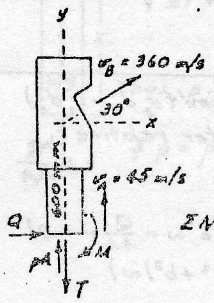
$v_o = Q/A_o = 0.125/0.0491 = 2.55$ m/s

Thus $R - (310 + 188.5)9.81 = 1000(0.125)(15.92 \cos 45° - 2.55)$

$= 1088$ N

$\underline{R = 5980 \text{ N}}$

10/23 $m' = 6$ kg/s; $pA = 1400(10^3)(7500)(10^{-6}) = 10\,500$ N



$\Sigma F_x = m'\Delta v_x$; $Q = 6(360 \frac{\sqrt{3}}{2} - 0)$

$\underline{Q = 1871 \text{ N}}$

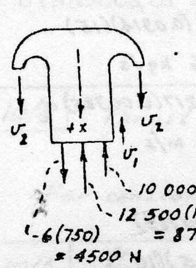
$\Sigma F_y = m'\Delta v_y$; $10\,500 - T = 6(360 \frac{1}{2} - 45)$

$\underline{T = 9690 \text{ N}}$

$\Sigma M_A = m'(v_2 d_2 - v_1 d_1)$; $M = 6(360 \frac{\sqrt{3}}{2} 0.6 - 0)$

$\underline{M = 1122 \text{ N·m}}$

10/24 $Q = 0.05\ m^3/s$; $v_1 = \dfrac{Q}{A_1} = \dfrac{0.05}{0.0125} = 4\ m/s$

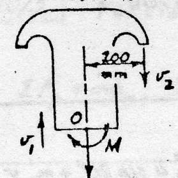


$v_2 = \dfrac{Q}{A_2} = \dfrac{0.05}{2(0.002)} = 12.5\ m/s$

$m' = 0.05(1030) = 51.5\ kg/s$

$\Sigma F_x = m'\Delta v_x$; $4500 - 875 - 10p = 1030(0.05)(12.5 - [-4])$

$\underline{p = 278\ kPa}$



For left side blanked off & with $Q = 0.05/2 = 0.025\ m^3/s$,

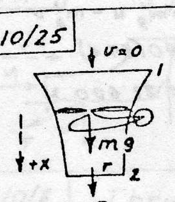
$v_2 = Q/A_2 = 0.025/0.002 = 12.5\ m/s$,

$m' = 51.5/2 = 25.8\ kg/s$

$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$

$M = 25.8(12.5)0.2 = \underline{64.4\ N\cdot m}$

10/25



mg = weight of helicopter = force on air stream

For system between sections 1 & 2,

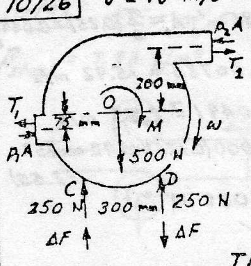
$\Sigma F_x = m'\Delta v_x$; $mg = \rho\pi r^2 v(v - 0)$

$v = \dfrac{1}{r}\sqrt{\dfrac{mg}{\pi\rho}}$

Power = rate of increase of kinetic energy

$P = \frac{1}{2}m'v^2 = m'v\dfrac{v}{2} = mg\dfrac{v}{2} = \underline{\dfrac{mg}{2r}\sqrt{\dfrac{mg}{\pi\rho}}}$

10/26 $v = 18\ m/s$



Power $P = M\omega$;

$M = \dfrac{40\,000}{900(2\pi/60)} = 424\ N\cdot m$

$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$

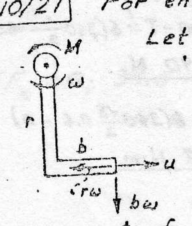
$424 + 0.3\Delta F = (20/60)(1000)[18(0.2) - (-18)(0.075)]$

$\Delta F = \dfrac{1650 - 424}{0.3} = 4090\ N$

Thus $C = 250 + 4090 = \underline{4340\ N\ up}$

$D = 4090 - 250 = \underline{3840\ N\ down}$

10/27 For entire system $\Sigma M = m'(v_2 d_2 - v_1 d_1)$



components of absolute velocity of water

Let u = velocity of water relative to nozzle $= \dfrac{Q}{4A}$

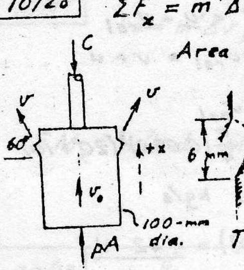
$m' = \rho Q$

$-M = \rho Q(r^2\omega + b^2\omega - \dfrac{Q}{4A}r - 0)$

$M = \rho Q(\dfrac{Qr}{4A} - (r^2 + b^2)\omega)$

For $M = 0$, $\omega_0 = \underline{\dfrac{Qr}{4A(r^2 + b^2)}}$

10/28 $\Sigma F_x = m'\Delta v_x$; $Q = A_0 v_0$; $v_0 = \dfrac{1.2/60}{\pi/4\,(0.1)^2} = 2.55\ m/s$



Area of discharge $0.006(0.5)\pi(0.1) = 942(10^{-6})\ m^2$

$v = \dfrac{1.2/60}{942(10^{-6})} = 21.2\ m/s$

$pA = 280(10^3)\frac{\pi}{4}(0.1)^2 = 2200\ N$

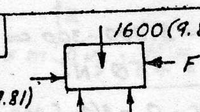
$m' = 1000(1.2/60) = 20\ kg/s$

Thus $2200 - C = 20(21.2\sin 60° - 2.55)$

$C = 2200 - 317 = 1882\ N$

& from levers, $\dfrac{25(9.81)(75 + b)}{75} = \dfrac{1882}{3}$, $\underline{b = 116.9\ mm}$

▸10/29



$160(9.81) = 1570\ N$

From Prob. 10/1

$F = \rho A(v - u)^2(1 + \cos 30°)$

$= 1000\frac{\pi}{4}(0.15)^2(1.866)(180 - u)^2$

$= 33.0(180 - u)^2\ N$

$\Sigma F = m\dot{u}$; $33.0(180 - u)^2 - 1570 = 1600\dot{u}$

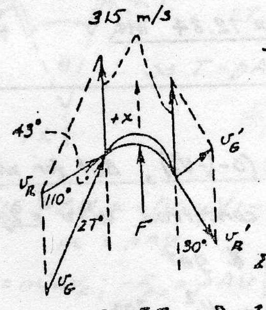
or $\displaystyle\int_0^u \frac{du}{0.0206(180 - u)^2 - 0.981} = \int_0^t dt$; integrate & get

$\dfrac{1}{2\sqrt{0.0206(0.981)}} \ln \dfrac{0.1422(180 - u) + 0.981}{0.1422(180 - u) - 0.981}\Bigg|_0^u = \displaystyle\int_0^4 dt = 4$

$3.52 \ln \dfrac{1 - 0.00535u}{1 - 0.00578u} = 4$, $\dfrac{1 - 0.00535u}{1 - 0.00578u} = 3.119$

Solve for u & get $\underline{u = 167.3\ m/s}$

▸10/30 Entrance: $\dfrac{v_G}{315} = \dfrac{\sin 110°}{\sin 43°}$, $v_G = 434\ m/s$



$\dfrac{v_R}{315} = \dfrac{\sin 27°}{\sin 43°}$, $v_R = 210\ m/s$

Exit: $v_R' = v_R$ (negligible frict

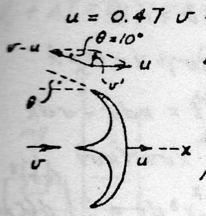
$v_{G_x}' = 315 - 210\frac{\sqrt{3}}{2} = 133.4\ m/s$

$|\Delta v_x| = 434\cos 27° - 133.4 = 253\ m/s$

$\Sigma F_x = m'\Delta v_x$; $\Sigma F_{all\ blades} = 15(253) = 38$

$P = \Sigma F v$; $P = 3800(315) = 1.197(10^6)\ W = \underline{1.197\ M}$

10/31



$v = \sqrt{2gh} = \sqrt{2(9.81)(300)} = 76.7$ m/s

$u = 0.47v = 36.1$ m/s

$\Delta v_x = [u-(v-u)\cos\theta] - v = -(v-u)(1+\cos\theta)$

$= -(76.7-36.1)(1+0.985) = -80.7$ m/s

Average tangential thrust per jet is

$F_x = m'\Delta v_x = 1000(76.7A)|-80.7| = 6.19(10^6)A$ N

where A = jet area, m^2

Theor. power $P = 6F_x u = 6(6.19)(10^6)A(36.1) = 1.340A(10^9)$ W

$\dfrac{\text{Actual power}}{\text{Theor. power}} = \dfrac{22(10^6)}{1.340A(10^9)} = 0.90(0.85)$, $A = 0.0215$ m^2

Thus $\pi d^2/4 = 0.0215$, $d = 0.1653$ m $= \underline{165.3 \text{ mm}}$.

$u = \dfrac{D}{2}\Omega$, $D = \dfrac{2u}{\Omega} = \dfrac{2(36.1)}{270(2\pi/60)} = \underline{2.55 \text{ m}}$

10/35

$m = 3160$ t

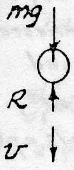
$T = m'u$; $5(7500) = \dfrac{2280}{160}u$

$\underline{u = 2630 \text{ m/s}}$

$\Sigma F = ma$; $5(7500) - 3160(9.81) = 3160a$

$\underline{a = 2.06 \text{ m/s}^2}$

10/36



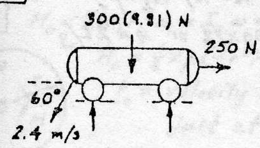
Relative velocity of attachment of mass is $u = v$. Thus with

$\Sigma F = m\dot{v} + \dot{m}u$

$mg - R = m\dot{v} + \dot{m}v = \dfrac{d}{dt}(mv)$

so $\Sigma F = \dfrac{d}{dt}(mv)$

10/37



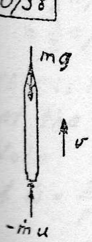
In x-dir. $\Sigma F_x = m\dot{v} + \dot{m}u$

$250 = 300a - 20(2.4\cos 60°)$

$a = \dfrac{250+24}{300}$

$\underline{a = 0.913 \text{ m/s}^2}$

10/38



$\Sigma F = ma$; $-\dot{m}u - mg = ma$

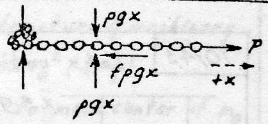
$-\dfrac{dm}{dt}u = m(a+g)$

$\displaystyle\int_{m_0}^{m} -\frac{dm}{m} = \int_0^t \frac{a+g}{u}\,dt$

$\ln\dfrac{m}{m_0} = -\dfrac{a+g}{u}t$, $\underline{m = m_0 e^{-\frac{a+g}{u}t}}$

10/39

Sol. I: entire chain



$\Sigma F_x = \dot{G}_x$; $P - f\rho g x = \dfrac{d}{dt}(0 + \rho x\dot{x})$

$= \rho(\dot{x}^2 + x\ddot{x})$

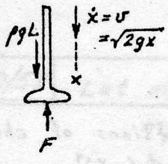
$\underline{a = \ddot{x} = \dfrac{P}{\rho x} - fg - \dfrac{\dot{x}^2}{x}}$

Sol. II: Eq. 191 for moving portion; $\Sigma F = m\dot{v} + \dot{m}u$

$P - f\rho g x = \rho x a + \rho\dot{x}\dot{x}$ where $u = \dot{x}$

$\underline{a = \dfrac{P}{\rho x} - fg - \dfrac{\dot{x}^2}{x}}$

10/40



$G_x = \rho(L-x)\sqrt{2gx} = \rho\sqrt{2g}\,(Lx^{1/2} - x^{3/2})$

$\Sigma F_x = \dot{G}_x$ for entire system

$\rho g L - F = \rho\sqrt{2g}\left(\frac{1}{2}Lx^{-1/2} - \frac{3}{2}x^{1/2}\right)\dot{x}$

$= \rho\sqrt{2g}\left(\frac{1}{2}L\sqrt{2g} - \frac{3}{2}\sqrt{2g}\,x\right)$

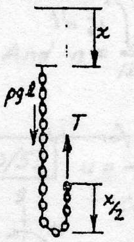
$= \rho g L - 3\rho g x$, $\underline{F = 3\rho g x}$

Alternatively,

$R = m'\Delta v = \rho\dot{x}(\dot{x}) = \rho(2gx) = 2\rho g x$

$\Sigma F_x = 0$; $F = \rho g x + 2\rho g x = 3\rho g x$

10/41



Take entire chain as system of constant mass

$\Sigma F_x = \dot{G}_x$; $\rho g l - T = \dfrac{d}{dt}\left(\rho\left[l - \frac{x}{2}\right]\dot{x} + 0\right)$

$= \rho\left(l - \frac{x}{2}\right)\ddot{x} - \rho\dfrac{\dot{x}^2}{2}$

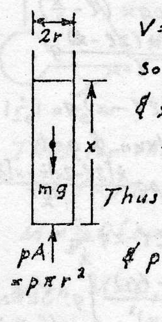
But $\ddot{x} = g$ & $\dot{x}^2 = 2gx$ so that

$\rho g l - T = \rho g l - \rho g\frac{x}{2} - \rho g x$

& $\underline{T = \dfrac{3\rho g x}{2}}$

$\Delta E = |\Delta V_g| = \rho g l\left(\frac{l}{2} + \frac{l}{2}\right) = \underline{\rho g l^2}$ energy loss

10/42



$\Sigma F_x = \dot{G}_x$; $pA - \rho g V = \dfrac{d}{dt}\left(\rho V\dfrac{\dot{x}}{2}\right) = \dfrac{\rho V}{2}\ddot{x}$

$V = \pi r^2 x$, $\dot{V} = \pi(2r\dot{r}x + r^2\dot{x}) = 0$

So $\dot{x}r = 2cx$ where $\dot{r} = -c$, constant

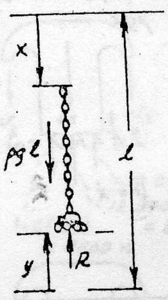
& $\ddot{x}r + \dot{x}\dot{r} = 2c\dot{x}$ or $\ddot{x} = \dfrac{3c\dot{x}}{r} = \dfrac{6c^2x}{r^2}$

$= \dfrac{6\pi c^2x^2}{V}$

Thus $p\dfrac{V}{x} - \rho g V = \dfrac{\rho V}{2}\,\dfrac{6\pi c^2x^2}{V}$

& $p = \rho g x + \dfrac{3\pi\rho c^2x^3}{V} = \underline{\rho g x\left(1 + \dfrac{3\pi c^2x^2}{gV}\right)}$

10/43 Take entire chain as system (constant mass)



$\ddot{x} = g$, $\ddot{y} = a$ $\qquad G_x = \rho(\ell - x - y)\dot{x}$

$\dot{x} = gt$, $\dot{y} = at$ $\qquad\quad - \rho(x+y)\dot{y}$

$x = \frac{1}{2}gt^2$, $y = \frac{1}{2}at^2$

$\Sigma F_x = \dot{G}_x$; $\rho g\ell - R = \rho[(-\dot{x}-\dot{y})\dot{x} + (\ell - x - y)\ddot{x}$

$\qquad - (\dot{x}+\dot{y})\dot{y} + (x+y)\ddot{y}]$

$\rho g \ell - R = \rho[-(\dot{x}+\dot{y})^2 - (x+y)(\ddot{x}+\ddot{y}) + \ell\ddot{x}]$

$= \rho[-(a+g)^2t^2 - (a+g)\frac{t^2}{2}(a+g) + \ell g]$

$= -\rho(a+g)^2t^2\frac{3}{2} + \rho g \ell$

So $\underline{R = \frac{3}{2}\rho(a+g)^2t^2}$

◄10/44 For airplane plus moving portions of chains

$\Sigma F = 0 = m\dot{v} + \dot{m}u = (m + 2\rho\frac{x}{2})\dot{v} + [2\frac{d}{dt}(\rho\frac{x}{2})]v$

$-(m+\rho x)dv = \rho v\,dx$ so $-\int_{v_0}^{v}\frac{dv}{v} = \int_0^x \frac{\rho\,dx}{m+\rho x}$

or $\ln\frac{v}{v_0} = \ln\frac{m}{m+\rho x}$, $v = v_0\frac{m}{m+\rho x} = \frac{v_0}{1+\rho x/m}$

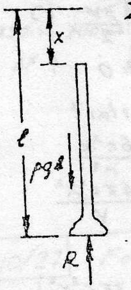
& for $x = 2L$, $\underline{v = \dfrac{v_0}{1 + \frac{2\rho L}{m}}}$

Also, $v = \dot{x} = \dfrac{v_0}{1+\frac{\rho x}{m}}$ so $\int_0^x\left(1 + \frac{\rho x}{m}\right)dx = \int_0^t v_0\,dt$

$v_0 t = x\left(1 + \frac{\rho x}{2m}\right)$ or

$\underline{x = \frac{m}{\rho}\left[\sqrt{1 + \frac{2v_0 t\rho}{m}} - 1\right]}$ for $+x$

◄10/45 For entire chain or rope of constant mass



$\Sigma F_x = \dot{G}_x$; $\rho g \ell - R = \frac{d}{dt}(\rho[\ell - x]\dot{x} + 0)$

$\Delta T + \Delta V = 0$; $\frac{1}{2}\rho(\ell - x)\dot{x}^2 = \rho g x(\ell - \frac{x}{2})$

$\dot{x}^2 = \frac{x(2\ell - x)}{\ell - x}g$

or $\dot{x}(\ell - x) = \frac{x}{\dot{x}}(2\ell - x)g$

Substitute into ΣF equation & get

$\rho g\ell - R = \rho g\frac{d}{dt}\left(\frac{x[2\ell - x]}{\dot{x}}\right) = \rho g\frac{\dot{x}[2\ell - 2x]\dot{x} - x[2\ell - x]\ddot{x}}{\dot{x}^2}$

$= 2\rho g(\ell - x) - \rho g x(2\ell - x)\frac{\ddot{x}}{\dot{x}^2}$

Differentiate $\dot{x}^2$ & get $\ddot{x} = \left[1 + \frac{x(\ell - x/2)}{(\ell - x)^2}\right]g$. Substitute

& get $\rho g\ell - R = 2\rho g(\ell - x) - \rho g x(2\ell - x)\dfrac{\left[1 + \frac{x(\ell - x/2)}{(\ell - x)^2}\right]g}{\frac{x(2\ell - x)}{\ell - x}g}$

Simplify, solve for R, & get

$\underline{R = \rho g x\frac{4\ell - 3x}{2(\ell - x)}}$; (Less than R are Prob. 10/43 for $x < \frac{2\ell}{3}$)

◄10/46 For falling part $\Sigma F = m\dot{v} + \dot{m}u$

where $\Sigma F = \rho g x$, $m = \rho x$, $\dot{m} = \rho v$, $u = v = \dot{x}$

Thus $\rho g x = \rho x\dot{v} + \rho v\dot{x}$, $gx\,dt = x\,dv + v\,dx$

or $gx\,dt = d(xv)$; $gx^2v\,dt = xv\,d(xv)$

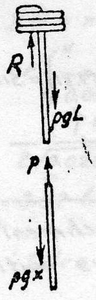
or $gx^2dx = \frac{1}{2}d([xv]^2)$, so $g\int_0^x x^2dx = \frac{1}{2}\int_0^{(xv)^2}d([xv]^2)$

$\frac{gx^3}{3} = \frac{1}{2}(xv)^2$, $\underline{v = \sqrt{\frac{2gx}{3}}}$

$a = \dot{v} = \sqrt{\frac{2g}{3}}\frac{1}{2}x^{-1/2}\dot{x} = \sqrt{\frac{2g}{3}}\frac{1}{2\sqrt{x}}\sqrt{\frac{2gx}{3}} = \underline{\frac{g}{3}}$ constant

$\Delta E = -\Delta V_g - \Delta T = +\frac{\rho g L^2}{2} - \frac{\rho L}{2}v^2_{x=L} = \frac{\rho g L^2}{2} - \frac{\rho g L^2}{3} = \underline{\frac{\rho g L^2}{6}}$

◄10/47 Entire system is conservative so $\Delta V_g + \Delta T = 0$



$-\rho g x\frac{x}{2} + \frac{1}{2}\rho x v^2 = 0$, $\underline{v = \sqrt{gx}}$, $a = \dot{v} = g/2$

For entire system

$\Sigma F_x = \dot{G}_x$; $\rho g L - R = \frac{d}{dt}(\rho x v) = \frac{d}{dt}(\rho\sqrt{g}\,x^{3/2}) = \frac{3}{2}\rho g x$

Thus $R = \rho g L - \frac{3}{2}\rho g x$, $\underline{R = \rho g(L - \frac{3}{2}x)}$

Explanation of $R = 0$ for $x = 2L/3$:

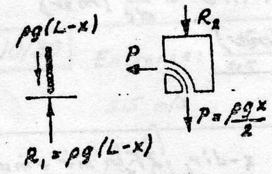
For vertical section $\Sigma F_x = ma_x$; $\rho g x - P = \rho x\frac{g}{2}$,

$P = \rho g x/2$. For idealized flow considered, a frictionless guide must be introduced

$\Sigma F_x = m'\Delta v_x$; $\rho g x/2 + R_2 = \rho v^2$,

$R_2 = \rho v^2 - \rho g x/2 = \rho(gx) - \rho g x/2$

$= \rho g x/2$



R = total force exerted on rope by platform & guide & is

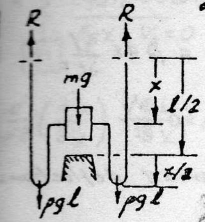
$R = R_1 - R_2 = \rho g(L - x) - \rho g x/2$

so $R = 0$ when $\rho g(L - x) = \rho g x/2$ or $x = 2L/3$

10/48

System is conservative during fall

$\Delta V_g + \Delta T = 0;\ \underbrace{-mgx - 2\rho g x(\ell/2 + x/4) + \frac{1}{2}m\dot{x}^2 + 2\frac{1}{2}\rho\left(\frac{\ell}{2} - x + \frac{x}{2}\right)\dot{x}^2}_{=0}$



or $[m + \rho(\ell - x)]\dot{x}^2 = 2g[mx + \rho x(\ell - \frac{x}{2})]$

Now for system $\Sigma F_x = \dot{G}_x$

$2\rho g\ell + mg - 2R = \frac{d}{dt}\left[m\dot{x} + 2\rho\left(\frac{\ell}{2} - x + \frac{x}{2}\right)\dot{x}\right]$

$= \frac{d}{dt}[(m + \rho[\ell - x])\dot{x}]$

Substitute 1st Eq. into 2nd & get

$2\rho g\ell + mg - 2R = 2\frac{d}{dt}\left[\frac{mgx + \rho g x(\ell - x/2)}{\dot{x}}\right]$

Upon simplification, $= 2[mg + \rho g(\ell - x)] - [m + \rho(\ell - x)]\ddot{x}$

or $mg + 2R - 2\rho g x = [m + \rho(\ell - x)]\ddot{x}$

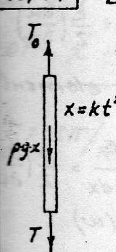
Take $\frac{d}{dt}(\dot{x}^2)$ from 1st Eq. & get $\ddot{x} = g\left[1 + \rho x \frac{m + \rho(\ell - \frac{x}{2})}{[m + \rho(\ell - x)]^2}\right]$

which shows that $\ddot{x} > g$.

Finally, substitute $\ddot{x}$ & get $R = \frac{\rho g}{2}\left\{(\ell - x) + x\frac{m + \rho(\ell - x/2)}{m + \rho(\ell - x)}\right\}$

10/49

Eq. 195; $\Sigma \underline{F} = m\ddot{\underline{r}}_e - \dot{m}\underline{u} - \frac{d^2}{dt^2}(m\underline{\rho}_e)$ for x-direction becomes



$\Sigma F_x = m\ddot{x} + \dot{m}\dot{x} - \frac{d^2}{dt^2}(m\frac{x}{2})$

where $m = \rho x = \rho k t^2$; $\underline{u}$ becomes $-\dot{x}$

Thus $\Sigma F_x = T + \rho g x - T_0$

$m\ddot{x} = \rho x(2k);\ \dot{m}\dot{x} = 2\rho k t(2kt) = 4\rho k^2 t^2$

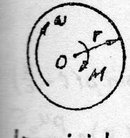
$\frac{d^2}{dt^2}(m\frac{x}{2}) = \frac{d^2}{dt^2}(\frac{1}{2}\rho k^2 t^4) = 6\rho k^2 t^2$

Substitute & get $T + \rho g x - T_0 = 2\rho k^2 t^2 + 4\rho k^2 t^2 - 6\rho k^2 t^2$

so $\underline{T + \rho g x - T_0 = 0}$

10/50

$H_0 = \Sigma m_i r_i^2 \omega = I_0\omega = \frac{1}{2}\pi\rho r^4\omega$; ρ = mass per unit disk area



$\dot{H}_0 = \frac{1}{2}\pi\rho(4r^3\dot{r}\omega + r^4\dot{\omega})$

$\dot{\underline{r}}_0$ = velocity of mass center of leaving particles just at separation. Magnitude is $r\omega$

$|\underline{r}_e| = r$

$|\underline{r}_e \times \dot{m}\dot{\underline{r}}_0| = \dot{m}r^2\omega$ C.W.; $\dot{m} = \frac{d}{dt}(\rho\pi r^2) = 2\rho\pi r\dot{r}$

so from Eq. 197, $\Sigma \underline{M}_0 = \dot{\underline{H}}_0 - \underline{r}_e \times \dot{m}\dot{\underline{r}}_0$

$M = \frac{1}{2}\pi\rho(4r^3\dot{r}\omega + r^4\dot{\omega}) - 2\rho\pi r\dot{r}(r^2\omega)$

$= \frac{1}{2}\pi\rho r^4\dot{\omega}$ so $\underline{\dot{\omega} = \frac{2M}{\rho\pi r^4}}$

{This is identical with application of $\Sigma M = I\dot{\omega}$ at the instant under consideration}

Note: Eq. $M_0 = \dot{H}_0$ gives $M = \frac{1}{2}\pi\rho(4r^3\dot{r}\omega + r^4\dot{\omega})$ & $\dot{\omega} = \frac{2M}{\rho\pi r^4} + \frac{4c\omega}{r}$ which is <u>wrong</u> since $\Sigma M_0 = \dot{H}_0$ is invalid for $m = f(t)$

10/51

Let m_0 = mass of rocket structure & machinery

m_f = mass of grain $= \rho\pi r_0^2(\ell - y)$

h_0 = distance from nozzle to mass center of m_0

Correction term $C = \frac{d^2}{dt^2}(mh) = \frac{d^2}{dt^2}\left(m_0 h_0 + m_f \frac{\ell + y}{2}\right)$

so $C = 0 + \frac{d^2}{dt^2}\left(\rho\pi r_0^2 \frac{[\ell - y][\ell + y]}{2}\right) = -\rho\pi r_0^2[\dot{y}^2 + y\ddot{y}]$

But $T = \dot{m}U = U\frac{d}{dt}(\rho\pi r_0^2[\ell - y]) = -U\rho\pi r_0^2\dot{y}$

so $n = \left|\frac{C}{T}\right| = \frac{\rho\pi r_0^2(\dot{y}^2 + y\ddot{y})}{U\rho\pi r_0^2\dot{y}} = \underline{\frac{y}{U}\left(\frac{\dot{y}}{y} + \frac{\ddot{y}}{\dot{y}}\right)}$

And $n = \frac{1.2}{1200}\left(\frac{0.050}{1.2} + \frac{0.150}{0.050}\right) = \underline{0.00304}$

10/52

Let m_0 = mass of rocket structure & machinery

m_f = mass of grain $= \rho\pi(r_0^2 - r^2)L$

h_0 = distance from nozzle to mass center of m_0

h_f = distance from nozzle to mass center of m_f

Correction term $C = \frac{d^2}{dt^2}(mh) = \frac{d^2}{dt^2}(m_0 h_0 + m_f h_f)$

so $C = 0 + \frac{d^2}{dt^2}(\rho\pi[r_0^2 - r^2]L h_f) = -2\rho\pi L h_f(\dot{r}^2 + r\ddot{r})$

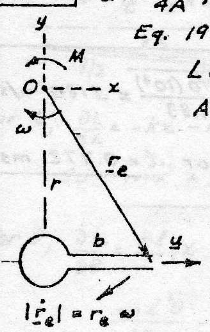
But $T = \dot{m}U = U\frac{d}{dt}(\rho\pi[r_0^2 - r^2]L) = -2\rho\pi L r\dot{r}U$

so $n = \left|\frac{C}{T}\right| = \frac{2\rho\pi L h_f(\dot{r}^2 + r\ddot{r})}{2\rho\pi L r\dot{r}U} = \underline{\frac{h_f}{U}\left(\frac{\dot{r}}{r} + \frac{\ddot{r}}{\dot{r}}\right)}$

And $n = \frac{2.4}{1200}\left(\frac{0.050}{0.025} + \frac{0.150}{0.050}\right) = \underline{0.01}$

10/53

$u = \frac{Q}{4A}$, $\dot{m} = -\rho Q$



Eq. 197, $\Sigma \underline{M}_0 = \dot{\underline{H}}_0 - \underline{r}_e \times \dot{m}\dot{\underline{r}}_0$

Let m = mass of water in tanks

Angular momentum about O due to m is $\underline{H}_{O_m} = mr^2\omega(-\underline{k})$

Pipes are always filled so their angular momentum is constant.

Thus $\dot{\underline{H}}_0 = \dot{m}r^2\omega(-\underline{k}) = \rho Q r^2\omega\underline{k}$

$\underline{r}_e \times \dot{m}\dot{\underline{r}}_0 = \underline{r}_e \times \dot{m}(\dot{\underline{r}}_e + \underline{u}) = \dot{m}(r_e^2\omega - ru)(-\underline{k})$ C.W.

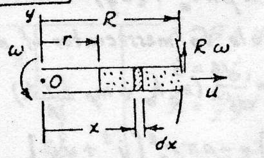
Thus for the clockwise sense $(-\underline{k})$

$-M = -\rho Q r^2\omega - \dot{m}(r_e^2\omega - ru)$

$M = \rho Q(ru - b^2\omega) = \underline{\rho Q\left(\frac{Qr}{4A} - b^2\omega\right)}$

With $M = 0$, $\underline{\omega = \omega_0 = \frac{Qr}{4Ab^2}}$

▲10/54 For 4 tubes $dH_O' = 4(\rho A\,dx)(x\omega)(x) = 4\rho A x^2 \omega\, dx$



$H_O' = 4\rho A \omega \int_r^R x^2 dx = \frac{4\rho A\omega}{3}(R^3 - r^3)$

so total $H_O = H_O' + I_O\omega$

$4\underline{r}_e \times \dot m \underline{\dot r}_o = 4R(R\omega)(\dot m)(\underline{k})$

$= 4R^2\omega(-\dot r\rho A)\underline{k}$

$\Sigma \underline{M}_O = \underline{\dot H}_O - \underline{r}_e \times \dot m \underline{\dot r}_o$

$0 = \frac{4}{3}\rho A\left[\dot\omega(R^3 - r^3) - 3\omega r^2 \dot r\right] + I_O\dot\omega$

$+ 4\rho A \dot r R^2 \omega$

A = cross-sectional area of tube
ρ = density of liquid
I_O = moment of inertia of disk & empty tubes

Simplify & get

$$\left[(R^3 - r^3) + \frac{3}{4\rho A}I_O\right]\dot\omega = 3\omega\dot r(r^2 - R^2)$$

or $$\int_{\omega_0}^{\omega}\frac{d\omega}{\omega} = 3\int_0^R \frac{r^2 - R^2}{(R^3 - r^3) + \frac{3I_O}{4\rho A}}dr$$

Now neglect mass of disk & hollow tubes

so $I_O \to 0$

$$\int_{\omega_0}^{\omega}\frac{d\omega}{\omega} = 3\int_0^R \frac{-(R+r)}{R^2 + Rr + r^2}dr = -3\int_0^1 \frac{1+x}{1+x+x^2}dx \quad \text{where } x = \frac{r}{R}$$

$$= -3\left[\frac{1}{\sqrt3}\tan^{-1}\frac{2x+1}{\sqrt3} + \frac12\ln(1+x+x^2)\right]_0^1$$

$$= -3\left[\frac{1}{\sqrt3}\tan^{-1}\frac{1}{\sqrt3} + \frac12 \ln 3\right] = -3\left[\frac{\pi}{6\sqrt3} + \frac{1.099}{2}\right] = -2.555$$

Thus $\ln\frac{\omega}{\omega_0} = -2.555$, $\omega = \omega_0 e^{-2.555}$, $\omega = \frac{\omega_0}{12.87}$

$\underline{\omega = 0.0777\,\omega_0}$

10/56 $2l = ct$ where $c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{210(10^6)}{7.83}} = 5180$ m/s

Thus $t = \frac{2(2)}{5180} = 0.772(10^{-3})$ s or $\underline{t = 0.772 \text{ ms}}$

10/57 $l = ct$, where $c = \sqrt{\frac{T}{\rho'}} = \sqrt{\frac{T}{\rho A}}$, ρ' = mass/length

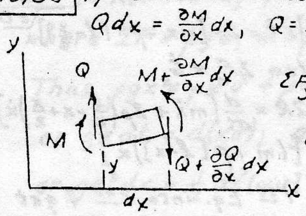
ρ = density

A = cross-sectional area

$\sqrt{\frac{T}{7830\frac{\pi}{4}(0.006)^2}} = \frac{l}{t} = \frac{30}{0.3}$, $\underline{T = 2210 \text{ N}}$

10/58 If rotational acceleration is neglected,

$Q\,dx = \frac{\partial M}{\partial x}dx$, $Q = \frac{\partial M}{\partial x}$



$\Sigma F_y = m\ddot y$; $-\frac{\partial Q}{\partial x}dx = \rho\,dx\frac{\partial^2 y}{\partial t^2}$

or $-\frac{\partial}{\partial x}\left(\frac{\partial M}{\partial x}\right) = \rho\frac{\partial^2 y}{\partial t^2}$

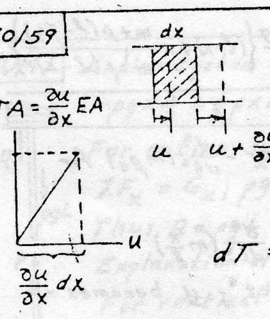
But for small deflections

$EI\frac{\partial^2 y}{\partial x^2} = M$

Thus $-\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) = \rho\frac{\partial^2 y}{\partial t^2}$

Or $\underline{\frac{\partial^4 y}{\partial x^4} + \frac{\rho}{EI}\frac{\partial^2 y}{\partial t^2} = 0}$

10/59 dV_e = elastic energy for element of volume A dx



$\sigma A = \frac{\partial u}{\partial x}EA$

$= \frac12\sigma A\frac{\partial u}{\partial x}dx$

$V_e = \frac{dV_e}{A\,dx} = \frac{\sigma}{2}\frac{\partial u}{\partial x} = \frac{E}{2}\left(\frac{\partial u}{\partial x}\right)^2$

dT = kinetic energy for element of volume A dx

$= \frac12\rho A\,dx\left(\frac{\partial u}{\partial t}\right)^2$, $T = \frac{dT}{A\,dx} = \frac{\rho}{2}\left(\frac{\partial u}{\partial t}\right)^2$

Now let $w = x - ct$ so sol. is $u = f(w)$

$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial w}\frac{\partial w}{\partial x} = \frac{\partial u}{\partial w}$; $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial w}\frac{\partial w}{\partial t} = -c\frac{\partial u}{\partial w}$

Thus $V_e = \frac{E}{2}\left(\frac{\partial u}{\partial w}\right)^2$ & $T = \frac{\rho}{2}c^2\left(\frac{\partial u}{\partial w}\right)^2$

But $c^2 = E/\rho$, so $\underline{V_e = T}$

▲10/60 V_e = elastic potential energy per unit volume

T = kinetic energy per unit volume

dU = total energy for length dx

$= (V_e + T)A\,dx$

From Prob. 10/59, $V_e = T = \frac{E}{2}\left(\frac{\partial u}{\partial x}\right)^2$; Also $u = u_0 \sin p\left(t - \frac{x}{c}\right)$

so $\frac{\partial u}{\partial x} = -\frac{pu_0}{c}\cos p\left(t - \frac{x}{c}\right)$ & $\sigma_0 = \sigma_{max} = \left|\left(\frac{\partial u}{\partial x}\right)\right|_{max.}(E) = \frac{pu_0 E}{c}$

$\therefore \frac{\partial u}{\partial x} = -\frac{\sigma_0}{E}\cos p\left(t - \frac{x}{c}\right)$ so $u_0 = \frac{c\sigma_0}{pE}$

Thus $dU = E\left(\frac{\partial u}{\partial x}\right)^2 A\,dx = \left[EA\frac{\sigma_0^2}{E^2}\cos^2 p\left(t - \frac{x}{c}\right)\right]dx$

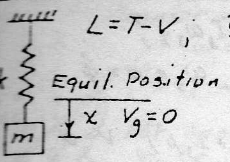
At a given x, the time average of dU is

$$d\bar U = \frac{1}{T}\int_0^T dU\,dt = \left\{\frac{p}{2\pi}\int_0^{2\pi/p}\frac{A\sigma_0^2}{E}\cos^2 p\left(t - \frac{x}{c}\right)dt\right\}dx$$

$$= \frac{pA\sigma_0^2}{2\pi E}\frac{1}{p}\left[\frac{p\left(t - \frac{x}{c}\right)}{2} + \frac14\sin 2p\left(t - \frac{x}{c}\right)\right]_0^{2\pi/p}$$

$= \frac{A\sigma_0^2}{2\pi E}[\pi]dx$ so $\dot U = \frac{d\bar U}{dt} = \frac{\sigma_0^2 Ac}{2E} = \frac{\sigma_0^2 A}{2\sqrt{\rho E}}$

10/62 $T=\frac{1}{2}m\dot{x}^2$; $V=V_e+V_g=\frac{1}{2}k\left(\frac{mg}{k}+x\right)^2-mgx$

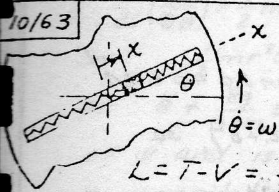


$L=T-V$, $\frac{\partial L}{\partial \dot{x}}=m\dot{x}$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=m\ddot{x}$

$\frac{\partial L}{\partial x}=-k\left(\frac{mg}{k}+x\right)+mg=-kx$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$; $\underline{m\ddot{x}+kx=0}$

10/63



For system of spring & slider,

$T=\frac{1}{2}m(\dot{x}^2+x^2\dot{\theta}^2)$

$V=\frac{1}{2}kx^2$

$L=T-V=\frac{1}{2}m(\dot{x}^2+x^2\dot{\theta}^2)-\frac{1}{2}kx^2$

$\frac{\partial L}{\partial \dot{x}}=m\dot{x}$; $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=m\ddot{x}$; $\frac{\partial L}{\partial x}=mx\dot{\theta}^2-kx$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$; $m\ddot{x}-mx\dot{\theta}^2+kx=0$

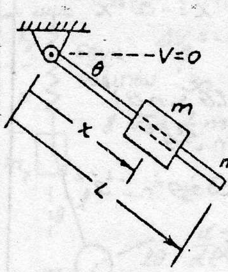
or $\underline{\ddot{x}+\left(\frac{k}{m}-\omega^2\right)x=0}$ where $\omega=\dot{\theta}$, constant

$\tau=2\pi/p=\underline{\dfrac{2\pi}{\sqrt{\frac{k}{m}-\omega^2}}}$, $\underline{\omega_{max}=\sqrt{k/m}}$

10/64

Collar: $T=\frac{1}{2}m(\dot{x}^2+x^2\dot{\theta}^2)$

$V=-mgx\sin\theta$



Rod: $T=\frac{1}{2}I\dot{\theta}^2=\frac{1}{2}m_0k^2\dot{\theta}^2$

$V=-m_0g\frac{L}{2}\sin\theta$

For system, $L=T-V$

$L=\frac{1}{2}m(\dot{x}^2+x^2\dot{\theta}^2)+\frac{1}{2}m_0k^2\dot{\theta}^2+\left(mx+m_0\frac{L}{2}\right)g\sin\theta$

$\frac{\partial L}{\partial \dot{x}}=m\dot{x}$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=m\ddot{x}$

$\frac{\partial L}{\partial x}=mx\dot{\theta}^2+mg\sin\theta$

$\frac{\partial L}{\partial \dot{\theta}}=mx^2\dot{\theta}+m_0k^2\dot{\theta}$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)=mx^2\ddot{\theta}+2mx\dot{x}\dot{\theta}+m_0k^2\ddot{\theta}$

$\frac{\partial L}{\partial \theta}=\left(mx+m_0\frac{L}{2}\right)g\cos\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0$, $j=x,\theta$

$j=x$: $m\ddot{x}-mx\dot{\theta}^2-mg\sin\theta=0$

$\underline{\ddot{x}-x\dot{\theta}^2=g\sin\theta}$

$j=\theta$; $mx^2\ddot{\theta}+2mx\dot{x}\dot{\theta}+m_0k^2\ddot{\theta}-\left(mx+m_0\frac{L}{2}\right)g\cos\theta=0$

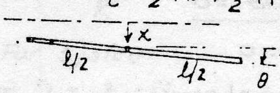
$\underline{\left(x^2+\frac{m_0}{m}k^2\right)\ddot{\theta}+2x\dot{x}\dot{\theta}=\left(x+\frac{m_0}{m}\frac{L}{2}\right)g\cos\theta}$

10/65 $T=\frac{1}{2}m\dot{x}^2+\frac{1}{2}\frac{1}{12}ml^2\dot{\theta}^2$

$V=V_e=\frac{1}{2}kx^2+\frac{1}{2}k\left(x-\frac{l\theta}{2}\right)^2$

$L=T-V$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=0$



For x; $\frac{\partial L}{\partial x}=-kx-k\left(x-\frac{l\theta}{2}\right)$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=\frac{d}{dt}(m\dot{x})=m\ddot{x}$

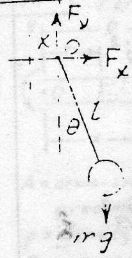
so $\underline{m\ddot{x}+2kx-k\frac{l\theta}{2}=0}$

For θ; $\frac{\partial L}{\partial \theta}=\frac{kl}{2}\left(x-\frac{l\theta}{2}\right)$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)=\frac{d}{dt}\left(\frac{1}{12}ml^2\dot{\theta}\right)=\frac{ml^2\ddot{\theta}}{12}$

$\frac{ml^2\ddot{\theta}}{12}-\frac{kl}{2}\left(x-\frac{l\theta}{2}\right)=0$

so $\underline{m\ddot{\theta}-6k\left(\frac{x}{l}-\frac{\theta}{2}\right)=0}$

10/66 $x = x_0 \sin \omega t\,;\ \dot{x} = x_0 \omega \cos \omega t\,;\ \ddot{x} = -\omega^2 x$



Velocity of m is v

$\therefore v^2 = (\dot{x} + l\dot{\theta}\cos\theta)^2 + (l\dot{\theta}\sin\theta)^2$

$= \dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2$

$T = \frac{1}{2}m\left[\dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2\right]$

$Q_x\,\delta x = F_x\,\delta x,\quad Q_x = F_x$

$Q_\theta\,\delta\theta = -mg\sin\theta\, l\,\delta\theta,\quad Q_\theta = -mgl\sin\theta$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j \quad$ where $j = x, \theta$

$x;\ \frac{d}{dt}m(\dot{x} + l\dot{\theta}\cos\theta) - 0 = F_x\,;$

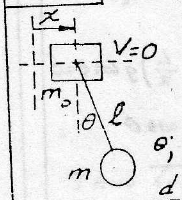
$\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta = F_x/m$

$\theta;\ \frac{d}{dt}m(l\dot{x}\cos\theta + l^2\dot{\theta}) + ml\dot{x}\dot{\theta}\sin\theta = -mgl\sin\theta$

$\ddot{x}\cos\theta + l\ddot{\theta} = -g\sin\theta,\quad \underline{\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{x_0}{l}\omega^2\sin\omega t\cos\theta}$

10/67 Velocity of m is $v^2 = (\dot{x} + l\dot{\theta}\cos\theta)^2 + (l\dot{\theta}\sin\theta)^2$

$= \dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2$



$T = \frac{1}{2}m(\dot{x}^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2) + \frac{1}{2}m_0\dot{x}^2$

$V = -mgl\cos\theta\,;\quad L = T - V$

$\theta;\ \frac{\partial L}{\partial \dot{\theta}} = m(l\dot{x}\cos\theta + l^2\dot{\theta})\,;\ \frac{\partial L}{\partial \theta} = m(-l\dot{x}\dot{\theta}\sin\theta - gl\sin\theta)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m(l\ddot{x}\cos\theta - l\dot{x}\dot{\theta}\sin\theta + l^2\ddot{\theta})$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0;\ m(l\ddot{x}\cos\theta - l\dot{x}\dot{\theta}\sin\theta + l^2\ddot{\theta}) + ml\sin\theta(\dot{x}\dot{\theta} + g) = 0$

$\underline{\ddot{x}\cos\theta + l\ddot{\theta} + g\sin\theta = 0}$

$x;\ \frac{\partial L}{\partial \dot{x}} = m(\dot{x} + l\dot{\theta}\cos\theta) + m_0\dot{x}\,;\ \frac{\partial L}{\partial x} = 0$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) + m_0\ddot{x}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0;\ \underline{(m + m_0)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = 0}$

For $\sin\theta = \theta,\ \cos\theta = 1,\ \dot{\theta}^2 = 0;\quad \ddot{x} + l\ddot{\theta} + g\theta = 0$

& $(m + m_0)\ddot{x} + ml\ddot{\theta} = 0.$ Eliminate $\ddot{x}$ & get

$\ddot{\theta} + \frac{m + m_0}{m_0}\frac{g}{l}\theta = 0,\qquad \tau = 2\pi\sqrt{\frac{m_0}{m + m_0}\frac{l}{g}}$

10/68 For bar $T = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)\,;$

where $\omega_1 = \omega_0\cos\phi,\ \omega_2 = -\omega_0\sin\phi,\ \omega_3 = p = \dot{\phi}$

Also let $\omega_0 = \dot{\theta}$

Now $T = \frac{1}{2}(I_1\omega_0^2\cos^2\phi + I_2\omega_0^2\sin^2\phi + I_3p^2),\ V = 0$

$\frac{\partial T}{\partial \theta} = 0,\ \frac{\partial T}{\partial \dot{\theta}} = \frac{\partial T}{\partial \omega_0} = (I_1\cos^2\phi + I_2\sin^2\phi)\omega_0$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \omega_0}\right) = 2(-I_1\cos\phi\sin\phi + I_2\sin\phi\cos\phi)\dot{\phi}\omega_0$

$= (I_2 - I_1)p\omega_0\sin 2\phi$

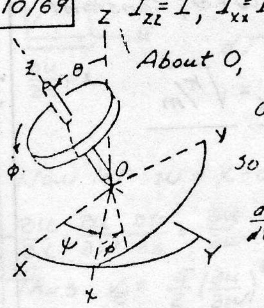
$Q_\theta\,\delta\theta = M\,\delta\theta \quad$ so $Q_\theta = M$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = Q_\theta \quad$ so $(I_2 - I_1)p\omega_0\sin 2\phi - 0 = M$

But $I_2 = \frac{1}{12}m(b^2 + c^2)$ & $I_1 = \frac{1}{12}m(a^2 + b^2)$, so

$\underline{M = \frac{1}{12}m(c^2 - a^2)p\omega_0\sin 2\phi}$

10/69 $I_{zz} = I,\ I_{xx} = I_{yy} = I_0\,,\ \Sigma\underline{M}_0 = \underline{i}\,\Sigma M_x + \underline{j}\,\Sigma M_y + \underline{k}\,\Sigma M_z$



About O, $T = \frac{1}{2}(I_0\omega_x^2 + I_0\omega_y^2 + I\omega_z^2)$

$\omega_x = \dot{\theta},\ \omega_y = \dot{\psi}\sin\theta\ \ \omega_z = \dot{\psi}\cos\theta + \dot{\phi}$

so $T = \frac{1}{2}(I_0\dot{\theta}^2 + I_0\dot{\psi}^2\sin^2\theta + I(\dot{\psi}\cos\theta + \dot{\phi})^2)$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j\ \ ;\ q_j = \theta, \phi, \psi$

For θ, $\quad Q_\theta\,\delta\theta = \Sigma M_x\,\delta\theta,\ \ Q_\theta = \Sigma M_x$

$\frac{\partial T}{\partial \theta} = I_0\dot{\psi}^2\sin\theta\cos\theta - I(\dot{\psi}\cos\theta + \dot{\phi})\dot{\psi}\sin\theta$

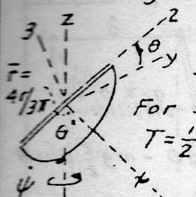
$\frac{\partial T}{\partial \dot{\theta}} = I_0\dot{\theta}$

Thus $\ I_0\ddot{\theta} - I_0\dot{\psi}^2\sin\theta\cos\theta + I(\dot{\psi}\cos\theta + \dot{\phi})\dot{\psi}\sin\theta = \Sigma M_x$

or $\Sigma M_x = I_0(\ddot{\theta} - \dot{\psi}^2\sin\theta\cos\theta) + I\dot{\psi}\omega_z\sin\theta$

(which is the first of Eqs. 166)

10/70 Let axes 1, 2, 3 be principal axes of half-disk
Angular velocity components of half-disk are



$\omega_1 = \dot\theta,\ \omega_2 = \dot\psi \sin\theta,\ \omega_3 = \dot\psi\cos\theta$

$I_1 = \frac{1}{2}mr^2,\ I_2 = \frac{1}{4}mr^2,\ I_3 = \frac{1}{4}mr^2$

For system

$T = \frac{1}{2}I_0\dot\psi^2 + \frac{1}{2}mr^2\left(\frac{1}{2}\dot\theta^2 + \frac{1}{4}\dot\psi^2\sin^2\theta + \frac{1}{4}\dot\psi^2\cos^2\theta\right)$

$= \frac{1}{2}I_0\dot\psi^2 + \frac{1}{4}mr^2\left(\dot\theta^2 + \frac{1}{2}\dot\psi^2\right)$

$V = mg\bar r(1-\cos\theta)$; $L = T - V$; $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_j}\right) - \frac{\partial L}{\partial q_j} = 0,\ q_j = \theta, \psi$

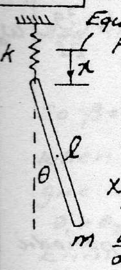
For θ; $\frac{\partial L}{\partial \theta} = -mg\bar r\sin\theta$, $\frac{\partial L}{\partial \dot\theta} = \frac{1}{2}mr^2\dot\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) = \frac{1}{2}mr^2\ddot\theta$

So $\frac{1}{2}mr^2\ddot\theta + mg\bar r\sin\theta = 0$

or $\ddot\theta + \frac{8g}{3\pi r}\sin\theta = 0$ which is independent of $\dot\psi$ and is the same equation as for a nonrotating support for m.

10/71 Take V = 0 at x = 0; $V_g = -mg\left(\frac{l}{2}\cos\theta + x\right)$



Equil. position $V_e = \left(mg + \frac{kx}{2}\right)x$, $V = -mg\frac{l}{2}\cos\theta + \frac{1}{2}kx^2$

$T = \frac{1}{2}m\bar v^2 + \frac{1}{2}\bar I\omega^2 = \frac{1}{2}m\left(\left[\dot x - \frac{l}{2}\dot\theta\sin\theta\right]^2 + \left[\frac{l}{2}\dot\theta\cos\theta\right]^2\right) + \frac{1}{2}\frac{1}{12}ml^2\dot\theta^2$

$L = T - V$

x; $\frac{\partial L}{\partial \dot x} = m\left(\dot x - \frac{1}{2}l\dot\theta\sin\theta\right)$; $\frac{\partial L}{\partial x} = -kx$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) = m\left(\ddot x - \frac{1}{2}l\ddot\theta\sin\theta - \frac{1}{2}l\dot\theta^2\cos\theta\right)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) - \frac{\partial L}{\partial x} = 0$; $\underline{m\left(\ddot x - \frac{1}{2}l\ddot\theta\sin\theta - \frac{1}{2}l\dot\theta^2\cos\theta\right) + kx = 0}$

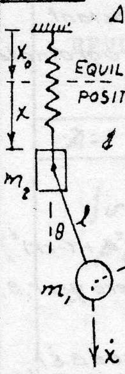
θ; $\frac{\partial L}{\partial \dot\theta} = \frac{1}{2}m\left(-\dot x l\sin\theta + \frac{1}{2}l^2\dot\theta\right) + \frac{1}{12}ml^2\dot\theta$

$\frac{\partial L}{\partial \theta} = \frac{1}{2}m\left(-l\dot x\dot\theta\cos\theta\right) - mg\frac{l}{2}\sin\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) = \frac{1}{2}m\left(-\ddot x l\sin\theta - \dot x\dot\theta l\cos\theta + \frac{1}{2}l^2\ddot\theta\right) + \frac{1}{12}ml^2\ddot\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) - \frac{\partial L}{\partial\theta} = 0$; $\underline{-\ddot x\sin\theta + \frac{2}{3}l\ddot\theta + g\sin\theta = 0}$

10/72 Take V = 0 at x = 0, θ = 0; V = ΔV



$\Delta V_e = (m_1 + m_2)gx + \frac{1}{2}kx^2$

$\Delta V_g = -m_2 gx - m_1 g(x - l[1-\cos\theta])$

so $V = \frac{1}{2}kx^2 + m_1 gl(1-\cos\theta)$

& $T = \frac{1}{2}m_2\dot x^2 + \frac{1}{2}m_1\left[(l\dot\theta\cos\theta)^2 + (\dot x - l\dot\theta\sin\theta)^2\right]$

$= \frac{1}{2}(m_1 + m_2)\dot x^2 + \frac{1}{2}m_1 l^2\dot\theta^2 - m_1 l\dot x\dot\theta\sin\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = 0$ where $L = T - V$, $q = x, \theta$

For x; $\frac{\partial L}{\partial x} = -kx$, $\frac{\partial L}{\partial \dot x} = (m_1 + m_2)\dot x - m_1 l\dot\theta\sin\theta$

So $(m_1 + m_2)\ddot x - m_1 l\ddot\theta\sin\theta - m_1 l\dot\theta^2\cos\theta + kx = 0$ ----(1)

For θ; $\frac{\partial L}{\partial\theta} = -m_1 l\dot x\dot\theta\cos\theta - m_1 gl\sin\theta$

$\frac{\partial L}{\partial\dot\theta} = m_1 l^2\dot\theta - m_1 l\dot x\sin\theta$

so $m_1 l^2\ddot\theta - m_1 l\ddot x\sin\theta - m_1 l\dot x\dot\theta\cos\theta + m_1 l\dot x\dot\theta\cos\theta + m_1 gl\sin\theta = 0$

or $l\ddot\theta - \ddot x\sin\theta + g\sin\theta = 0$ ----(2)

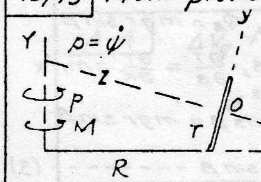
For x & θ small, neglect $\dot x^2, \dot\theta^2, \dot x\dot\theta$; let $\sin\theta = \theta$, $\cos\theta = 1$

Eqs. 1 & 2 become

$(m_1 + m_2)\ddot x - m_1 l\theta\ddot\theta + kx = 0$ ----(1a)

$l\ddot\theta - \theta\ddot x + g\theta = 0$ ----(2a)

10/73 From previous analysis (Prob. 7/31) the components of angular velocity of wheel are



$\omega_x = 0,\ \omega_y = p\cos\theta,\ \omega_z = p\left(\sin\theta + \frac{R}{r}\right)$

For disk $I = I_z = \frac{1}{2}mr^2$

$\frac{1}{2}I = I_y = \frac{1}{4}mr^2$

For axle $I_1 = I_Y$

For system

$T = \frac{1}{2}I_1 p^2 + \frac{1}{2}m(R + r\sin\theta)^2p^2 + \frac{1}{2}\left[\frac{I}{2}p^2\cos^2\theta + I\left(\sin\theta + \frac{R}{r}\right)^2p^2\right]$

$= \frac{1}{2}\left\{I_1 + m\left[\frac{3}{2}R^2 + 3Rr\sin\theta + \frac{3}{2}r^2\sin^2\theta + \frac{1}{4}r^2\cos^2\theta\right]\right\}p^2$

$= \frac{1}{2}Kp^2$ where $K = \{\cdots\cdots\} = \left\{I_1 + m\left(\frac{3}{2}[R + r\sin\theta]^2 + \frac{r^2}{4}\cos^2\theta\right)\right\}$

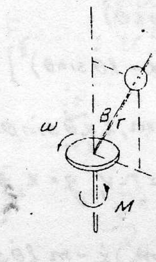
$\frac{\partial T}{\partial\psi} = 0$, $\frac{\partial T}{\partial\dot\psi} = \frac{\partial T}{\partial p} = Kp$, $\frac{d}{dt}\left(\frac{\partial T}{\partial p}\right) = K\dot p$; $Q_\psi\,\delta\psi = M\,\delta\psi$ so $Q_\psi = M$

Thus $\frac{d}{dt}\left(\frac{\partial T}{\partial p}\right) - \frac{\partial T}{\partial\psi} = Q_\psi$ gives

$K\dot p = M$, so

$M = \underline{\left\{I_1 + m\left[\frac{3}{2}(R + r\sin\theta)^2 + \frac{1}{4}r^2\cos^2\theta\right]\right\}\dot p}$

10/74



Given: $\dot{\beta} = k$, constant, ω = constant

Replace spherical coordinates R, θ, ϕ by $r, \theta, -\beta$ where $\dot{\theta} = \omega$, $\dot{\phi} = -\dot{\beta} = -k$.

Take $V = 0$ at $r = 0$

$V = mgr\cos\beta$;

$T = \frac{1}{2}m(\dot{r}^2 + r^2\omega^2\sin^2\beta + (-rk)^2)$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q$; where $q_j = r, \theta, \beta$

For r; $Q_r\delta r = -mg\cos\beta\,\delta r$,

$Q_r = -mg\cos\beta$

$\frac{\partial T}{\partial r} = mr(\omega^2\sin^2\beta + k^2)$

$\frac{\partial T}{\partial \dot{r}} = m\dot{r}$, $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{r}}\right) = m\ddot{r}$

Thus $m\ddot{r} - mr(\omega^2\sin^2\beta + k^2) = -mg\cos\beta$

or $\underline{\ddot{r} - r\omega^2\sin^2\beta - rk^2 + g\cos\beta = 0}$ -------- (1)

For θ; $Q_\theta\delta\theta = M\delta\theta$, $Q_\theta = M$

$\frac{\partial T}{\partial \theta} = 0$, $\frac{\partial T}{\partial \dot{\theta}} = \frac{\partial T}{\partial \omega} = mr^2\omega\sin^2\beta$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \omega}\right) = m\omega(2r\dot{r}\sin^2\beta + 2r^2\dot{\beta}\sin\beta\cos\beta)$, ω const.

$= 2mr\omega\sin\beta(\dot{r}\sin\beta + r\dot{\beta}\cos\beta)$

Thus $2mr\omega\sin\beta(\dot{r}\sin\beta + r\dot{\beta}\cos\beta) - 0 = M$

or $\underline{M = (2mr\omega\sin\beta)\frac{d}{dt}(r\sin\beta)}$ ---------- (2)

For β; $Q_\beta\delta\beta = mg\sin\beta\,r\delta\beta$, $Q_\beta = mgr\sin\beta$

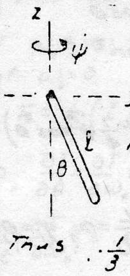
$\frac{\partial T}{\partial \beta} = mr^2\omega^2\sin\beta\cos\beta$, $\frac{\partial T}{\partial \dot{\beta}} = \frac{\partial T}{\partial k} = mr^2k$

Thus $2mr\dot{r}k - mr^2\omega^2\sin\beta\cos\beta = mgr\sin\beta$

or $\underline{2k\dot{r} = (r\omega^2\cos\beta + g)\sin\beta}$ -------- (3)

10/75

$T = \frac{1}{2}I_0\dot{\psi}^2 + \frac{1}{2}\frac{1}{3}ml^2\dot{\theta}^2 + \frac{1}{2}\frac{1}{3}m(l\sin\theta)^2\dot{\psi}^2$



$V = mg\frac{l}{2}(1-\cos\theta)$

$L = T - V$; $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$, $q_j = \theta, \psi$

For θ; $\frac{\partial L}{\partial \theta} = \frac{1}{3}ml^2\dot{\psi}^2\sin\theta\cos\theta - mg\frac{l}{2}\sin\theta$

$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{3}ml^2\dot{\theta}$

Thus $\frac{1}{3}ml^2\ddot{\theta} - \frac{1}{3}ml^2\dot{\psi}^2\sin\theta\cos\theta + mg\frac{l}{2}\sin\theta = 0$

or $\ddot{\theta} - \dot{\psi}^2\sin\theta\cos\theta + \frac{3g}{2l}\sin\theta = 0$ ------ (1)

For ψ; $\frac{\partial L}{\partial \psi} = 0$; $\frac{\partial L}{\partial \dot{\psi}} = I_0\dot{\psi} + \frac{1}{3}ml^2\sin^2\theta\,\dot{\psi}$

Thus $\frac{d}{dt}\left\{\left(I_0 + \frac{1}{3}ml^2\sin^2\theta\right)\dot{\psi}\right\} = 0$,

so $\left(I_0 + \frac{1}{3}ml^2\sin^2\theta\right)\dot{\psi} = C$, a constant

Now $\theta = 0$ when $\dot{\psi} = \omega_0$, so $C = I_0\omega_0$ | Thus $\dot{\psi} = \dfrac{I_0\omega_0}{I_0 + \frac{1}{3}ml^2\sin^2\theta}$

For θ small & I_0 large, $\dot{\psi} \simeq \omega_0$, & Eq. 1 becomes $\ddot{\theta} - \omega_0^2\theta + \frac{3g}{2l}\theta = 0$ or $\ddot{\theta} + \left(\frac{3g}{2l} - \omega_0^2\right)\theta = 0$

where $\sin\theta \to 0$, $\cos\theta \to 1$.

Solution is $\theta = \theta_0\sin pt$ where $p^2 = \frac{3g}{2l} - \omega_0^2$

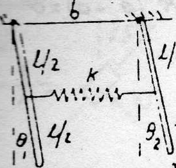
$\dot{\theta} = \Omega_0$ for $t = 0$, so $\theta_0 = \Omega_0/p$.

Thus $\theta = \frac{\Omega_0}{p}\sin pt$.

Period $\underline{\tau = \frac{2\pi}{p} = 2\pi\Big/\sqrt{\frac{3g}{2l} - \omega_0^2}}$

When $\omega_0 > \sqrt{\frac{3g}{2l}}$, motion is no longer periodic & rod tends to swing out

10/76 Let $V=0$ when $\theta_1=\theta_2=0$;



$$V_e=\frac{1}{2}k\left(\frac{\ell}{2}\sin\theta_2-\frac{\ell}{2}\sin\theta_1\right)^2$$

$$V_g=mg\frac{\ell}{2}\left[(1-\cos\theta_2)+(1-\cos\theta_1)\right]$$

For small angles $\sin\theta\approx\theta$

$\cos\theta=1-\frac{\theta^2}{2}$

so $V_e=\frac{1}{8}k\ell^2(\theta_2-\theta_1)^2$, $V_g=mg\frac{\ell}{4}(\theta_2^2+\theta_1^2)$

$$V=\frac{1}{8}k\ell^2(\theta_2-\theta_1)^2+\frac{1}{4}mg\ell(\theta_1^2+\theta_2^2)$$

$$T=\frac{1}{2}\left(\frac{1}{3}m\ell^2\right)\dot\theta_1^2+\frac{1}{2}\left(\frac{1}{3}m\ell^2\right)\dot\theta_2^2=\frac{1}{6}m\ell^2(\dot\theta_1^2+\dot\theta_2^2)$$

With $L=T-V$ & $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\xi}\right)-\frac{\partial L}{\partial\xi}=0$, $\xi=\theta_1,\theta_2$,

For θ_1; $\frac{\partial L}{\partial\theta_1}=\frac{1}{4}k\ell^2(\theta_2-\theta_1)-\frac{1}{2}mg\ell\theta_1$

$\frac{\partial L}{\partial\dot\theta_1}=\frac{1}{3}m\ell^2\dot\theta_1$, $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta_1}\right)=\frac{1}{3}m\ell^2\ddot\theta_1$

Thus $\frac{1}{3}m\ell^2\ddot\theta_1-\frac{1}{4}k\ell^2(\theta_2-\theta_1)+\frac{1}{2}mg\ell\theta_1=0$ - - - - - (1)

For θ_2; $\frac{\partial L}{\partial\theta_2}=\frac{-1}{4}k\ell^2(\theta_2-\theta_1)-\frac{1}{2}mg\ell\theta_2$

$\frac{\partial L}{\partial\dot\theta_2}=\frac{1}{3}m\ell^2\dot\theta_2$, $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta_2}\right)=\frac{1}{3}m\ell^2\ddot\theta_2$

Thus $\frac{1}{3}m\ell^2\ddot\theta_2+\frac{1}{4}k\ell^2(\theta_2-\theta_1)+\frac{1}{2}mg\ell\theta_2=0$ - - - - - (2)

Add (1) & (2) & get with $\beta=\theta_1+\theta_2$

$\frac{1}{3}m\ell^2\ddot\beta+\frac{1}{2}mg\ell\beta=0$ or $\ddot\beta+\frac{3}{2}\frac{g}{\ell}\beta=0$

So $\beta=\theta_1+\theta_2=A\sin pt+B\cos pt$ where $p^2=\frac{3g}{2\ell}$

When $t=0$, $\theta_1=0$, $\theta_2=\theta_0$ & $\dot\theta_1=\dot\theta_2=0$. These conditions give $B=\theta_0$ & $A=0$ so

$\theta_1+\theta_2=\theta_0\cos pt$ & $\ddot\theta_1+\ddot\theta_2=-\theta_0p^2\cos pt$.

Eliminate θ_2 from Eq. (2) & get

$\ddot\theta_1+\mu^2\theta_1=C^2\cos pt$ where $\mu^2=\frac{3}{2}\left(\frac{k}{m}+\frac{g}{\ell}\right)$

Sol. is $\quad C^2=\frac{3k\theta_0}{4m}$

$\theta_1=D\sin\mu t+E\cos\mu t+F\cos pt$

where $F=C^2/(\mu^2-p^2)=\theta_0/2$

Boundary conditions $\theta_1=0$ & $\dot\theta_1=0$ for $t=0$ give

$D=0$ and $E=-F=-\frac{\theta_0}{2}$ Thus

$$\theta_1=\frac{\theta_0}{2}(\cos pt-\cos\mu t)$$

where $p=\sqrt{\frac{3g}{2\ell}}$, $\mu=\sqrt{\frac{3}{2}\left(\frac{k}{m}+\frac{g}{\ell}\right)}$

APPENDIX A

REVIEW PROBLEMS

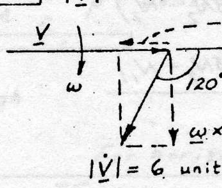
A-1 For constant acceleration a,

$\int v\,dv=a\int ds$, $v^2=2as$

$F=ma$, $s=\frac{v^2}{2F/m}=\frac{v^2m}{2F}=\frac{(270/3.6)^2\,115}{2(4)\,90}$

$=\underline{898\text{ m}}$

A-2 $|\underline{V}|=10$ units

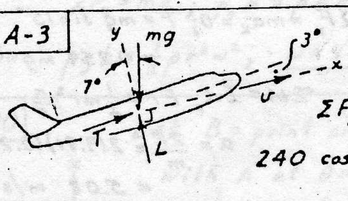


$\frac{d}{dt}|\underline{V}|=-6\sin30°=\underline{-3\text{ units/s}}$

$\omega=\frac{|\underline{\omega}\times\underline{V}|}{|\underline{V}|}=\frac{6\cos30°}{10}$

$=\underline{0.520\text{ rad/s}}$

A-3



$m=90$ t, $T=240$ kN

$\Sigma F_x=ma_x$,

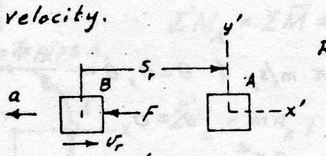
$240\cos3°-90(9.81)\sin7°=90a$

$\underline{a=1.467\text{ m/s}^2}$

A-4 $\Delta V_g+\Delta T=0$; $-\rho gL\left(\frac{h}{2}+\frac{L}{2}\right)+\frac{1}{2}\rho L v^2=0$

$v^2=g(L+h)$, $\underline{v=\sqrt{g(L+h)}}$

A-5 A Newtonian coordinate system may be attached to A since it moves with constant velocity.



Rel. to x'-y' $v_r=150$ km/h

$v_r^2=2as_r$ with constant a

$a=\frac{150^2}{2(15)}=750\text{ km/h}^2$

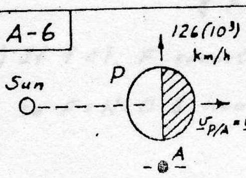
or $a=\frac{750/3.6}{3600}=0.0579\text{ m/s}^2$

$F=ma$; $F=800(0.0579)=\underline{46.3\text{ N}}$

$v_r=at$; $t=\frac{150}{750}=0.2$ h or 720 s

so $I=\int F\,dt=Ft=46.3(720)=33\,300$ N·s $=\underline{33.3\text{ kN·s}}$

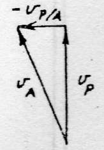
A-6



$\underline{v}_A=\underline{v}_P+\underline{v}_{A/P}=\underline{v}_P-\underline{v}_{P/A}$

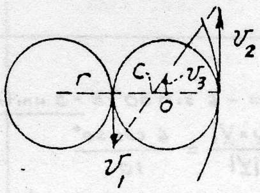
$v_A=\sqrt{(126)^2+(22.2)^2}\,(10^3)$

$=\underline{127.9(10^3)\text{ km/h}}$



A-7 Potential energy loss of rope equals kinetic energy gain of successively smaller portion of rope. At end of motion, A impacts cylinder with infinite velocity, assuming conservative system, and energy is dissipated in impact losses.

A-8



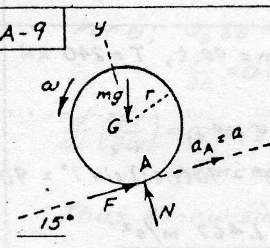
$$\frac{v_2 - v_3}{r} = \frac{v_1 + v_2}{2r}, \quad v_3 = \frac{1}{2}(v_2 - v_1)$$

$$v_1 = rN_1, \quad v_2 = 3rN_2, \quad v_3 = 2rN_3$$

$$2N_3 = \frac{1}{2}(3N_2 - N_1)$$

$$\underline{N_3 = \frac{1}{4}(3N_2 - N_1)}$$

A-9



$$\Sigma F_x = ma_x = 0; \quad F = mg \sin 15° = 0.259\,mg$$

$$\Sigma \bar{M} = \bar{I}\alpha; \quad Fr = \frac{1}{2}mr^2 \frac{a}{r}$$

$$a = \frac{2F}{m} = 2(9.81)(0.259) = \underline{5.08 \text{ m/s}^2}$$

A-10 Use polar coordinates where

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ is replaced by $-a_x$ &

$a_r = \ddot{r} - r\dot{\theta}^2$ is replaced by $+a_y$

$r = 2.4 + y = 2.4 + 0.15 \sin \pi t$ m, $\quad \theta = \frac{\pi}{4} \sin \frac{\pi}{2} t$

$\dot{r} = \dot{y} = 0.15\pi \cos \pi t \qquad \dot{\theta} = \frac{\pi^2}{8} \cos \frac{\pi}{2} t$

$\ddot{r} = \ddot{y} = -0.15\pi^2 \sin \pi t \qquad \ddot{\theta} = \frac{-\pi^3}{16} \sin \frac{\pi}{2} t$

For $t = 2$ s,

$r = 2.4$ m, $\dot{r} = 0.15\pi$ m/s, & $\theta = 0$, $\dot{\theta} = -\frac{\pi^2}{8}$ rad/s

$\ddot{r} = 0 \qquad \ddot{\theta} = 0$

$a_y = a_r = 0 - 2.4\left(\frac{\pi^4}{64}\right) = -\frac{3\pi^4}{80}$ m/s²

$a_x = -a_\theta = -(2.4)(0) - 2(0.15\pi)(-\pi^2/8) = \frac{3\pi^3}{80}$ m/s²

$$\underline{\mathbf{a} = \frac{3\pi^3}{80}(\mathbf{i} - \pi \mathbf{j}) \text{ m/s}^2}$$

A-11 $\Delta V_e + \Delta T = 0; \quad \frac{1}{2}C(\theta^2 - \theta_0^2) + \frac{1}{2}mk^2(\dot{\theta}^2 - 0) = 0$

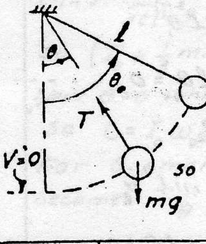
$$\dot{\theta}^2 = \frac{C}{mk^2}(\theta_0^2 - \theta^2)$$

Power $P = M\dot{\theta} = C\theta \frac{1}{k}\sqrt{\frac{C}{m}}\sqrt{\theta_0^2 - \theta^2}$

$$\frac{dP}{d\theta} = \frac{C}{k}\sqrt{\frac{C}{m}}\left(\frac{\theta_0^2 - 2\theta^2}{\sqrt{\theta_0^2 - \theta^2}}\right) = 0 \text{ for max. } P \text{ so } \underline{\theta_m = \theta_0/\sqrt{2}}$$

$$P = \frac{C\theta_0}{\sqrt{2}}\frac{1}{k}\sqrt{\frac{C}{m}}\theta_0\sqrt{1 - \frac{1}{2}} = \underline{\frac{C}{2k}\sqrt{\frac{C}{m}}\theta_0^2}$$

A-12



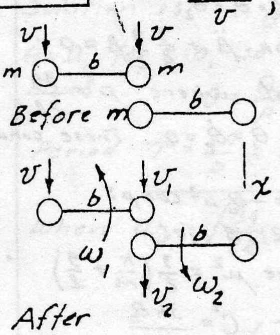
$$\Sigma F_n = ma_n; \quad T - mg\cos\theta = ml\dot{\theta}^2$$

$$\Delta T + \Delta V = 0; \quad \frac{1}{2}ml^2\dot{\theta}^2 + mgl[(1 - \cos\theta) - (1 - \cos\theta_0)] = 0$$

$$\frac{l\dot{\theta}^2}{2g} + (\cos\theta_0 - \cos\theta) = 0$$

$$\text{so } T = mg\left(\frac{l\dot{\theta}^2}{g} + \cos\theta\right) = \underline{mg(3\cos\theta - 2\cos\theta_0)}$$

A-13



$e = \frac{v_2 - v_1}{v}; \quad \Sigma G_x = $ constant

$$2mv = m(v + v_1) + mv_2$$

$$v = v_1 + v_2$$

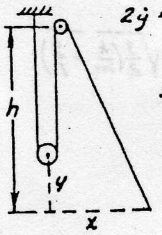
Solve for v_1 & v_2 & get

$$v_1 = \frac{v}{2}(1 - e), \quad v_2 = \frac{v}{2}(1 + e)$$

$$\omega_1 = \frac{v - v_1}{b} = \underline{\frac{v}{2b}(1 + e)}$$

$$\omega_2 = \frac{v_2}{b} = \underline{\frac{v}{2b}(1 + e)} = \omega_1$$

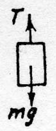
A-14



$\sqrt{x^2 + h^2} + 2(h - y) = 3h, \quad 2y = \sqrt{x^2 + h^2} - h$

$$2\dot{y} = \frac{x\dot{x}}{\sqrt{x^2 + h^2}}, \quad 2\ddot{y} = \frac{\dot{x}^2}{\sqrt{x^2 + h^2}} - \frac{(x\dot{x})^2}{\sqrt{(x^2 + h^2)^3}}$$

$$\text{so } \ddot{y} = \frac{h^2\dot{x}^2}{2\sqrt{(x^2 + h^2)^3}} = \frac{h^2 v^2}{2\sqrt{(x^2 + h^2)^3}}$$

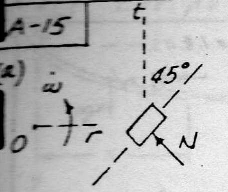


$$\Sigma F_y = ma_y; \quad T - mg = ma_y$$

$$\underline{T = mg\left(1 + \frac{h^2 v^2}{2g\sqrt{(h^2 + x^2)^3}}\right)}$$

A-15

(a)



Let B on disk be coincident with A

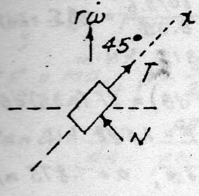
$\underline{a}_A = \underline{a}_B + \underline{a}_{rel}$

a_A in dir. of force N

$a_B = r\dot{\omega}$ From diag.

$\underline{a_{rel} = r\dot{\omega}/\sqrt{2}}$

(b)



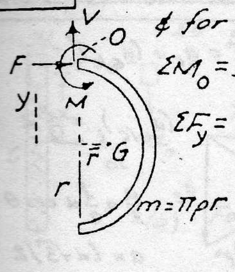
$\Sigma F_x = ma_x; \quad \underline{T = mr\dot{\omega}/\sqrt{2}}$

(c) For $T = 0$, $\omega \neq 0$ $\quad \Sigma F_x = ma_x; \quad 0 = m\left(\frac{r\dot{\omega}}{\sqrt{2}} - \frac{r\omega^2}{\sqrt{2}}\right)$

$\dot{\omega} = \omega^2, \quad \underline{\omega = \sqrt{\dot{\omega}}}$

A-16 For complete ring $I_O = (2m)r^2 + (2m)r^2 = 4mr^2$

& for half ring $I_O = 2mr^2 = 2\pi\rho r^3$

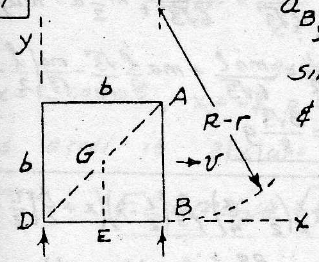


$\Sigma M_O = I_O\alpha; \quad \underline{M = 2\pi\rho r^3\alpha}$

$\Sigma F_y = m\bar{a}_y; \quad V = \pi\rho r(\bar{r}\alpha) = \pi\rho r\left(\frac{2r}{\pi}\right)\alpha$

$\underline{V = 2\rho r^2\alpha}$

A-17



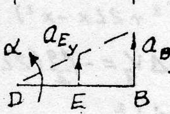
$a_{B_y} = \frac{v^2}{R-r}, \quad a_{E_y} = \frac{1}{2}a_{B_y}$

Since $v_{B_y} = 0, \ \omega = 0$

$\& \ (a_{G/E})_n = (a_{G/E})_y = 0$

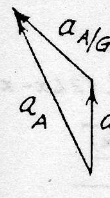
Thus $a_{G_y} = a_{E_y} = \frac{1}{2}\frac{v^2}{R-r}$

$\Sigma F_x = 0$ so $a_{G_x} = 0$



$\alpha = a_{B_y}/\overline{DB} = \frac{v^2/b}{R-r}$

$\underline{a}_A = \underline{a}_G + (\underline{a}_{A/G})_t$



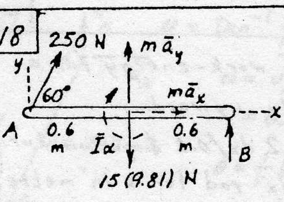
$a_{A/G} = (a_{A/G})_t = \overline{AG}\alpha_{AG} = \frac{b}{\sqrt{2}}\frac{v^2/b}{R-r} = \frac{1}{\sqrt{2}}\frac{v^2}{R-r}$

$a_G = \frac{1}{2}\frac{v^2}{R-r}$

$\underline{a}_A = \frac{v^2}{R-r}\left(-\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\underline{i} + \left[\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + \frac{1}{2}\right]\underline{j}\right)$

$= \underline{\frac{v^2}{R-r}\left(-\frac{1}{2}\underline{i} + \underline{j}\right)}$

A-18

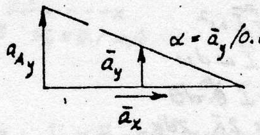


$\Sigma M_B = \bar{I}\alpha + m\bar{a}_y d$

$250\frac{\sqrt{3}}{2}1.2 - 15(9.81)(0.6)$

$= \frac{1}{12}15(1.2)^2\frac{\bar{a}_y}{0.6} + 15\bar{a}_y(0.6)$

$\bar{a}_y = 14.29 \text{ m/s}^2, \ \alpha = \frac{14.29}{0.6} = \underline{23.8 \text{ rad/s}^2}$



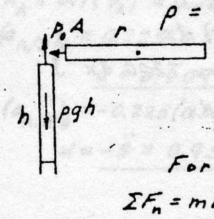
$\Sigma F_y = m\bar{a}_y; \ B - 15(9.81) + 250\frac{\sqrt{3}}{2}$

$= 15(14.29)$

$\underline{B = 145.0 \text{ N}}$

A-19 Let A = cross-sectional area of tubes

ρ = density of water



For vertical section

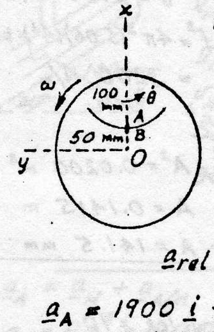
$p_o A = \rho g A h, \quad p_o = \rho g h$ (negative pressure)

For horizontal section

$\Sigma F_n = ma_n; \ p_o A = \rho A r \frac{r}{2}\omega^2, \quad p_o = \frac{1}{2}\rho r^2\omega^2$

Thus $\rho g h = \frac{1}{2}\rho r^2\omega^2, \quad \underline{\omega = \sqrt{2gh}/r}$

A-20 Let B = point on disk coincident with A at $\theta = 0$.



$\underline{a}_A = \underline{a}_B + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_B = -50(10)^2\underline{i} + 50(25)\underline{j}$ mm/s²

$2\underline{\omega}\times\underline{v}_{rel} = 2(10\underline{k})\times(100)(3)(-\underline{j})$

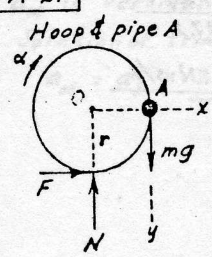
$= 6000\ \underline{i}$ mm/s²

$\underline{a}_{rel} = 100(3^2)\underline{i} - 100(5)\underline{j}$ mm/s²

$\underline{\underline{a}_A = 1900\ \underline{i} + 750\ \underline{j}}$ mm/s²

A-21

Hoop & pipe A



$\Sigma M_A = \Sigma \bar{M} = 0; \ Nr - Fr = 0, \ F = N$ regardless of f

$\left.\begin{array}{l}\Sigma F_x = ma_x; \ F = ma_{A_x} \\ \Sigma F_y = ma_y; \ mg - N = ma_{A_y}\end{array}\right\}$

(a) If $f > 1$, F may equal N, & hoop will not slip; $a_{A_x} = r\alpha, \ a_{A_y} = r\alpha$

Thus, $mg - mr\alpha = mr\alpha, \quad \underline{\alpha = \frac{g}{2r}}$

& $F = mg/2 = N$

(b) If $f < 1$, F cannot equal $N > 0$

so $F = N = 0$ & hoop slips with $a_{A_y} = g$

so $\underline{\alpha = \frac{g}{r}}$

A-22 Use the equation of work-energy for a small displacement dx upward.

Drum turns through angle $2\,dx/0.2$ & pinion turns through angle $\frac{0.3}{0.1}\frac{2\,dx}{0.2} = 30\,dx$ rad for x in metres.

$dU = dT + dV$; $T = \Sigma \frac{1}{2} m v^2 + \Sigma \frac{1}{2}\bar{I}\omega^2$

$$dT = \Sigma m v\,dv + \Sigma \bar{I}\omega\,d\omega$$
$$= \Sigma m\ddot{x}\,dx + \bar{I}\ddot{\theta}\,d\theta$$
$$= \Sigma m\ddot{x}\,dx + \bar{I}\frac{2\ddot{x}}{r}\frac{2\,dx}{r}$$
$$= \left[450 + 100(0.15)^2\frac{4}{(0.2)^2}\right]\ddot{x}\,dx$$
$$= 675\,\ddot{x}\,dx$$
$$dV = \Sigma mg\,dx = 450(9.81)\,dx$$
$$dU = M\,d\theta = 240(30\,dx) = 7200\,dx \text{ J}$$

So $7200\,dx = 675\,\ddot{x}\,dx + 450(9.81)\,dx$

$$\ddot{x} = \frac{2780}{675} = \underline{4.13 \text{ m/s}^2}$$

A-23 During impact $\Delta H_O = 0$

$f = 4$ Hz

$0.06(300)r = (5+0.06)vr$, $v = 3.56$ m/s

$f = \frac{p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, $k = 4\pi^2 m f^2 = 4\pi^2(5.06)(4^2)$
$= 3200$ N/m

$V_{e\,max.} = T_{max.}$; $\frac{1}{2}kA^2 = \frac{1}{2}mv^2$

$\frac{1}{2}3200A^2 = \frac{1}{2}5.06(3.56)^2$, $A^2 = 0.0200$ m^2

$A = 0.1415$ m

or $\underline{A = 141.5 \text{ mm}}$

For damped motion

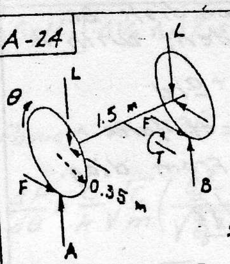
$\ln\frac{x_0}{x_n} = nb\tau$, n = No. of cycles = 10

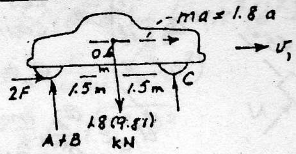
$b\tau = \frac{c}{2m}\frac{2\pi}{p} = \frac{\pi c}{m}\sqrt{\frac{m}{k}} = \frac{\pi c}{\sqrt{mk}}$

Thus $\ln\frac{1}{0.6} = 10\frac{\pi c}{\sqrt{5.06(3200)}}$, $0.511 = \frac{31.42c}{127.2}$

$\underline{c = 2.07 \text{ N·s/m}}$

A-24





For 80 km/h, speed of propeller shaft is $3.7\frac{v}{r} = 3.7\frac{80/3.6}{0.35} = 235$ rad/s

Power $P = \omega T$, $T = \frac{75}{235} = 0.319$ kN·m

For rear wheels, $dU = 0$, $0.319(3.7\,d\theta) = 2F(0.35)\,d\theta$

$F = 1.688$ kN

For entire car, $\Sigma F = ma$; $2(1.688) = 1.8a$, $a = 1.875$ m/s^2

$\Sigma M_C = m\bar{a}d$; $3(A+B) - 1.8(9.81)(1.5) = 2(1.688)(0.6)$

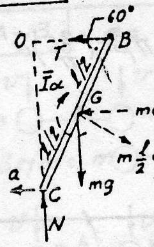
$A+B = 9.50$ kN --- (1)

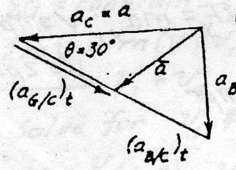
For rear-wheel assembly, $\Sigma M_{\text{propeller shaft}} = 0$

$(B-A)\frac{1.5}{2} = 0.319$, $B - A = 0.426$ kN -- (2) Solve (1)&(2)& get

$\underline{B = 4.96 \text{ kN}, A = 4.54 \text{ kN}}$

A-25 $a_B = a_C + (a_{B/C})_t$, $\bar{a} = a_C + (a_{G/C})_t$





$(a_{G/C})_t = \frac{l}{2}\alpha$ so

$\frac{a}{2} = \frac{l}{2}\alpha\cos\theta$

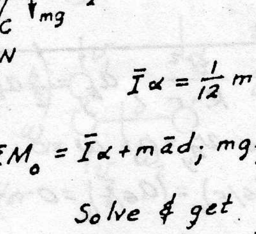
$a = l\alpha\sqrt{3}/2$

$\bar{I}\alpha = \frac{1}{12}ml^2\frac{2a}{l\sqrt{3}} = \frac{mal}{6\sqrt{3}}$; $m\frac{l}{2}\alpha = ma/\sqrt{3}$

$\Sigma M_O = \bar{I}\alpha + m\bar{a}d$; $mg\frac{l}{2}\frac{1}{2} = \frac{mal}{6\sqrt{3}} + ma\frac{l}{2}\frac{\sqrt{3}}{2} - \frac{ma}{\sqrt{3}}\left(\frac{l}{2} - \frac{l}{2}\frac{1}{2}\right)$

Solve & get $a = \frac{3\sqrt{3}}{8}g$

A-26 $V_{g_2} = -\rho g\frac{L}{2}\frac{L}{4} - 2\rho g\frac{x}{2}\left(\frac{L}{2}+\frac{x}{4}\right) - \rho g\left(\frac{L}{2}-x\right)\left(x + \frac{1}{2}\left[\frac{L}{2}-x\right]\right)$



ρ = mass per unit length

$V_{g_2} = -\frac{\rho g}{4}(L^2 + 2Lx - x^2)$

$V_{g_1} = -\rho g\frac{L^2}{4}$; $\Delta V_g = -\frac{\rho g}{4}(2Lx - x^2)$

$\Delta T = \frac{1}{2}\rho\frac{L-x}{2}\dot{x}^2$

$\Delta T + \Delta V_g = 0$;

$\frac{1}{2}\rho\frac{L-x}{2}\dot{x}^2 - \frac{\rho g}{4}(2Lx - x^2) = 0$

so $\underline{\dot{x} = \sqrt{\frac{gx(2L-x)}{L-x}}}$

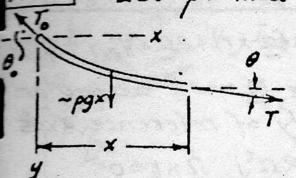
As $x \to L$, $\dot{x} \to \infty$

For $x = L$, $\Delta V_g = -\rho g\frac{L^2}{4}$ & $\rho g\frac{L^2}{4}$ represents the kinetic energy concentrated in the last element o falling rope. Impact occurs as the last element is b to rest & the energy $\rho g\frac{L^2}{4}$ is lost during impact.

A-27

Let ρ = mass of rope per unit length.

Assume $\bar{x} = L/2$



For complete rope ($x = L$, $T = 0$)

$\Sigma F_y = 0$; $T\sin\theta_0 = \rho g L$

$\Sigma F_x = ma_x$; $T\cos\theta_0 = \rho L \frac{L}{2}\omega^2$

For section of length x,

$\Sigma F_y = 0$; $T\sin\theta + \rho g x = \rho g L$, $T\sin\theta = \rho g(L-x)$

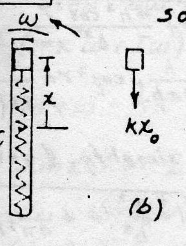
$\Sigma F_x = ma_x$; $\rho L \frac{L}{2}\omega^2 - T\cos\theta = \rho x \frac{x}{2}\omega^2$, $T\cos\theta = \frac{\rho\omega^2}{2}(L^2-x^2)$

$$\tan\theta = \frac{dy}{dx} = \frac{\rho g(L-x)}{\frac{\rho\omega^2}{2}(L^2-x^2)} = \frac{2g}{\omega^2}\frac{1}{L+x}$$

$$\int_0^y dy = \frac{2g}{\omega^2}\int_0^x \frac{dx}{L+x}, \quad \underline{y = \frac{2g}{\omega^2}\ln\left(1+\frac{x}{L}\right)}$$

A-28

(a) Accel. in dir. of force is $x_0\omega^2 - \ddot{x}$

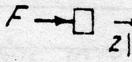


so $\Sigma F_x = ma_x$; $kx_0 = m(x_0\omega^2 - \ddot{x})$

$$\underline{\ddot{x} = -\left(\frac{k}{m}-\omega^2\right)x_0} \quad \text{where } \frac{k}{m} > \omega^2$$

(b) For general position $\ddot{x} + p^2 x = 0$

where $p^2 = \frac{k}{m} - \omega^2$



$x = A\sin pt + B\cos pt$

$x = x_0$ & $\dot{x} = 0$ when $t = 0$

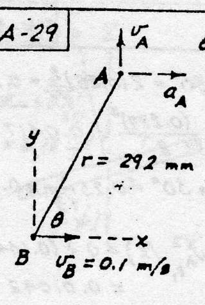
gives $B = x_0$ & $A = 0$

so $x = x_0\cos pt$, $\dot{x} = -x_0 p\sin pt$, $|\dot{x}|_{x=0} = x_0 p$

Thus accel is $2|\dot{x}|\omega = 2x_0\omega\sqrt{\frac{k}{m}-\omega^2}$

$F = ma$, $\underline{F = 2mx_0\omega\sqrt{\frac{k}{m}-\omega^2}}$

A-29



$\theta = \tan^{-1}\frac{250}{150}$, $\sin\theta = 0.857$, $\cos\theta = 0.514$

$u = v_A\sin\theta - v_B\cos\theta = \dot{r}$

$= 0.1\frac{5}{3}0.857 - 0.1(0.514) = \underline{0.0772 \text{ m/s}}$

$\dot{\theta} = \frac{1}{r}(v_A\cos\theta + v_B\sin\theta)$

$= \frac{1}{0.292}[0.15(0.514) + 0.1(0.857)]$

$= \underline{0.559 \text{ rad/s CCW}}$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $\underline{a}_B = \underline{0}$

$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 = \dot{u} - 0.292(0.559)^2 = \dot{u} - 0.0910$ m/s²

$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.292\ddot{\theta} + 2(0.0772)(0.559) = 0.292\ddot{\theta} + 0.0863$ m/s²

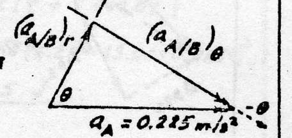
$a_A = 0.1(5/3)^2 = 0.225$ m/s² $\rightarrow$

$(a_{A/B})_r = 0.225(0.514) = \dot{u} - 0.0910$

$\underline{\dot{u} = 0.207 \text{ m/s}^2}$

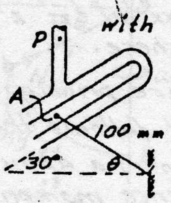
$(a_{A/B})_\theta = -0.225(0.857) = 0.292\ddot{\theta} + 0.0863$

$\underline{\alpha = -\ddot{\theta} = 0.958 \text{ rad/s}^2}$



A-30

Let M = point on slotted link coincident with A at instant shown.

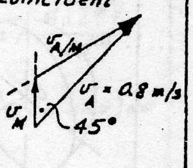


$v_M = v_P$ & $a_M = a_P$

$\underline{v}_A = \underline{v}_M + \underline{v}_{A/M}$

$v_A = R\dot{\theta} = 0.1(8) = 0.8$ m/s

Solution of triangle gives



$v_P = v_M = 0.8\frac{\sqrt{3}-1}{\sqrt{6}}$, $\underline{v_P = 0.239 \text{ m/s up}}$

$\underline{a}_A = \underline{a}_M + \underline{a}_{A/M}$

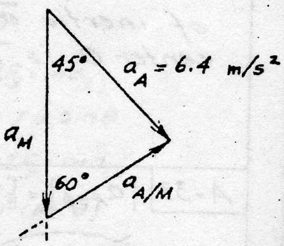
$a_A = R\dot{\theta}^2 = 0.1(8^2) = 6.4$ m/s²

$a_{A/M}$ is along slot

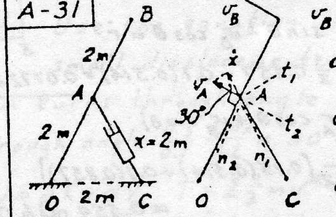
a_M is vertical

Solution of triangle gives

$a_M = a_P = \underline{7.14 \text{ m/s}^2 \text{ down}}$



A-31



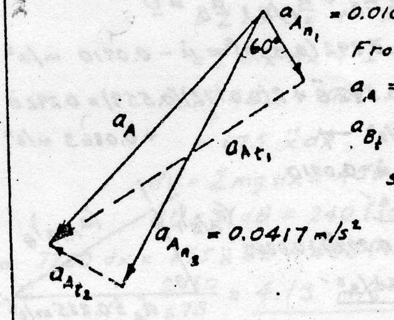
$v_B = 2\dot x/\cos 30° = 2(0.25)\frac{2}{\sqrt3} = 0.577\ \frac{m}{s}$

$a_{A_{n_2}} = \frac{v_A^2}{AO} = \frac{(0.289)^2}{2} = 0.0417\ m/s^2$

$v_{A_{t_1}} = \dot x \tan 30° = 0.25\frac{1}{\sqrt3} = 0.1443\frac{m}{s}$

$a_{A_{n_1}} = -(\ddot x - v_{A_{t_1}}^2/x) = 0 + (0.1443)^2/2$

$= 0.01042\ m/s^2$



$a_{A_{n_1}} = 0.01042\ m/s^2$

From vector solution

$a_A = 0.0434\ m/s^2$

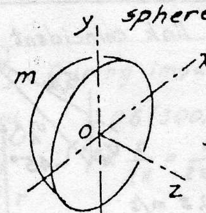
$a_{B_t} = 2a_{A_t}$; $a_{B_n} = 2a_{A_n}$

so $a_B = 2a_A = 0.0867\ m/s^2$

in the same direction as $\underline{a}_A$.

$a_{A_{n_2}} = 0.0417\ m/s^2$

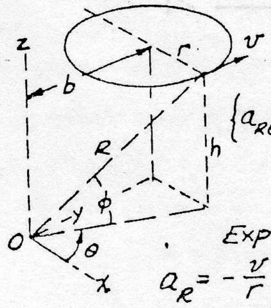
A-32 Let I = moment of inertia of complete sphere about a diametral axis.



By symmetry $I_{xx} = I_{yy} = I_{zz} = \frac{1}{2}I$ for the half-sphere.

Since x, y, z axes are principal axes through O, it follows that the ellipsoid of inertia about O is a sphere. Hence, moments of inertia about all axes through O are identical and equal $\frac{2}{5}mr^2$. Similarly for a hemispherical shell where moment of inertia about any axis through its center O is $\frac{2}{3}mr^2$.

A-33 $\{a_{R\theta\phi}\} = [T_\phi][T_\theta]\{a_{xyz}\}$; $a_x = -v^2/r$, $a_y = 0$, $a_z = 0$

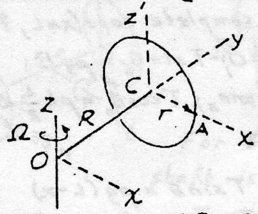


$$\{a_{R\theta\phi}\} = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{bmatrix} \begin{Bmatrix} -v^2/r \\ 0 \\ 0 \end{Bmatrix}$$

Expand & get

$$\left.\begin{aligned} a_R &= -\frac{v^2}{r}\cos\phi\cos\theta \\ a_\theta &= \frac{v^2}{r}\sin\theta \\ a_\phi &= \frac{v^2}{r}\sin\phi\cos\theta \end{aligned}\right\} \text{Ans.}$$

A-34 $\underline{a}_A = \underline{a}_C + \dot{\underline{\Omega}}\times\underline{r} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}) + 2\underline{\Omega}\times\underline{v}_{rel} + \underline{a}_{rel}$



where $\underline{\Omega} = \omega\underline{k}$ = angular velocity of reference axes

$\underline{a}_C = -R\Omega^2\underline{j}$, $\dot{\underline{\Omega}}\times\underline{r} = 0$

$\underline{\Omega}\times(\underline{\Omega}\times\underline{r}) = \Omega\underline{k}\times(\Omega\underline{k}\times r\underline{i})$

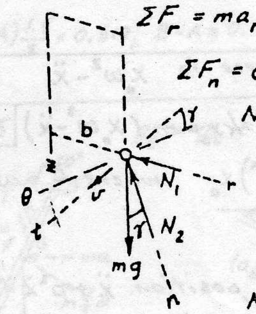
$= -r\Omega^2\underline{i}$

$2\underline{\Omega}\times\underline{v}_{rel} = 2\Omega\underline{k}\times R\Omega\underline{k} = 0$

$\underline{a}_{rel} = -\frac{(R\Omega)^2}{r}\underline{i}$

Thus $\underline{a}_A = -R\Omega^2\underline{j} - r\Omega^2\underline{i} - \frac{(R\Omega)^2}{r}\underline{i} = -\Omega^2\left(\frac{r^2+R^2}{r}\underline{i} + R\right.$

A-35 $\Delta V_g + \Delta T = 0$; $-mgh + \frac{1}{2}mv^2 = 0$, $v^2 = 2g$



$\Sigma F_r = ma_r$; $N_1 = m\frac{(v\cos\gamma)^2}{b} = \frac{2mgh}{b}\cos$

$\Sigma F_n = 0$; $N_2 - mg\cos\gamma = 0$

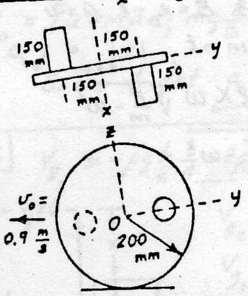
$N = mg\cos\gamma\sqrt{1 + \frac{4h^2\cos^2\gamma}{b^2}}$

But $\tan\gamma = \frac{h}{2\pi b}$, $\cos^2\gamma = \frac{1}{1+\left(\frac{h}{2\pi\right.}}$

substitute, simplify, & get

$N = \frac{mg}{1+\left(\frac{h}{2\pi b}\right)^2}\sqrt{1+\frac{h^2}{b^2}\left(4+\frac{1}{4\pi^2}\right)}$

A-36 $\omega_x = v_0/r = 0.9/0.2 = 4.5$ rad/s; $\omega_y = \omega_z =$



$H_x = I_{xx}\omega_x$, $H_y = -I_{xy}\omega_x$

$H_z = -I_{xz}\omega_x$

$I_{xx} = \frac{1}{2}mr^2 + 2m'd^2$

$= \frac{1}{2}24(0.2)^2 + 2(4)(0.$

$= 0.660\ kg\cdot m^2$

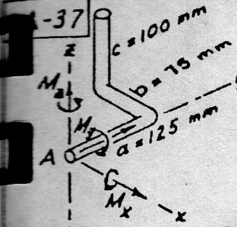
$I_{xy} = 0 + 2(4)(0.15)(0.075)$

$= 0.0900\ kg\cdot m^2$

$\underline{H}_o = 0.660(4.5)\underline{i} - 0.0900(4.5)\underline{j}$

$\underline{H}_o = 2.97\underline{i} - 0.405\underline{j}\ kg\cdot m^2/s$

A-37



$I_{xz} = \rho c \frac{c}{2}(-b) = -\frac{\rho b c^2}{2}$, ρ = unit mass

$I_{yz} = \rho c a\left(\frac{c}{2}\right) = \frac{\rho a c^2}{2}$

$I_{zz} = \frac{1}{3}\rho a^3 + \rho b\left(\frac{1}{12}b^2 + a^2 + \left[\frac{b}{2}\right]^2\right) + \rho c(b^2 + a^2)$

$= \rho\left[\frac{a^3}{3} + \frac{b^3}{3} + ba^2 + cb^2 + ca^2\right]$

Substitute $a = 0.125$ m, $b = 0.075$ m, $c = 0.100$ m, $\rho = 1.2$ kg/m & get

$I_{xz} = -0.450(10^{-3})$ kg·m²

$I_{yz} = 0.750(10^{-3})$ kg·m²

$I_{zz} = 4.91(10^{-3})$ kg·m²

$M_x = -I_{xz}\dot{\omega}_z = 0.450(10^{-3})(10^3) = 0.450$ N·m

$M_z = I_{zz}\dot{\omega}_z = 4.91(10^{-3})(10^3) = 4.91$ N·m

$M = \sqrt{M_x^2 + M_z^2} = \sqrt{0.450^2 + 4.91^2} = \underline{4.93 \text{ N·m}}$

A-38 Using the Lagrangian approach,

$T = \frac{1}{2}m(\dot{x}^2 + x^2\omega^2) + \frac{1}{2}I\omega^2$, $V = \frac{1}{2}kx^2$, $L = T - V$, $\omega = \dot{\theta}$

$\frac{\partial L}{\partial \theta} = 0$, $\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \omega} = mx^2\omega + I\omega$

$\frac{d}{dt}(mx^2\omega + I\omega) - 0 = 0$ ---(1)

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$

$\frac{\partial L}{\partial x} = mx\omega^2 - kx$, $\frac{\partial L}{\partial \dot{x}} = m\dot{x}$

$m\ddot{x} - mx\omega^2 + kx = 0$ -----(2)

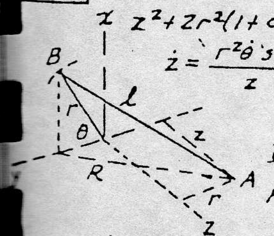
From (1) $(mx^2 + I)\omega$ = const C; $(mx_0^2 + I)\omega_0 = C$

$(mx^2 + I)\omega = (mx_0^2 + I)\omega_0$ [conservation of angular momentum]

Substitute into (2) & get

$$\ddot{x} + \left\{\frac{k}{m} - \left[\frac{mx_0^2 + I}{mx^2 + I}\right]^2\omega_0^2\right\}x = 0$$

A-39 $z^2 + r^2(1+\cos\theta)^2 = R^2 = \ell^2 - r^2\sin^2\theta$



$z^2 + 2r^2(1+\cos\theta) = \ell^2$, $2z\dot{z} + 2r^2(-\dot{\theta}\sin\theta) = 0$

$\dot{z} = \frac{r^2\dot{\theta}\sin\theta}{z} = \frac{r^2 p \sin\theta}{\sqrt{\ell^2 - 2r^2(1+\cos\theta)}}$

$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r}_{A/B}$

For $\theta = \pi/2$, $\underline{v}_A = \dot{z}\underline{k} = \frac{r^2 p}{\sqrt{\ell^2 - 2r^2}}\underline{k}$

$\underline{v}_B = -rp\underline{j}$; $\underline{r}_{A/B} = -r\underline{i} - r\underline{j} + \sqrt{\ell^2 - 2r^2}\,\underline{k}$

Thus $\frac{r^2 p}{\sqrt{\ell^2 - 2r^2}}\underline{k} = -rp\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -r & -r & \sqrt{\ell^2 - 2r^2} \end{vmatrix}$

Expand, equate like coefficients of $\underline{i}, \underline{j}, \underline{k}$ & get

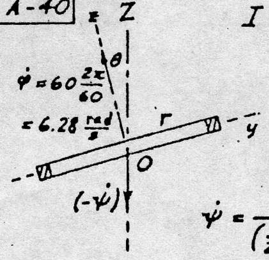
$\omega_z = -\omega_y \frac{\sqrt{\ell^2 - 2r^2}}{r}$, $\omega_x = \omega_y - \frac{rp}{\sqrt{\ell^2 - 2r^2}}$

Also $\underline{\omega}\cdot\underline{r}_{A/B} = 0$ so $-r\omega_x - r\omega_y + \sqrt{\ell^2 - 2r^2}\,\omega_z = 0$

Combine & get

$\omega_x = \frac{rp(r^2 - \ell^2)}{\ell^2\sqrt{\ell^2 - 2r^2}}$, $\omega_y = \frac{r^3 p}{\ell^2\sqrt{\ell^2 - 2r^2}}$, $\omega_z = -\frac{r^2 p}{\ell^2}$

A-40



$I = I_{zz} = mr^2$, $I_0 = I_{xx} = I_{yy} = \frac{1}{2}mr^2$

For precession with zero moment

$\dot{\psi} = \frac{I\dot{\varphi}}{(I_0 - I)\cos\theta} = \frac{\dot{\varphi}}{\left(\frac{I_0}{I} - 1\right)\cos\theta}$, Eq. 17A

$\dot{\psi} = \frac{60}{(\frac{1}{2} - 1)\cos 15°} = -124.2$ rev/min or $\dot{\psi} = -13.01$ rad/s

($+\dot{\psi}$ is in $+Z$-dir.)

$\omega_x = 0$, $\omega_y = \dot{\psi}\sin\theta$, $\omega_z = \dot{\varphi} + \dot{\psi}\cos\theta$

$\underline{H} = \frac{1}{2}mr^2(0)\underline{i} + \frac{1}{2}mr^2(\dot{\psi}\sin\theta)\underline{j} + mr^2(\dot{\varphi} + \dot{\psi}\cos\theta)\underline{k}$

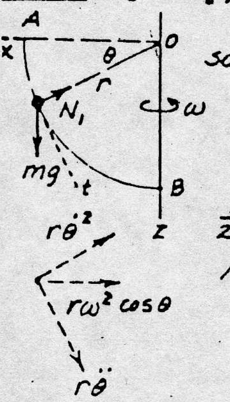
$\underline{\omega} = (0)\underline{i} + \dot{\psi}\sin\theta\,\underline{j} + (\dot{\varphi} + \dot{\psi}\cos\theta)\underline{k}$

$T_{rot} = \frac{1}{2}\underline{\omega}\cdot\underline{H} = \frac{1}{4}mr^2\left[(\dot{\psi}\sin\theta)^2 + 2(\dot{\varphi} + \dot{\psi}\cos\theta)^2\right]$

$= \frac{1}{4}8(0.300)^2\left[(13.01)^2(0.2588)^2 + 2(6.28 - 13.01\times0.9659)^2\right]$

$= 0.18[11.34 + 78.96] = \underline{16.25 \text{ J}}$

A-41 $\Sigma F_t = ma_t$; $mg\cos\theta = m(r\ddot{\theta} + r\omega^2\cos\theta\sin\theta)$



Also $\dot{\theta}d\dot{\theta} = \ddot{\theta}d\theta$

so $g\cos\theta - r\omega^2\cos\theta\sin\theta = r\frac{d(\frac{1}{2}\dot{\theta}^2)}{d\theta}$

$\int_0^{\dot{\theta}} d(\tfrac{1}{2}\dot{\theta}^2) = \int_0^{\theta}\left(\frac{g}{r}\cos\theta - \omega^2\cos\theta\sin\theta\right)d\theta$

$\frac{1}{2}\dot{\theta}^2 = \frac{g}{r}\sin\theta - \frac{\omega^2}{2}\sin^2\theta$

Now let $\dot{\theta} = 0$ for $\theta = \pi/2$ & get

$\omega^2 = \frac{2g}{r}\frac{1}{\sin\pi/2} = \frac{2g}{r}$

so $\underline{\omega = \sqrt{2g/r}}$

Coriolis term is a vector in the y-direction

$2\underline{u}\times\underline{\omega} = 2\omega r\dot{\theta}\sin\theta\,\underline{j}$

In y-dir, $\Sigma F_y = ma_y$;

$N_2 = m2\omega r\dot{\theta}\sin\theta$

In r-dir, $\Sigma F_r = ma_r$;

Thus $mg\sin\theta - N_1 = m(-r\dot{\theta}^2 - r\omega^2\cos^2\theta)$

$N_1 = m(g\sin\theta + 2g\sin\theta - r\omega^2\sin^2\theta + r\omega^2\cos^2\theta)$

$= m(3g\sin\theta + r\omega^2\cos 2\theta)$

For $\omega = \sqrt{2g/r}$, N_1 & N_2 become

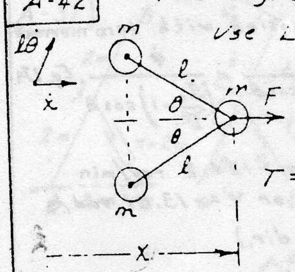
$\underline{N_1 = mg(3\sin\theta + 2\cos 2\theta)}$

$N_2 = 2m\sqrt{\frac{2g}{r}}\,r\sin\theta\sqrt{\frac{2g}{r}\sin\theta - \frac{2g}{r}\sin^2\theta}$

$\underline{N_2 = 4mg(\sin^{3/2}\theta)\sqrt{1 - \sin\theta}}$

A-42 Two degrees of freedom x, θ

Use Lagranges Eqs.



$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = F_j \quad , \; j = x, \theta$$

$$T = \frac{1}{2} m\dot{x}^2 + 2\frac{1}{2} m\left[(\dot{x} + l\dot{\theta}\sin\theta)^2 + (l\dot{\theta}\cos\theta)^2\right]$$

For x; $Q_x \delta x = F\delta x$, $Q_x = F$; $\frac{\partial T}{\partial x} = 0$

$$\frac{\partial T}{\partial \dot{x}} = m\dot{x} + 2m(\dot{x} + l\dot{\theta}\sin\theta)$$

so $m\ddot{x} + 2m\ddot{x} + 2ml(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) - 0 = F$

& for $\dot{\theta} = 0$, $3m\ddot{x} + 2ml\ddot{\theta}\sin\theta = F$ - - - - - (1)

For θ; $Q_\theta \delta\theta = 0$, $Q_\theta = 0$;

$$\frac{\partial T}{\partial \theta} = 2m\left[(\dot{x} + l\dot{\theta}\sin\theta)l\dot{\theta}\cos\theta - l^2\dot{\theta}^2\cos\theta\sin\theta\right]$$

$$= 0 \quad \text{for } \dot{\theta} = 0$$

$$\frac{\partial T}{\partial \dot{\theta}} = 2m\left[(\dot{x} + l\dot{\theta}\sin\theta)l\sin\theta + l^2\dot{\theta}\cos^2\theta\right]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = 2m\left[l\ddot{x}\sin\theta + l\dot{x}\dot{\theta}\cos\theta + l^2\ddot{\theta}\sin^2\theta + 2l^2\dot{\theta}^2\sin\theta\cos\theta + l^2\ddot{\theta}\cos^2\theta - 2l^2\dot{\theta}^2\cos\theta\sin\theta\right]$$

& for $\dot{\theta} = 0$, $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = 2m\left[l\ddot{x}\sin\theta + l^2\ddot{\theta}\right]$

Thus for $\dot{\theta} = 0$, $2m\left[l\ddot{x}\sin\theta + l^2\ddot{\theta}\right] - 0 = 0$ ... (2)

Combine (1) & (2) & get $m\ddot{x}(3 - 2\sin^2\theta) = F$

$$\text{or} \quad \ddot{x} = \frac{F/m}{3 - 2\sin^2\theta}$$